CE-430 & MS-441 **COHESIVE FRACTURE AND SCALING**

30 lecture hours, MWF Instructor: Prof. Zdeněk P. Bažant, Illinois Registered Structural Engineer

Introduction

Historical Perspective. Overview of main concepts.
Reasons for fracture analysis and size effect. Relevance to concrete, rock, ice, etc.

2. Essentials of Linear Elastic Fracture Mechanics (LEFM)

Energy release rate and fracture energy
Westergaard's solution of crack in infinite plane.
Asymptotic near tip fields, modes I, II, III, stress intensity factors, fracture toughness.
Rice's *J*-integral for energy flux. Determination of LEFM parameters.
Energy-based calculation of displacements from stress intensity factors.
Size effects in fracture mechanics vs. plasticity, scaling of energy release.
Failures at very small cracks. Asymptotic far-away field and crack interactions.

3. Nonlinear Fracture Mechanics and Cohesive Crack Model

Fracture process zone and crack bridging zone in concrete, rocks, ceramics, etc.
Fracture process zone size and Irwin's characteristic length.
Models of Dugdale and Barenblatt. Cohesive (fictitious) crack model of Hillerborg.
Compliance formulation of cohesive crack model.
Softening stress-displacement relations, and relation to fracture energy.
Work-of-fracture method for measuring fracture energy. Jenq-Shah 2-parameter model.
Crack-band model. Strain softening laws and their regularization. Material char. length.

4. Equivalent LEFM, *R*-Curves and Fracture Stability

Resistance curve (*R*-curve). Crack equilibrium and energy analysis of fracture stability.
R-curve determination as the envelope of fracture equilibrium curve for different sizes.
Fracture analysis based on *R*-curves. Snapback instability at ligament tearing.

5. Scaling and Size Effect

Geometrically similar structures, power scaling laws. Size effect as a principal characteristic of quasibrittle fracture. Exper. evidence for concrete, rock, ice, composites.
Energy derivation of size effect law for quasibrittle materials. Asymptotic analysis.
Transitional size. Experimental evidence for concrete, rock, ceramics, fiber composite.
Determination of fracture energy and process zone length from size effect measurements.
Size effect on apparent fracture toughness or fracture energy.
Calculation of *R*-curve from size effect data. Applications to various types of structures.
Brittleness and size effect in structural design.
Size effect in cohesive crack models for concrete, rocks, composites, ice, wood. Slopes.

6. Statistical Fracture Mechanics

Statistical Strength Theory and Size Effect.
QUIZ (1 hour, written). **TAKE-HOME EXAM** assigned.
Weibull's weakest link theory. Extreme value distribution. Stability postulate of extreme value statistics. Statistical size effect. Finite weakest link model for quasibrittle materials.
Distributions of strength, static lifetime, and Evans law.

7. Effects of Time, Rate, Creep, Temperature, Load Cycling and Distributed Damage

Rate and thermal effects in fracture. Effect of creep, and influence on size effect.
Generalization of cohesive crack model.
Fatigue crack growth in quasibrittle materials, Paris law and its size dependence.
Nonlocal continuum modeling of damage localization. Material models for damage.
stability of crack systems, and variational microplane concept.

PREREQUISITES: Solid background in mathematics and mechanics at the undergraduate level, including calculus, differential equations, mechanics, mechanics of materials, elements of continuum mechanics and theory of structures.

Recommended Texts: 1) Z.P. Bažant and J. Planas (1998). Fracture and Size Effect in Concrete and Other Quasibrittle Materials, CRC Press, Bocca Raton, FL. 2) Z.P. Bažant (2002). Scaling of Structural Strength, 2nd ed., Elsevier, London.

