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# Creep of Concrete

Under sustained load, concrete continues to deform. This phenomenon is called creep. The creep, along with the instantaneous (or initial elastic) strain, is defined as the strain difference between loaded and load-free specimens. Exposed to drying, the load-free specimens exhibit drying shrinkage (typically reaching 0.0002 to 0.0004, and in poor concrete even 0.0008). After many years, the creep strain typically attains values 2 to 6 times larger than the initial elastic strain. After unloading, a partial creep recovery takes place. The creep at drying greatly exceeds the sum of the shrinkage and the creep of a sealed specimen. The difference, called the drying creep (or Pickett effect), represents a hygro-mechanical coupling between strain and water content changes. Imbibition of water causes swelling (normally much less than the drying shrinkage) or, if concrete has dried, a partial reversal of shrinkage.

Drying shrinkage (Fig. 1 left and middle) is caused by compressive stresses in the microstructure balancing changes in the capillary tension and surface tension on pore walls as well as the disjoining pressure in hindered adsorbed water layers. The chemical processes of portland cement hydration cause autogeneous shrinkage, which is observed in sealed specimens and amounts to only about 5% of drying shrinkage (but much more in high strength concretes). Creep is caused by slips due to bond ruptures (with restorations at adjacent sites) in the hardened portland cement paste. The paste is strongly hydrophylic, having a disordered colloidal microstructure, porosity about 0.4 to 0.55, and an enormous internal surface area—about 500 m<sup>2</sup>/cm<sup>3</sup>; its main component is the tri-calcium silicate hydrate gel (3 CaO · 2 SiO<sub>3</sub> · 3 H<sub>2</sub>O, in short C<sub>3</sub>S<sub>2</sub>H<sub>3</sub>); it forms crystalline sheets and needles of colloidal dimensions, weakly bound by van der Waals forces.

The physical mechanism and modeling are still being debated. The model in Eqs. (2)–(11) which follow is not the only one available but fits the data best.

**Constitutive Law:** At service stresses in structures, which are less than about one half of the strength, and in absence of cracking, the creep strain depends on stress linearly. It may be fully characterized by the compliance function  $J(t, t')$  (Fig. 1 right), defined as the strain  $\epsilon$  at time (or age)  $t$  caused by a unit uniaxial stress  $\sigma = 1$  applied at age  $t'$ . As  $t'$  increases, the creep diminishes. This phenomenon, called aging, causes that  $J$  does not depend only on the time lag  $t - t'$ . At variable stress  $\sigma(t)$ , each stress increment  $d\sigma(t')$  applied at time  $t'$  produces strain history  $d\epsilon(t) = J(t, t')d\sigma(t')$ . By superposition principle (introduced by Boltzmann and Volterra), one gets the linear (uniaxial) creep law

of aging viscoelasticity:

$$\epsilon(t) = \int_{t_1}^t J(t, t')d\sigma(t') + \epsilon^0(t) \quad (1)$$

where  $\epsilon^0$  denotes the sum of shrinkage strain  $\epsilon_{sh}$  and thermal expansion (and smeared cracking strain, if any). The integral is the Stieltjes integral, which admits histories  $\sigma(t)$  with jumps; for time intervals with no jump, one may set  $d\sigma(t') = [d\sigma(t')/dt']dt'$ . When history  $\epsilon(t)$  is prescribed, then (1) represents a Volterra integral equation for  $\sigma(t)$ , which is not integrable analytically for realistic forms of  $J(t, t')$ , although numerical integration is easy. The solution  $\sigma(t)$  for strain  $\epsilon = 1$  imposed at age  $\hat{t}$  (and for  $\epsilon^0 = 0$ ) is called the relaxation function  $R(t, \hat{t})$ ;  $R(t, \hat{t}) \approx 0.992/J(t, \hat{t}) - 0.115\psi/J(t, t - 1\text{day})$ ,  $\psi = J(\bar{t}, \hat{t})/J(t, \bar{t}) - 1$ ,  $\bar{t} = (\hat{t} + t)/2$ . Generalizing Eq. (1) according to principle of superposition and taking isotropy into account yields a triaxial stress-strain relation, in which the shear and bulk compliance functions are  $J_G(t, t') = 2(1 + \nu)J(t, t')$  and  $J_K(t, t') = 3(1 - 2\nu)J(t, t')$  ( $\nu \approx 0.18 = \text{Poisson ratio}$ ). At high stress, the creep law appears to be nonlinear (Fig. 2) but (1) remains valid if the strain due to time-dependent growth of distributed microcracks is included in  $\epsilon^0(t)$ .

