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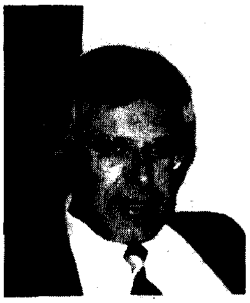
Nonstationary Long-Time Processes Causing Loss of Serviceability

Processus évolutif à long terme provoquant la diminution de l'aptitude au service

Nicht-Stationäre Langzeitprozesse und reduzierte Gebrauchstauglichkeit

Zdenek P. BAZANT
Prof. of Civil Eng.
Northwestern University
Evanston, IL, U.S.A.



Born and educated in Prague, Dr. Bažant joined the faculty of Northwestern University in 1969 where he has been Professor since 1973 and served as Structural Engineering Coordinator and as Director of the Center for Concrete and Geomaterials.

SUMMARY

The article presents a broad but nonexhaustive overview of the state-of-the-art and modern research directions in mathematical modeling of long-time processes engendering loss of serviceability of concrete structures, and to highlight some selected recent advances. Particular attention is given to creep and the effects of moisture and temperature, the fracture aspects of long-time damage, physically based models for freeze-thaw durability and for corrosion of steel, and probabilistic modeling – an essential ingredient of serviceability analysis.

RÉSUMÉ

L'article présente une revue, large mais non exhaustive, des directions de la recherche moderne dans la modélisation mathématique de processus à long terme provoquant la diminution de l'aptitude au service de structures en béton armé. Quelques progrès récents sont présentés. Une attention particulière est donnée au fluage et aux effets de l'humidité et de la température, aux aspects de rupture suite à des dommages anciens, à des modèles physiques pour l'étude de la durabilité sous l'effet de cycles gel-dégel et pour la corrosion de l'acier ainsi que des modèles probabilistes – un élément essentiel pour l'étude de l'aptitude au service.

ZUSAMMENFASSUNG

Der Vortrag gibt einen breiten, jedoch nicht umfassenden Überblick über den Kenntnisstand und die modernen Entwicklungen in der mathematischen Beschreibung langdauernder Prozesse, die den Gebrauchszustand von Massivbauten beeinträchtigen können; weiter beleuchtet er einige ausgewählte Neuentwicklungen. Besonders werden Kriechen, Wirkungen von Feuchte und Temperatur, die Brucherscheinungen bei Langzeitschädigung, physikalisch begründete Frost-Tau-Beständigkeit, Stahlkorrosion und Wahrscheinlichkeitsmodelle behandelt, die wesentliche Bestandteile einer Gebrauchsfähigkeitsberechnung sind.

1. INTRODUCTION

Concrete construction, an over-quarter-billion-dollar industry in the United States, is often plagued by long-time serviceability problems. All too often, slender prestressed concrete bridges, pavements, ocean structures, dams, containments, tanks, building structures, etc. suffer excessive deflections, cracking, corrosion or other serviceability impairments and have to be either closed or repaired well before the end of their initially projected design life. The cost to the society is tremendous, and in fact greatly exceeds in strictly economic terms the cost of catastrophic failures due to mispredicted safety margin. These economic costs are not only reflected in the actual damages but also in wrong economic decisions in the selection of design alternatives. For example, in competitive bidding one type of structure may appear to cost 20% less than another type of structure but should not be selected if its design life should turn out to be 30% shorter than that of the other alternative. Concrete structures of excellent serviceability, of course, can be and have been designed, but at the present we are still far from being able to achieve in structural design the optimum balance between the conflicting requirements of serviceability, economy of construction and maintenance, high performance and safety against catastrophic collapse.

The salient aspect of the problem is that serviceability depends on many physical-chemical processes, each of which is influenced by numerous factors. It is impossible to make close predictions and decide which processes and factors dominate in a given case unless a mathematical model which takes all these processes and factors into account is developed and computationally implemented. The processes which affect long-time serviceability or durability are in general nonstationary in time and involve:

1. Chemical aging, i.e., a change of material properties by long-time chemical processes such as cement hydration;

2. Creep, both viscoelastic and viscoplastic;

3. Effects of variable moisture content and water diffusion through the pores;

4. Effects of temperature variations;

5. Fracture processes, which involve development of cracking and other progressive damage as well as the bond deterioration;

6. Fatigue, which affects primarily interface bond;

7. Freeze-thaw durability;

8. Corrosion of steel and concrete; and

9. Chemical attack, such as alkali-cement reaction, penetration of chloride ions, sea water attack, carbonation, etc.

The intent of this lecture is to present a broad apercu of the current state-of-the-art and research directions in computational mechanics of concrete creep and other serviceability problems and then focus on some selected recent advances in mathematical models of several physical processes which determine long-term serviceability and have been investigated at the author's institution. Some basic probabilistic methods which need to be incorporated in such analyses will be also reviewed.

2. STATE-OF-THE-ART AND RESEARCH DIRECTIONS IN CREEP AND SHRINKAGE

The literature on concrete creep and shrinkage is vast and continues growing at an increasing pace; see Fig. 1 which shows the plot of the annual numbers of publications on material behavior models and on probabilistic models which