720-422 – Inelastic Analysis of Structures – Syllabus

Quarter length course, 30 class hours
MWF, 30 class hours

Instructor: Prof. Zdeněk P. Bažant
Illinois Registered Structural Engineer

1. **<u>Plastic Beams and Frames Under Uniaxial Stress</u>**
   Uniaxial stress-strain relations. Plastic collapse of steel or concrete beams and frames.
   Lower and upper bound theorems of limit analysis (proof). Uniqueness.Virtual work method.
   Methods of limit analysis (incl. computer methods) for beams, frames, trusses and grillages.
   Nonproportionally increasing loads.
   Optimum limit design. Incremental collapse and cyclic collapse. Shakedown.
   Melan's lower bound (static) and Koiter's upper bound (kinematic) theorems. Virtual work.
2. **<u>Plasticity, Yield Surfaces in Three-Dimensions, Combined Loading of Beams</u>**
   Combined bending moment and normal force, and normality.
   Review of cartesian tensors and stress invariants. Von Mises, Tresca, Drucker-Praguer and Mohr-Coulomb yield surfaces. Perfect plasticity. Yield surfaces, flow rule and normality.
   Loading surfaces for hardening plasticity. Drucker's postulate and second-order work.
   Normality and friction. Nonassociated flow.
   Plastic pure torsion. Upper and lower bounds for combined torque and bending moment.
3. **<u>Multi-Dimensional Plastic Structures</u>**
   Yield line theory of plates. Work at yield hinges.
   Upper and lower bounds for plates (rectangular, circular, polygonal).
   Yield lines in reinforced concrete plates. Johansen criterion.
   Upper bound for two-dimensional plasticity problems by via virtual work.
   Lower bounds for two-dimensional plasticity problems via equlibrium.
   Slip line theory (for Tresca yield criterion).
   Plastic punching, foundation failure and slope failure. Virtual work, effect of friction.
4. **<u>Time-Dependent Inelastic Behavior of Concrete Structures</u>**.
   Creep and shrinkage properties, principle of superposition in time.
   QUIZ (60 minutes, written)
   Age-adjuststed effectivemodulus method.
   Comments on restraints, structural system changes, nonhomogeneity, creep buckling.
   Moisture and thermal effects, shrinkage and drying creep. Design recommendations.

FINAL EXAMINATION (120 minutes, written)

<u>PREREQUISITES:</u> Solid background in mathematics and mechanics at the undergraduate level, including calculus, differential equations, mechanics, mechanics of materials, elements of continuum mechanics and theory of structures.

<u>Text:</u> 1. M. Jirásek and Z.P. Bažant, *Inelastic Analysis of Structures*, J. Wiley & Sons, 2002
2. Z.P. Bažant, *Inelastic Analysis of Structures* (Lecture Notes), Northwestern University, 1977, 1992, 2010.

Other Sources:

Z.P. Bažant, ed., *Mathematical Modeling of Creep and Shrinkage of Concrete*, J. Wiley, 1982.
J. Lubliner, *Plasticity Theory*, Macmillan, New York, 1990 (531.38 L, 929p).
W.F. Chen and H. Zhang, *Structural Plasticity*, Springer-Verlag 1991.
Z.P. Bažant and F.H. Wittmann, eds., *Creep and Shrinkage in Concrete Structures,* J. Wiley 1982.
W.F. Chen and D.J. Han, *Plasticity for Structural Engineers*, Springer, NY, 1988 (620.111233c518p).
P.G. Hodge, *Plastic Analysis of Structures,* McGraw-Hill, 1959.
M.R. Horne, *Plastic Theory of Structures*, M.I.T. Press, 1971.
C.R. Calladine, *Engineering Plasticity*, Pergamon Press, 1969.
W. Johnson and P.B. Mellor, *Engineering Plasticity*, Van-Nostarnd-Reinhold, 1973.
Z.P. Bažant and L. Cedolin, *Stability of Structures: Elastic, Inelastic, Fracture and Damage Theories*, Oxford University Press, 1991 (Ch. 8, 10, 13).
W.F. Chen, *Constitutive Equations for Engineering Materials,* Vol. 2 Elsevier, Amsterdam, 1994.
W.F. Chen, I. Sokal, *Plastic Design and Second-Order Analysis of Frames*, Springer Verlag 1995.
J.A. Kamenjarzh, *Limit Analysis of Solids and Structures*, CRC Press, Boca Raton 1996.
B. Druyanov, R. Nepershin, *Problems of Technological Plasticity*, Elsevier 1994.
R. Park, W.L. Gamble, *Reinforced Concrete Slabs*, J. Wiley 1980.
S. Kaliszky, *Plasticity Theory and Engineering. Applications,* Adademiai Kiado, Budapest, 1989.