The value  $J(t', t') = q_1$  corresponding to extrapolation of creep curves to zero load duration may be considered as age independent. The conventional Young's elastic modulus, implied as  $E(t') = 1/J(t' + \delta, t')$  where usually  $\delta \in (0.0001 \text{ s}, 10 \text{ min.})$ , increases with age  $t'$ . A realistic form of  $J(t, t')$  (Bažant et al. 1997; bold curves in Fig. 1 right) may be conveniently expressed by its rate:

$$\dot{J}(t, t') = v^{-1}(t) \dot{C}_g(\theta) + 1/\eta_f \quad (2)$$

$$v^{-1}(t) = q_2 (\lambda_0/t)^m + q_3 \quad (3)$$

$$\dot{C}_g(\theta) = \frac{n\theta^{n-1}}{\lambda_0^n + \theta^n}, \quad \theta = t - t' \quad (4)$$

where  $\dot{\phantom{x}} = \partial/\partial t$ ;  $\theta$ =load duration;  $\lambda_0 = 1 \text{ day}$ ,  $m = 0.5$ ,  $n = 0.1$ ;  $q_2, q_3$  = dimensionless constants;  $C_g(\theta)$  = (age-independent) compliance function for delayed elasticity of the cement gel (hardened cement paste without its pores);  $v(t)$  = volume of gel per unit volume of concrete, growing due to hydration; and  $\eta_f$  = effective viscosity for the flow of concrete. By integration,  $C_g(\theta) = \ln[1 + (\theta/\lambda_0)^n]$ . As for  $J(t, t')$ , it can be obtained only by numerical integration (however, for computer structural analysis in short time steps, the rate  $\dot{J}(t, t')$  suffices and in fact allows a simpler algorithm). For creep of sealed specimens, called the basic creep,

$$1/\eta_f = q_4/t \quad (\text{at constant } w) \quad (5)$$

where  $q_4$  = dimensionless constant. Then the flow part of  $J(t, t')$  is simply  $q_4 \ln(t/t')$ . Eqs. (4), (3) and (5) are the simplest formulae satisfying the asymptotic conditions that  $\dot{J}$  for both short and long times  $\theta$ , and the aging rate (given by  $dv^{-1}(t)/dt$ ), must be power functions (due to self-similarity conditions, ensuing from the absence of any characteristic time).

**Variable Environment:** At variable  $w$  (mass of water per unit volume of concrete), a physically realistic constitutive relation may be based on the idea of micro-prestress  $S$ , considered to be a dimensionless measure of the stress peaks at the creep sites in the microstructure. The micro-prestress is produced by chemical volume changes and by changes in the disjoining pressures in the hindered adsorbed water layers (up to ten water molecules, or 2.7 nm, in thickness) confined between sheets of calcium silicate hydrates. The disjoining pressures must vary with the relative humidity  $h$  in the capillary pores as well as temperature  $T$ , so as to maintain thermodynamic equilibrium (equality of chemical potentials). The rate of bond breakages may be assumed to be a quadratic function of the level of micro-prestress, which leads to the expression

$$1/\eta_f = q_4 S \quad (6)$$

The micro-prestress is not appreciably affected by the applied load. It relaxes in time and its evolution at each point of a concrete structure may be solved from the differential equation

$$\dot{S} + c_0 S^2 = c_1 \left| \dot{T} \ln h + T \dot{h}/h \right| \quad (7)$$

where  $c_0, c_1 =$  positive constants (the absolute value ensures that  $S$  could never be negative and reflects the fact that not only drying and cooling but also wetting and heating accelerate creep, the latter activating different creep sites than the former). The fact that changes of  $w$  or  $h$  produce new micro-prestress peaks and thus activate new creep sites explains the drying creep effect. A part of this effect, however, is caused by the fact that microcracking in a companion load-free specimen makes its overall shrinkage less than the shrinkage in an uncracked (compressed) specimen, thus increasing the difference between the two (which defines creep).