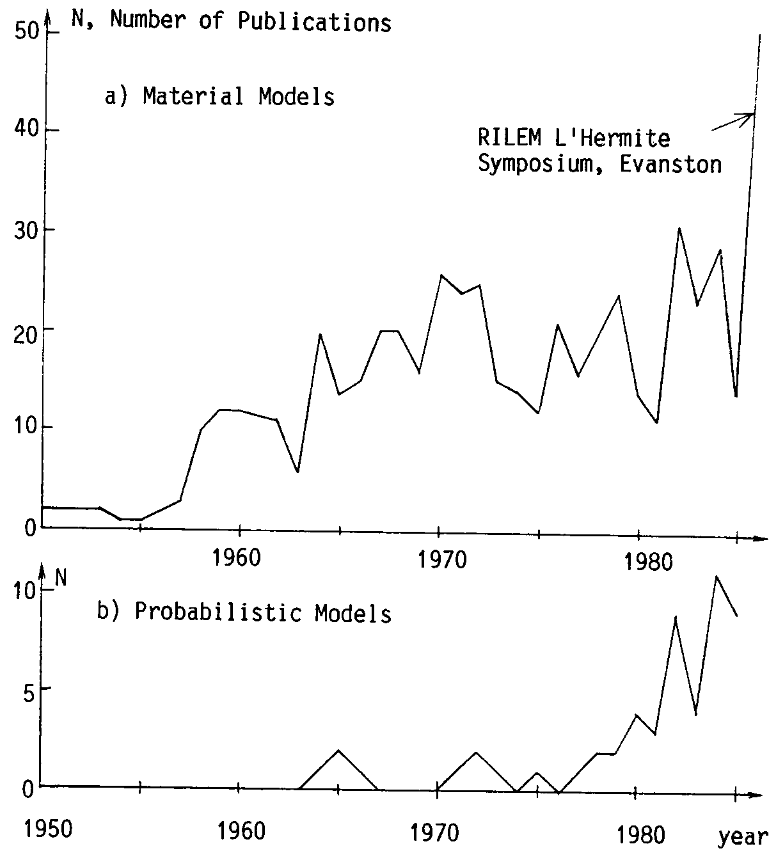


Fig. 1 - Plot of the annual number of publications on mathematical modeling of creep and shrinkage and their effect in structures, extracted from RILEM State-of-the-Art Report [1].

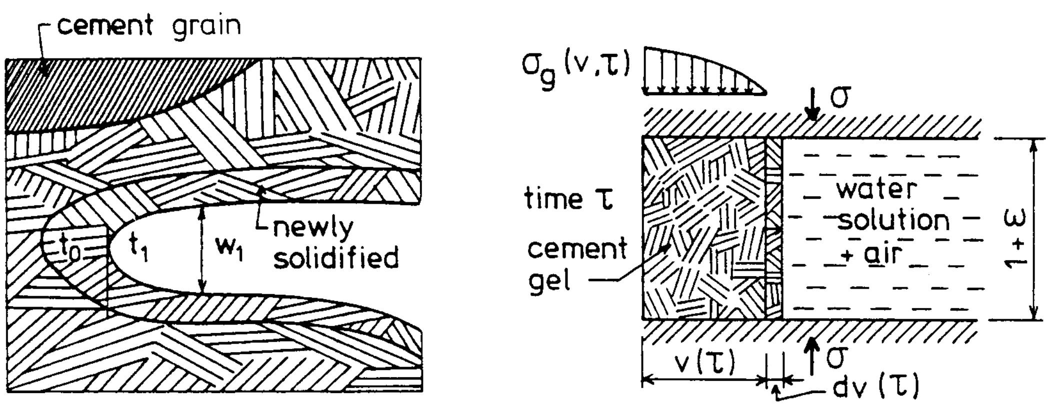


Fig. 2 - Model of Solidification of Portland Cement from Ref. 22.

were referenced in a recent state-of-art report (1986) in which a total of 1036 works were cited. As seen from today's perspective, the evolution of creep and shrinkage theory can be subdivided into three phases:

In Phase I, which ended roughly fifteen years ago, elementary phenomenologic data have been acquired and simplified linear models, realistic for only a limited range of time and environmental conditions, were developed. In Phase II, which might be regarded as nearly complete by now, the experimental base was greatly extended and systematically compiled, especially with regard to the effects of temperature and humidity, and a comprehensive theory of linear analysis with aging has been formulated. In Phase III, which we are now entering, a coherent and realistic general theory must be developed if we should be able to derive significant benefits from the availability of powerful computers. This theory no doubt (1) needs to be nonlinear, (2) needs to be based on physical processes on the microstructural level, and (3) must cover not only deterministic but also probabilistic aspects of long-time predictions.

The theory needs to be also extended to some challenging new problems, for example the problem of radioactive waste disposal. In this regard, questions are being raised as to what is the creep at variable elevated temperatures for periods spanning into thousands of years. The empirical formulas still in use in the current code recommendations are, of course, useless for such long times, and the only hope is a consistent rational theory based on the knowledge of physical mechanisms.

The fact that the prediction of creep and shrinkage in structures involves great uncertainty and random variability has been thought to negate the need for an accurate, sophisticated theory. The contrary conclusion, however, is justified. The random variability can, and must, be taken into account by a probabilistic design approach. The probabilistic design, however, makes sense only if the deterministic theory which corresponds to the mean behavior is sound, for otherwise the probabilistic analysis does not deal with truly random errors but with systematic errors disguised as random ones. Improvement of the constitutive relations for creep appears to be a prerequisite for significantly reducing the statistical error of the predictions of creep effects in structures, which is often enormous for the present models but can in fact be rather low in the laboratory.

With the linear theory of concrete creep we have probably arrived almost as far as we can go. The current state-of-the-art [1] may be briefly described as follows:

1. The structural analysis can be carried out for a general compliance function as measured, including the aging effect. The latest formulas for the compliance function, such as the log-double power law and the triple power law [1] can approximate the measured compliance data almost as closely as the scatter of data permits one to discern. Although the theoretical basis of the linear analysis is the principle of superposition for arbitrary variable stress and strain histories, it is most effective for computer solutions to convert the compliance function within the computer program into a rate-type model which may be interpreted by a spring-dashpot rheologic model with age-dependent viscosities and elastic moduli. The humidity and temperature effects may be most realistically incorporated in this rate-type form as influences on the viscosities of the spring-dashpot model [1].

2. In many applications, especially for simple structures and for simple humidity and temperature histories, sufficiently accurate linear solutions can be obtained by the age-adjusted effective modulus method [2,3,1,10,6]. Its error with regard to the exact solutions of linear integral equations is insignificant in comparison to the error of the linearity assumption itself.