Syllabus of Course 720-424

30 Lecture Hours

STABILITY OF STRUCTURES

Zdeněk P. Bažant
Illinois Registered Structural Engineer

DAY/DATE HOURS SUBJECT

I. Buckling of Columns

Euler load, adjacent equilibrium, and bifurcation (1.2).
Differential equation of beam-column (1.3).
Critical loads of perfect columns with various end conditions (1.4). Imperfect columns and the Southwell plot (1.5). Code specifications (1.6). Effect of shear (1.7p).

II. Buckling of Frames

Stiffness and flexibility matrices of beam columns (2.1). Critical loads of frames (2.2).
Large regular frames (2.4). Built-up columns (2.7p).
Postcritical behavior (2.6p). High arches (2.8p).

III. Dynamic Analysis of Stability

Vibrations of columns and divergence (3.1).
Nonconservative loads and flutter (3.2p).
Pulsating loads and parametric resonance (3.3p).
Definition of stability (3.5).
Theorems of Lagrange-Dirichlet and Liapunov. Energy analysis (3.7p).

IV. Energy Methods for Discrete and Continuous Structures

Potential energy of discrete elastic systems (4.2p).
Bifurcation buckling (4.3). Snapthrough and flat arches (4.4p).
Large-deflection postcritical behavior—by energy analysis.
Imperfection sensitivity (4.5 p, 4.6 p, 2.6 p, 1.9r). Variational methods (5.1).
Beam on elastic foundation and axisymmetric shell buckling—variationally (5.2).
Differential equation and potential energy of plates (7.1, 7.2p).
Rayleigh and Timoshenko quotients (5.3p, 5.4p, 5.5p).
Large deflections of columns via energy method (5.9p).

V. Thin walled Beams, Plates, and Shells

Potential energy and differential equation (6.1p).
Axial-torsional buckling and lateral buckling by energy analysis (6.2p, 6.3p).
Buckling of plates (7.3p).

1 hour **QUIZ (60 minutes, written)**

Buckling of cylindrical shells, imperfections and knock-down factors (7.5, 7.7p).

VI. Non-Elastic Stability Problems

Reduced and tangent modulus loads (8.1 p).
Steel and concrete codes—inelastic effects (8.4c, 8.5c).
Effect of creep (9.2c, 9.3c). Localization due to softening—energy analysis (13.2c).

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2 hours **FINAL EXAMINATION (written)**

p = partial coverage only, c = comments only, r = read only

PREREQUISITES: Mathematics and mechanics at the undergraduate level, including calculus, differential equations, mechanics (statics and dynamics), mechanics of materials and theory of structures.

TEXT: Zdeněk P. Bažant and Luigi Cedolin, *Stability of Structures: Elastic, Inelastic Fracture and Damage Theories,* 3rd ed., World Scientific, New York 2010 (or 2nd ed., Dover Publications 2003 or 1st ed., Oxford University Press 1991).

COURSE CONDUCT: Each homework is assigned automatically as soon as the subject has been covered in the class. The homeworks are due within one week of the assignment. The grade will be based on a written final (2 hours, 2/3 weight) and a written midterm quiz (1 hour, 1/3 weight) (no use of book nor notes). The grade, calculated from the weighted average of the final quiz and the midterm quiz, can be adjusted up or down according to the homeworks. A further upward adjustment of the grade (but no downward adjustment) can be made based on the knowledge demonstrated in class discussions. All the adjustments cannot exceed ½ of the grade. If some required homeworks are missing, the grade will have to be Y (incomplete). Retaking a missed quiz or exam requires a serious excuse such as illness.