The concept of micro-prestress is also needed to explain the stiffening due to aging. One physical cause of aging is that the hydration products gradually fill the pores of hardened cement paste, as reflected in function  $v(t)$  in (3). But hydration ceases after about one year, yet the effect of age at loading  $t'$  is strong even after many years. The explanation is that the peak microstresses relax with age, which reduces the number of creep sites and the rate of bond breakages.

At variable environment, time  $t$  in (3) must be replaced by equivalent hydration time  $t_e = \int \beta_h \beta_T dt$  where  $\beta_h =$  decreasing function of  $h$  (0 if  $h <$  about 0.85) and  $\beta_h \propto e^{-Q_h T/R}$ ,  $Q_h/R \approx 2700$  K. In (4),  $\theta = t - t'$  must be replaced by  $t_r - t'_r$  where  $t_r = \int \psi_h \psi_T dt =$  reduced time, capturing the effect of  $h$  and  $T$  on creep viscosity;  $\psi_h =$  function of  $h$  decreasing from 1 at  $h = 1$  to about 0.1 at  $h = 0$ ;  $\psi_T \propto e^{-Q_v T/R}$ ,  $Q_v/R \approx 5000$  K.

The evolution of distributions  $h(\mathbf{x}, t)$  ( $\mathbf{x} =$  coordinate vector) may be considered uncoupled from the stress and deformation problem and may be solved numerically from the diffusion equation  $\dot{h} = \text{div}[C(h)\text{grad } h] + \dot{h}_s(t_e)$  where  $h_s(t_e) =$  self-desiccation caused by hydration (which is mild in normal concretes but strong

in high strength concretes),  $C(h) =$  diffusivity, which decreases about 20 times as  $h$  drops from 100% to 60%. The field of free (unrestrained) shrinkage strain rates

$$\dot{\epsilon}_{sh} = k_{sh} \dot{h} \quad (8)$$

where  $k_{sh} =$  shrinkage coefficient. Since  $\dot{\epsilon}_{sh}$  at various points are incompatible, the calculation of the overall shrinkage of structures as well as test specimens is a stress analysis problem, in which creep and cracking must be taken into account.

For finite element structural analysis in time steps, it is advantageous to convert the constitutive law to a rate-type form. This may be achieved by approximating  $C_g(\theta)$  with a Kelvin chain model (or the associated relaxation function with a Maxwell chain model). The history integrals such as (1) then disappear from the constitutive law, the history being characterized by the current values of the internal state variables (the partial strains or stresses of the Maxwell or Kelvin chain).

Conversion to a rate-type form is also convenient for introducing the effect of variable temperature, which affects the Kelvin chain viscosities (according to Arrhenius law) and also the rate of hydration, as captured by  $t_e$ . For a three-dimensional tensorial generalization of (2)–(8), see Bažant et al. (1997).

**Approximate Cross-Section Response at Drying:** Although multidimensional finite element calculations of creep and moisture diffusion are nowadays feasible, simplified one-dimensional analysis of concrete beams or girders still reigns in practice. In that approach, one needs as input the average cross-sectional compliance function  $\bar{J}(t, t', t_0)$  (Fig. 1 right, light lines) and average shrinkage function  $\bar{\epsilon}_{sh}(t, t_0)$  of the cross section (Fig. 1 left and middle) ( $t_0 =$  age at start of drying). The algebraic expressions for such average characteristics are considerably more complicated and inaccurate than the constitutive law for a material point because of ignoring the differences due to cross section geometry, reinforcement and loading (compressive or tensile axial force, bending moment, shear, torque, etc.). The following approximations (Bažant and Baweja 2000), partly based (under strong simplifications) on the foregoing constitutive relations, have been derived and their coefficients optimized by fitting a large test data bank; for environmental humidities  $h_e$  below 98%:

$$\bar{\epsilon}_{sh}(t, t_0) = -\epsilon_{sh\infty} k_h S(t), \quad k_h = 1 - h_e^3 \quad (9)$$

$$S(t) = \tanh \sqrt{\frac{t - t_0}{\tau_{sh}}}, \quad \tau_{sh} = k_t (k_s D)^2 \quad (10)$$

where  $D = 2v/s =$  effective thickness,  $v/s =$  volume-to-surface ratio,  $k_t = 1$  for normal (type I) cement;  $k_s =$  shape factor (e.g., 1.0 for a slab, 1.15 for a cylinder); and  $\epsilon_{sh\infty} \approx \epsilon_{s\infty} E(607)/(E(t_0 + \tau_{sh}))$ ,  $\epsilon_{s\infty} =$  constant;  $E(t) \approx E(28)\sqrt{4 + 0.85t} =$  age dependence of Young's modulus (all times are in days). (2), (3) and (4) apply except that  $1/\eta_f$  must be replaced by

$$\frac{1}{\eta_f} = \frac{q_4}{t} + q_5 \frac{\partial}{\partial t} \sqrt{F(t) - F(t_0)} \quad (11)$$

where  $F(t) = \exp\{-8[1 - (1 - h_e)S(t)]\}$  and  $t'_0 = \max(t', t_0)$ . The form of the expression for shrinkage half-time  $\tau_{sh}$  is based on the diffusion theory. Function ‘tanh’ in (9) is the simplest function satisfying two asymptotic conditions ensuing from the diffusion theory: (1) for short times  $\bar{\epsilon}_{sh} \propto \sqrt{t - t_0}$ , and (2) the final shrinkage must be approached exponentially. Generalizations for the temperature effect have also been made.

Empirical formulae have been developed for predicting the parameter values in the foregoing equations on the basis of concrete strength and some parameters of the concrete mix. However, they are very crude, leading to prediction errors with coefficients of variation 23% for creep and 34% for drying. These high uncertainties can be drastically reduced by updating certain coefficients of the formulae according to short-time creep and shrinkage tests of the given concrete. However, the evolution of weight loss of the drying test specimens must be measured simultaneously (or else the updating problem is ill-conditioned). A fully rational prediction of concrete creep and shrinkage properties from its composition is a formidable problem, not yet resolved satisfactorily.

**Engineering Applications:** Nonuniform creep and restrained shrinkage may cause deleterious cracking in bridges, buildings and other structures, which may trigger or promote other processes endangering durability. Creep nonuniformity is caused by differences in the histories of pore humidity and temperature, age, and concrete type. In slender structures, creep may cause collapse due to long-time instability. Transfer of compressive stresses from creeping and shrinking concrete to steel bars or other non-creeping parts (e.g. stone cladding or masonry liner) may induce their buckling or compression failure. The creep effects are particularly important for prestressed concrete structures (because of their slenderness and high flexibility), and are paramount in safety analysis of nuclear reactor containments and vessels. The stress relaxation caused by creep is useful in mitigating the effects of shrinkage, cooling, differential creep and differential settlements, but undesirable in the case of a stress loss in prestressing tendons. At high temperature exposure, as in fire or postulated nuclear reactor accidents, creep is very large and plays a major role. In preliminary design of structures, simplified calculations use the creep coefficient  $\varphi(t, t') = E(t')J(t, t') - 1 = \epsilon_{creep}/\epsilon_{initial}$ . The change of structure state from time  $t_1$  of initial loading to time  $t$  can be estimated by elastic analysis in which Young’s modulus  $E$  is replaced by the so-called age-adjusted effective modulus  $E''(t, t_1) = [E(t_1) - R(t, t_1)]/\varphi(t, t_1)$ .

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Zdeněk P. Bažant

[Northwestern University, Evanston, Illinois, USA]

### Captions:

Fig. 1. Top: Typical time evolution of shrinkage, and of basic creep and drying creep compliances, for various slab thicknesses  $D$ , environmental humidities  $h_e$ , and ages at loading  $t'$ .

Fig. 2 Creep isochrones.