On the other hand, the classical effective modulus method, which is not appreciably simpler than the age-adjusted one, is sufficient only for those cases where the stresses are nearly constant in time.

3. The linearity assumption fails not only above the service load range but also when moisture effects and cracking dominate and when the stress history includes large changes of stress after a long period of creep. When the linear creep law is converted to the rate-type form to make numerical computations more efficient, the conversion algorithm yields viscosities and elastic moduli which may violate thermodynamic restrictions, having negative values for some (short) time periods [1,3]. After long studies it appears that this theoretical difficulty cannot be overcome within the realm of a linear theory and is probably caused by the fact that we are trying to use a linear theory for what is inherently a nonlinear phenomenon. Another dubious consequence of the linearity assumption is that the creep recovery curves may be obtained as nonmonotonic, due to the fact that the measured creep curves for different ages at loading may diverge. Most likely, this phenomenon is a manifestation of a deviation from linearity. The most important deviation of creep from linearity is caused by simultaneous drying or wetting and, as recently transpired, also by simultaneous temperature changes.

4. In the current code recommendations [6,7] as well as many computational studies, the creep and shrinkage in the entire cross section of a beam or plate is characterized by a single mean compliance function and mean shrinkage function. This is a source or large error as well as complexity. The mean cross section compliance function for a drying environment in reality depends on the cross section shape as well as the type of loading (e.g., bending, compression, tension, torsion). With this practice, the coefficient of variation of the errors of the long-time prediction of the compliance function and shrinkage function apparently cannot be reduced below about 18% even if the most sophisticated formulation is used, and about 40% if the simple formulations of the type of code recommendatons are used [8,1]. The formulation of the true material creep and shrinkage properties applicable to a point rather than the entire cross section is much simpler and can also be made with a much smaller error. The cross section must then, of course, be subdivided into layers or finite elements, and the creep and shrinkage of each such layer or element must then be based on the calculated humidity and temperature for the same element rather than on the mean environmental humidity and temperature.

With regard to the characterization of the creep and shrinkage properties for structural analysis, the current thinking [1] might be characterized as follows:

1. The standard recommendation should specify the compliance function rather than the creep coefficient or specific creep defining only the creep part of strain. The reason is that, due to significant short-time creep, the definition of the elastic modulus is ambiguous, depending strongly on the duration of measurement. The elastic modulus values recommended in the codes are suitable only for elastic analysis and are rather different from the elastic modulus vaues which correctly correspond to the recommended values of the creep coefficients.

2. If the entire cross section of a beam or plate is characterized by one mean compliance function, this function should be formulated as the sum of the basic creep compliance and the additional compliance due to drying. The reason is that the shapes of the curves of these two compliance functions are rather different; the drying compliance has a final asymptotic value while the basic one apparently does not, and the shape and values of the drying compliance curves depend on the cross section size and shape while those of

the basic compliance do not.

3. For physical reasons, as well as for reasons of convergence of numerical solutions, the constitutive equation or compliance function must satisfy the principle of continuity. This principle dictates that the strain responses for any two stress histories that are infinitely close to each other must be *also infinitely close. E.g., if one strain history is caused by constant* stress σ_1 applied from time t_1 to time t_2, and another strain history by a constant stress σ_0 applied from t_0 to t_1 and constant stress σ_1 applied from t_1 to t_2, the in the limit $\sigma_0 \rightarrow 0$ the second strain history must approach the first stress history.

4. Although a separation of creep into a reversible (delayed) component and an irreversible (flow) component can be introduced if desired, it is neither required nor justified by thermodynamics [1,10]. The reason is that in aging materials the reversible creep component cannot be characterized as a unique function of the thermodynamic state variables. Rather, it depends stronly on the age at unloading, the age at first loading, the recovery duration, the moisture history, etc.

5. The existing test data do not reveal the existence of any final asymptotic value for basic creep [1]. The question of its existance would be moot if a slope decrease (in log-time) were assumed to begin after the end of the lifetime of the structure, but this is not the case for various formulas in use, or given in standard recommendations. Therefore, it is desirable to change these formulas, especially since no particular simplification of structural computations is achieved by specifying the compliance function that has a final asymptotic value.

6. Among the simplest formulas for the *basic creep compliance function*, a power function of the load duration, with an exponent about 1/8, appears to be the best. However, measurements indicate a transition from the power law for short load durations to a logarithmic law for long load durations. The time of this transition increases as the age at loading increases while the strain value at the transition seems to be roughly the same for all ages at loading. The classical Ross' hyperbola as well as Dischinger's exponential *are adequate only for creep durations of approximately one year, as originally* derived, and their use for long-time extrapolations is incorrect, experimentally unjustified. The dependence of the basic creep strain at constant stress duration on the age at loading approximately follows an inverse power function [1].

7. Unlike thermorheologically simple materials, the compliance function of concrete unfortunately cannot be expressed as a function of a single time variable, the reduced time. Numerous attempts in this regard were unconvincing since they considered only very limited test data. Among other simplifications of the constitutive relation for creep, the strain hardening and time hardening (concepts in which the creep rate decline is expressed as a function of the current creep strain or the time elapsed from the first loading) are alone insufficient to describe the behavior of concrete under variable stress [1].

8. The constitutive relation for creep and shrinkage should be based to the maximum possible extent on mathematical formulations for the various physical processes involved in creep and shrinkage. The physical considerations and mechanism theories which have so far yielded information on the compliance function and the constitutive relation include [1]:

a) The activation energy theory, which governs the creep rate, as well as the *rate of aging (hydration), and moisture diffusivity.*

b) Diffusion theory for moisture and temperature effects.

c) Theories of capilarity, surface adsorption (free and hindered) and diffusion between the small and large pores in cement paste.

d) Thermodynamic restrictions.

e) Aging interpreted as time variability of the composition of a mixture of nonaging components.

f) Fracture mechanics and cracking models.

g) Basic restrictions of continuum mechanics, including the principles of objectivity, tensorial invariance, form invariance, continuity, etc.; and

h) Stochastic process modeling.

Since proposals for physical mechanisms whose consequences for the stress-strain relation or compliance function are not (or cannot) be formulated mathematically are of little use as they can be neither proved nor disproved. It is essential to require that proposals for new mechanism models be described in mathematical terms.

9. The diffusion theory of moisture transport, whose relevance is confirmed by experimental evidence, indicates that: (1) the half-time of shrinkage as well as drying creep is proportional to the square of the cross section size, to the drying diffusivity, and to the shape factor that results from weight loss measurements [8,1], and (2) the shrinkage function as well as the drying creep term should initially evolve as the square root of drying duration [35,8]. For better accuracy, however, corrections need to be introduced due to the spoiling effects of aging, cracking or strain-softening, and creep due to residual stresses. Calculations of the distributions of water content in structures at various times require the use of a diffusion equation which is strongly nonlinear, principally due to a strong decrease in diffusivity as the specific water content decreases [1].

10. The effect of temperature on creep is two-fold: The effect on creep rate, modeled through the dependence of the viscosity coefficients in a rate-type model on temperature, and the effect on the aging rate (hydration) [1,10]. These two effects counteract each other and have to be described by separate activation energy expressions. Sometimes one effect dominates, sometimes the other. Activation energy also governs the dependence of moisture diffusivity on temperature, which in turn determines the effect of temperature on shrinkage as well as drying creep.

11. Creep and shrinkage measurements provide strong experimental evidence that the increase of creep due to simultaneous drying, called also the Pickett effect, is the result of two physical phenomena: (1) Cracking or strain-softening due to residual stresses, which causes that the shrinkage observed on a companion load free specimen is much less than the true material shrinkage which takes place in a compressed creep specimen [12,13], and (2) a cross effect between creep rate and pore humidity rate, which can be described either as stress induced shrinkage or as a dependence of creep viscosities on pore humidity rates [13,1]. Explanation of the latter cross effect can be provided in terms of an effect of the diffusion of water between small and large pores in hardened cement paste on the rate of bond ruptures which cause creep. This diffusion must be caused by differences in chemical potentials of water in the small and large pores, which further indicate that there should also be a cross effect between the creep rate and the rate of temperature. Such a cross effect provides an explanation for the transitional thermal creep, i.e., an acceleration of creep due to simultaneous change of temperature [1].

12. Since thermodynamics deals only with substances invariable in time, the aging (hydration) should be mathematically described as a consequence of a

varying composition of a mixture of reacting constituents which are themselves time invariant (nonaging).

After this succint review of the current state of creep modeling and computational approach, we will now focus attention on the mathematical modeling of several physical phenomena which are important for creep as well as serviceability in general.

3. SOME RECENT ADVANCES

3.1 Basic Creep Model Based on Solidification Process

As already mentioned, a physically based model for the role of aging in concrete creep should be based on some suitably idealized formulation for the solidification process of portland cement in which the volume growth of the hydrated volume fraction of cement is the basic variable [22,10,1]. The hydration process is imagined to gradually deposit layers of tricalcium silicate hydrates and other chemical products of hydration on the surfaces of the pores filled with water (Fig. 2). From the mechanics viewpoint, the essential aspect of this process is that at the time when each infinitesimal layer of hydrated material is attached to the existing solid microstructure, it must be essentially stress free, and stress is introduced into it only by the subsequent deformation. Under the assumption that the hydrated material as deposited has time invariant properties, the aforementioned property led [22] to the following form of the uniaxial stress-strain relation for basic creep:

$$\dot{\varepsilon}(t) = \frac{\dot{\sigma}(t)}{E} + \frac{1}{v(t)} \int_0^t \dot{J}_g(t - t')\, dt' \tag{1}$$

in which ε, σ = strain and stress, t = time = current age of concrete, superimposed dots denote derivatives with respect to time, E = time-independent elastic modulus, v(t) = volume fraction of the hydrated cement, and $\dot{J}_g(t - t')$ = rate of the compliance function for the hydrated material, which was originally considered to be free of any aging effect, and thus to be a function of a single variable, the time lag t - t'. This function has led to the triple power law which was found capable to describe very well the basic creep curves for concrete but still to have some shortcomings due to its neglect of nonlinearities and deviations from the principle superposition in time.

In recent studies (Bažant and Prasannan, in preparation), it transpires that this original creep model based on solidification (Eq. 1) should be extended in two respects: (1) the creep rate should be made to depend nonlinearly on stress in the same form as generally used in viscoplasticity, and (2) the compliance function of the material should depend on the time lag $\theta - \theta'$ of some reduced time θ rather than of the actual time t. The latter modification is indicated by taking into account the recently established fact that, in addition to volume growth, the hydrated material undergoes progressive polymerization. The polymerization stiffens the microstructure and thus causes increasing resistance to creep. These two physical aspects lead to the following modification of Eq. 1:

$$\dot{\varepsilon}(t) = \frac{\dot{\sigma}(t)}{E} + \frac{F[\sigma(t)]}{v(t)} \int_0^t \Phi[\theta(t) - \theta(t')]dt' \tag{2}$$

in which F and Φ = positive smooth continuous monotonic functions, Φ representing the rate of compliance with regard to the reduced time θ, which can be defined as $\theta(t) = t^s$, typically $s \approx 2/3$. For the growth of volume fraction one may approximately use the function:

$$v(t) = (t^{-m} + \alpha)^{-1},$$

with m, α = positive constants. Aproximately, $\Phi = (\theta - \theta')^{n-1}$, with $n \sim 0.1$.

According to the numerical studies of Prasannan, Eq. 2 not only describes very well the available test data on concrete creep curves at various ages at loading, but also describes approximately correctly the nonlinear dependence of creep on stress, the deviations from the superposition principle observed in creep recovery tests as well as in step-wise increasing step histories. Furthermore, the recovery curves are not obtained as nonmonotonic, and the creep curves at various ages at loading do not exhibit divergence.

The principal advantage of Eq. 2, however, is that the rate-type approximation, which can be obtained by expanding function Φ into Dirichlet series, corresponds to a spring-dashpot model in which only the viscosities are age-dependent while the elastic moduli are independent of reduced time θ (although they depend on the actual age t). This brings about a considerable simplification of creep structural analysis. Furthermore, based on the expansion of function Φ in Dirichlet series, one can give explicit expressions for the elastic moduli of the spring-dashpot model. These moduli are always positive, which removes the problem that in the existig models with age-dependent elastic moduli the values of these moduli can be obtained as negative for some short periods of time, as already mentioned. Moreover, due to the constancy of the elastic moduli for the spring-dashpot model, there is no numerical advantage in preferring the Maxwell chain model to the Kelvin chain model, and one may use the latter spring-dashpot model which has a more direct correspondence to creep data.

3.2 Stress-Induced Shrinkage and Thermal Expansion

Another formulation for creep which is based on a physical concept for the creep process and appears to greatly improve the description of the existing data for creep at simultaneous drying, is the idea of stress-induced shrinkage and thermal expansion. It is hypothesized that the viscosities in a spring-dashpot model, e.g., the Kelvin chain, depend on the rate of flow of water between small and large pores in the hydrated cement paste [13]. This idea has led for the μ-th unit of the Kelvin chain model to the stress-strain relation:

$$\frac{\dot{\sigma}}{E_\mu} + \frac{1}{\eta_\mu}(1 + k|\dot{h}|)\sigma = \dot{\varepsilon}_\mu + \kappa_\mu \dot{h} \tag{3}$$

in which ε_μ = partial strain of the Kelvin chain unit, η_μ = viscosity for this unit, κ_μ = coefficient for the portion of the shrinkage rate attributable to this unit, and r = positive constant. In this equation, the term $r|\dot{h}|$ may be considered as a term introducing a dependence of the overall viscosity on the pore humidity rate. It can be also shown that this dependence is a special case of a thermodynamic model based on the hypothesis of the movement of hindered adsorbed water proposed by Bažant in 1972 [11]. Algebraic rearrangement of Eq. 3 leads to an equivalent differential equation:

$$\frac{\dot{\sigma}}{E_\mu} + \frac{1}{\eta_\mu}\sigma = \dot{\varepsilon}_\mu + \kappa_\mu \dot{h} - \bar{\kappa}_\mu \dot{h}, \qquad \bar{\kappa}_\mu = \frac{k\sigma}{\eta_\mu} \operatorname{sign} \dot{h} \tag{4}$$

in which $\kappa_\mu \dot{h}$ may be regarded as the stress induced shrinkage, representing an

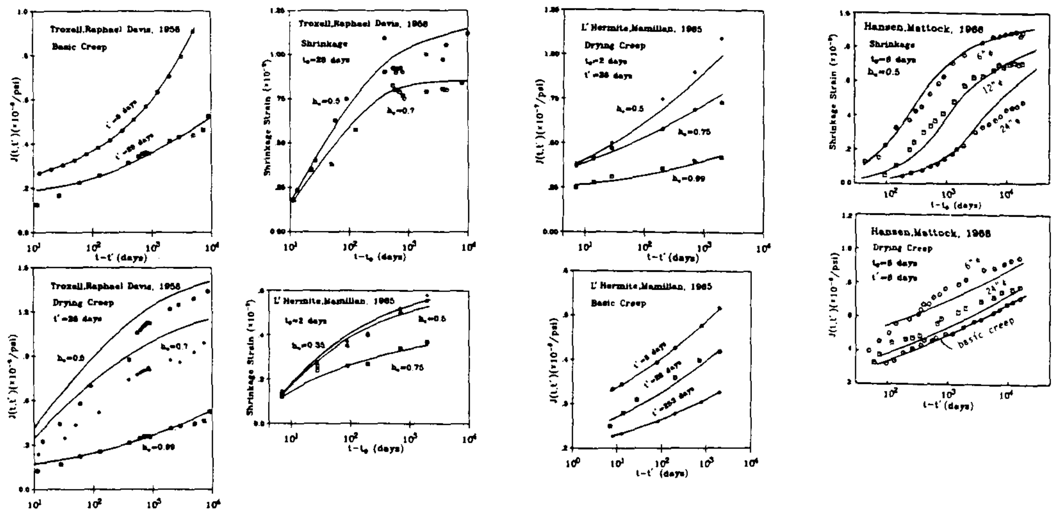
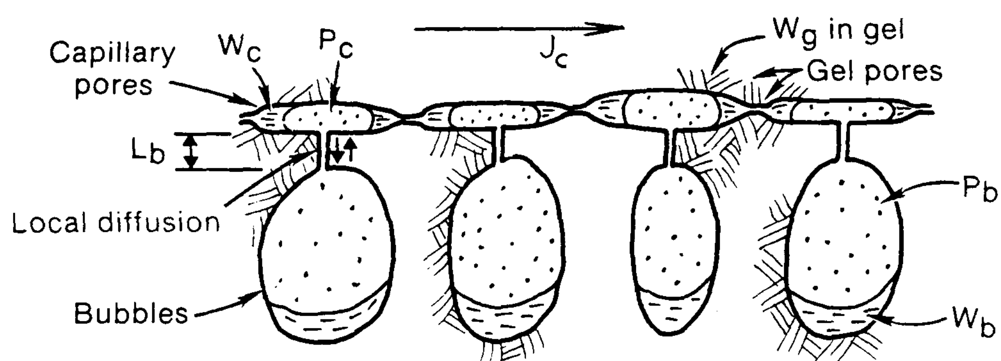


Fig. 3 - Stress-Induced Shrinkage Model with Cracking Compared with Previous Test Data (after Ref. 13).



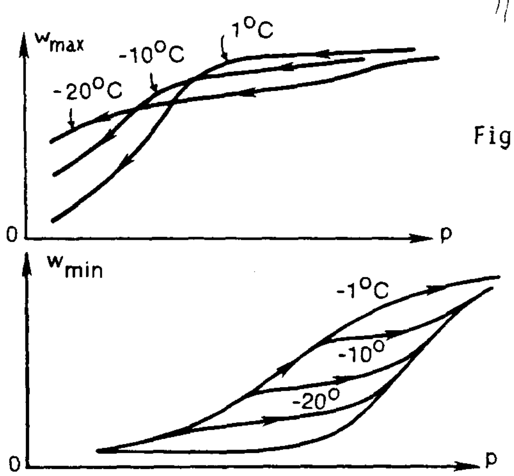


Fig. 4 - Example of calculated sorption-desorption isotherms for frozen concrete at various temperatures, used in a mathematical model for freeze-thaw damage, and a schematic picture of the macroscopic and local diffusion processes in the transfer of water in freezing concrete (after Ref. 37).

increased shrinkage when σ is compressive (negative) and $\dot{h}$ is negative (drying).

Based on thermodynamic arguments, a similar effect on viscosities is exerted by the rate of temperature T. As shown by Bažant, this leads to the phenomenon of stress-induced thermal expansion, a concept proposed by Thelandersson [14] by direct interpretation of test data and Bažant and Chern [13] as a special case of Bažant's 1972 aforementioned model [11].

In generalization to multiaxial stresses, the stress-induced shrinkage as well as the stress induced thermal expansion ceases to be isotropic, making possible for example stress-induced shrinkage shear or stress-induced thermal shear strain. It may be noted that variations of the pore humidity must also be reflected in a dependence of η_μ and of κ_μ on h. Furthermore, these quantities also depend on the equivalent age of concrete and on temperature. Finally, the solidification model in the form of Eq. 2 is more easily combined with the stress-induced shrinkage and thermal expansion than with previous models for the effect of humidity and temperature variations. For comparisons of the stress-induced shrinkage and thermal dilatation with numerous test results [13].

3.3 Fracture and Damage Phenomena Due to Nonuniform Shrinkage and Creep

As clearly demonstrated by Wittmann and Roelfstra [12] and also assumed in the modeling of Bažant and Wu [15] and others, the magnitude of creep at drying is strongly affected by cracking. The increase of creep in drying was explained by Wittmann and Roelfstra [12] as an apparent phenomenon, caused by the fact that the shrinkage observed on a shrinkage specimen is much less than the true shrinkage of the material, being strongly reduced by cracking, while the shrinkage which takes place in a compressed specimen is roughly equal to the true material shrinkage due to the fact that the compression stress prevents cracking. Subsequently, it has been concluded [13] that while this phenomenon is very important for explaining the drying creep effect (Pickett effect), it cannot explain it fully, and probably more than half of the drying creep effect must be attributed to a dependence of the creep viscosity on $\dot{h}$ as described above. In any case, though, cracking plays an essential role in any creep calculation at simultaneous drying as well as wetting, and by implication also at variable temperature.

Cracking is in creep calculations normally modeled as strain softening, i.e., as the decrease of stress at increasing strain (which is more correctly considered as a gradual decrease rather than an abrupt drop). The problem of strain softening has recently generated considerable discussions with regard to mesh sensitivity, convergence, and strain-localization instabilities. The problem is that the modeling must allow for the localization of cracking into distinct fractures, a problem which requires the use of fracture mechanics. The difficulties are due to the fact that neither the classical linear fracture mechanics, nor the stress-strain relation in the classical form, provides an adequate model for the distributed cracking phenomenon.

In recent studies of concrete fracture modeling [15] it has been found that the aforementioned problems can be circumvented if the concept of a nonlocal continuum is introduced into the model of cracking, or generally distributed damage. In a nonlocal continuum, in contrast to the classical, local continuum, one uses quantities which are obtained by averaging of some continuum variables over a characteristic volume, V_r, of the material centered around each given point of the structure. It has been also found that the classical nonlocal continuum, in which the strain is considered to be a nonlocal property, has certain theoretical defficiencies, and that the appropriate model for strain-softening should consist of a nonlocal continuum in which the strain is local. This implies that the elastic strain as well as the stress have local definitions and the only state variables which are permitted to be nonlocal are those which govern distributed cracking (damage). The relation to fracture mechanics consists in the fact that the energy dissipated by cracking per unit volume of the cracked material gives the fracture energy when multiplied by the characteristic volume V_r. This approach, which was formulated by Bažant, Pijaudier-Cabot and Lin [17,18] was demonstrated to converge properly in various strain softening problems as the mesh is refined. It was proven to yield a finite energy dissipation due to cracking while the previous models for strain softening could yield in the limit of zero mesh size a vanishing energy dissipation, which was physically inadmissible.

Thus, computer analysis of creep and shrinkage with cracking may be carried out in the usual manner with the smeared cracking approach. However, the fracturing strain ε^{fr} which appears in the smeared cracking models (or the crack band model) must be processed through an averaging integral as follows:

$$\bar{\varepsilon}^{fr} = \frac{1}{V_r} \int \alpha \, \varepsilon^{fr} \, dV \tag{5}$$

in which $\bar{\varepsilon}^{fr}$ denotes the nonlocal fracturing strain which must be used in the constitutive equation instead of ε^{fr}, V = volume, and α = a given function of space coordinates which represents a weighting function for the averaging. Based on the experience recently gained in studies of strain softening in general, it is likely that this approach should also provide a consistent model for strain-softening in the case of creep.

It may be noted that expansion of the integral in Eq. 5 into a Taylor series yields for the nonlocal fracturing strain an approximation which is roughly equivalent to considering the strength limit at the body surface to depend on the gradient of strain. This concept is in fact a classical idea which was proposed already by L'Hermite [16]. From his observations of shrinkage cracking in specimens of various sizes he concluded that realistic predictions can be obtained only if the strength limit is considered to be approximately inversely proportional to the gradient of shrinkage strain. Similar ideas also appeared in the modeling of ductility of reinforced concrete beams under seismic loading (e.g., Karsan and Jirsa, [20]. Recently, this approach has been developed much more rigorously for triaxial constitutive relations by Floegl and Mang [19] and Schreyer and Chen [21], although in no relation to concrete creep or shrinkage. It may be that these gradient formulations would provide another workable alternative to the nonlocal formulation indicated in Eq. 5.

3.4 Freeze-Thaw Durability

The serviceability of concrete structures exposed to freeze-thaw cycles is a theoretically rather complex problem where mathematical analysis can bring significant improvement in understanding. It might seem that the current state-of-the-art, which relies on the use of air entrainment to endow concrete with adequate freeze-thaw resistance, is satisfactory. However, the air entrainment reduces appreciably the strength of concrete, and in principle it would be preferable to achieve adequate freeze-thaw durability without having the large pores produced by air entrainment. Furthermore, as new types of cements and admixtures are being developed, it is important to understand how their different properties affect freeze-thaw durability.

As shown by Powers and others [38], the damage to concrete due to freezing is principally caused by the volume expansion of ice as it freezes. This volume expansion produces pressures in the pores, which can be relieved by water diffusion to other pores that are not saturated, such as air bubbles created by air entrainment, or unsaturated larger capillary pores. This consideration indicates that a concrete whose degree of saturation is below a certain critical value should be resistant to freeze-thaw damage, as proposed by Fagerlund [39]. An important point is that water can freeze only in relatively large pores, while it never freezes in the cement gel pores of molecular dimensions which contain essentially absorbed water. Thus, control of porosity of concrete offers the possibility of providing adequate freeze-thaw resistence without creating air-entrained bubbles.

To gain better understanding, a mathematical model for freeze-thaw damage has been developed jointly at Northwestern University and W. R. Grace Co. [37]. This model is based on determination of the isotherms of specific water content versus pore pressure for various temperatures in the freezing range. These isotherms are theoretically predicted from the known isotherms at room temperature, taking into account the volume expansion of ice on freezing, as well as the depression of the freezing point as a function of the size of the pore, and the melting of ice as a result of pore pressure which exceeds the crystal pressure of ice for a given temperature. The formulation of the desorption-sorption isotherms for the freezing range is based on the pore size distribution function, and also takes into account the hysteresis due to possible partial saturation of pores with ice.

The equation of state of the pore water is then used in a diffusion model to calculate pore pressures. For this purpose, a double diffusion model is developed [37], which distinguishes between the macroscopic diffusion of water through concrete, and local diffusion between smaller and larger pores (the local diffusion provides an important mechanism for the relief of pressures due to freezing). A schematic idealization of the diffusion processes is depicted in Fig. 4.

A computer program which calculates the sorption isotherms in the freezing range and solves the diffusion problem with a time variation of the pore pressure distributions throughout a concrete specimen has been written by J. C. Chern and has been used in some limited analytical studies which indicate that the essential aspects of the problem as currently understood can be modeled mathematically. The use of this mathematical model reduces the problem of freeze-thaw durability to the problem of durability of concrete under stresses, normally of cyclic nature, which are produced by the pore pressures. A complete analysis should include the calculation of cracking and fracture produced by the pore pressures. The analysis requires coupling the aforementioned program with a computer program for cracking and fracture of a concrete structure.

While this mathematical model is still preliminary, it offers considerable promise for the understanding of the numerous factors which affect freee-thaw damage. Aside from the material properties, the freeze-thaw damage is also affected by various structural mechanics factors; e.g., a superimposed compressive stress can block the freeze-thaw damage, the size and shape of specimens must have considerable effect because it controls the rate of cooling of concrete in the interior as well as the rate of escape of water from the regions of high pore pressure, creep must have an effect since it reduces the self-stresses produced by freezing, etc. All these mechanics factors have so far been neglected in the estimates of freeze-thaw resistance of structures.

3.5 Corrosion of Reinforcement in Concrete

Although the physical processes involved in the corrosion of steel in concrete and their governing equations seem to be relatively well understood [40,41,46] at the present, the interaction of these processes is very complex and calls for a comprehensive mathematical model which would make it possible to handle the numerous factors that affect the problem. Computer calculations based on such a mathematical model will probably be the best means for realistic predictions of corrosion resistance under a variety of circumstances and applications. An attempt for a comprehensive mathematical model of corrosion damage based on the known physical processes involved has already been made [43] and some elementary applications have been demonstrated, with the particular view of concrete structures exposed to sea water. This mathematical model describes interaction between the following physical processes:

1. Diffusions of oxygen, chloride ions and pore water through the concrete cover of reinforcement;

2. Chemical reaction producing ferrous hydroxide near the surface of the steel bar in concrete;

3. The depassivation of the steel bar caused by a critical concentration of chloride ions;

4. The cathodic and anodic electric potentials as functions of the concentrations of oxygen and of ferrous hydroxide, as described by Nernst's equation;

5. The polarization of electrodes due to changes in the concentration of oxygen and of ferrous hydroxide;

6. The flow of electric current through the electrolyte in pores of concrete;

7. The mass sinks or sources of oxygen, ferrous hydroxide and hydrated red rust at electrodes, based of Faraday law;

8. The rate of rust production, based on reaction kinetics.

Some of the processes involved are geometrically illustrated in Fig. 5. This figure also shows some typical idealized distributions of stresses which are caused by the volume expansion associated with conversion of iron into rust. The prediction of corrosion damage should ultimately be based on the calculation of the variations of stress distributions caused by rust formation, and the cracking or fracture which these stresses cause. To follow the corrosion process after the start of cracking, the model needs to further take into account the effect of cracks on the rate of transport of oxygen, chloride ions, water, and ferrous hydroxide.

Applicatility of the mathematical model for corroision has been demonstrated by various simplified examples [43], in which approximate estimates of the effective resistence of the corrosion cells have been made and the oxygen and chloride ion transports through concrete cover has been considered as quasi-stationary and one-dimensional, reducing the corrosion problem to an ordinary differential equation in time. For determination of the extent of cathodic and anodic areas and of the thickness of the rusting layer, a variational principle of maximum corrosion current (effectively equivalent to the principle of maximum entropy production) has been formulated. It was shown that the process can be controlled by various factors, e.g., by oxygen diffusion towards the cathode, by oxygen diffusion towards the anode, by limiting voltage, by chloride ion diffusion, by pH, by cover thickness, etc.

It was concluded that, without calculations, intuitive predictions of corrosion based on consideration of only one process involved in corrosion are in general inadequate. Further development of models of this kind, their experimental verification and calibration are an important problem for future research in computational methods for predicting long-time serviceability of concrete structures.

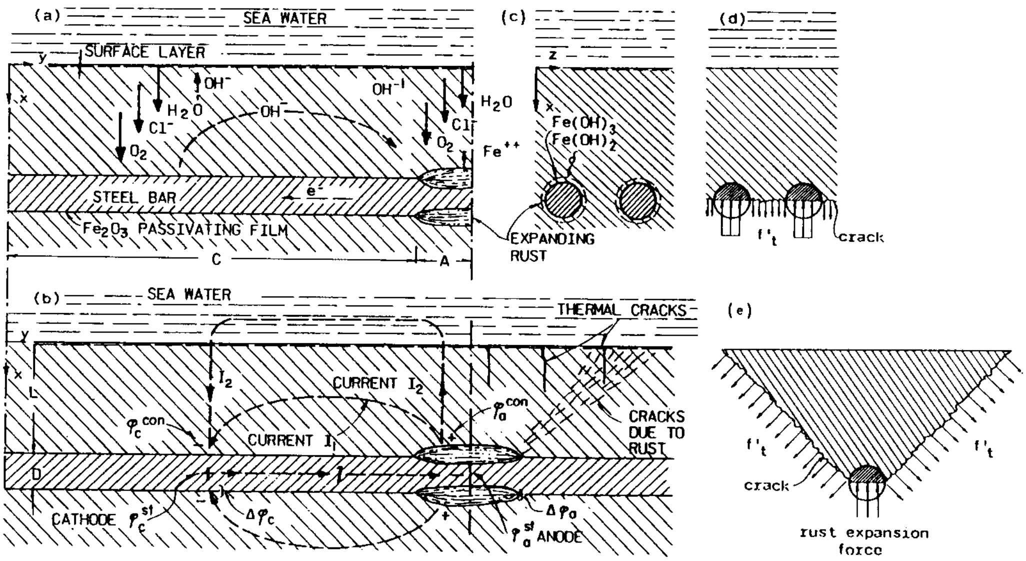


Fig. 5 - A simplified picture of the basic processes involved in steel corrosion and cracking of concrete cover (after Ref. 43).

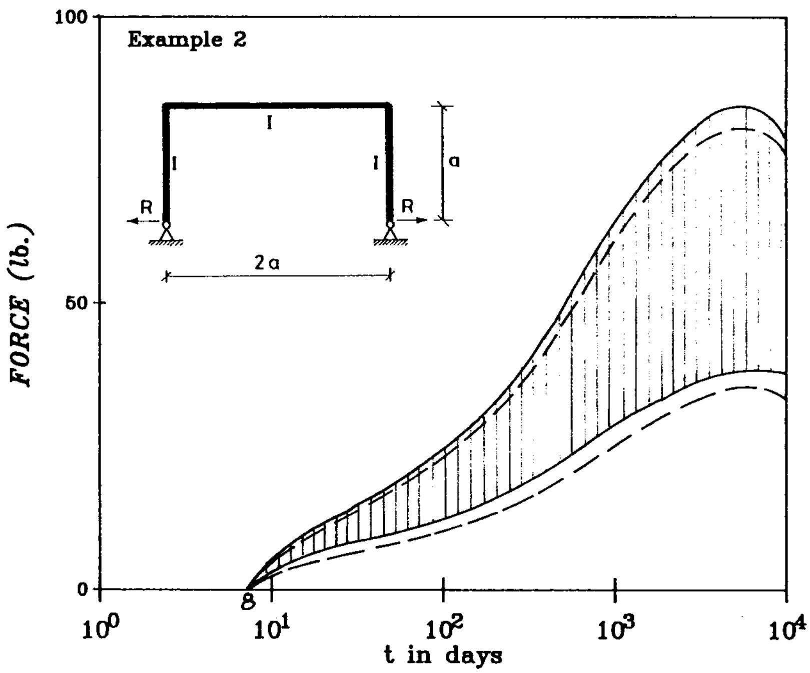


Fig. 6 - Example of a scatter band of mean ± standard deviation (68% probability limits) for shrinkage stresses in a concrete portal frame, obtained by latin hypercube sampling (after Ref. 33).

4. PROBABILISTIC ASPECTS OF SERVICEABILITY AND BAYESIAN UPDATING

Although prediction errors in the serviceability problems usually do not cause loss of human life (creep buckling excepted), they are important in economic terms, especially since the influencing factors and parameters are normally far more uncertain than the strength. This is certainly typical of creep and shrinkage prediction, for which it has been shown that the prediction errors (confidence limits) which are exceeded with a 10% probability are about ± 70% for the current code recommendations of ACI or CEB-FIP [1]. For these simple formulations, the major part of this error is due to the model error, i.e., the fact that the form of the creep and shrinkage law does not have a realistic form. However, even with the most sophisticated and comprehensive prediction model, the BP Model [8], these prediction errors are about ± 31% of the mean prediction. The major part of this error is due to the uncertainty of the effect of concrete composition and curing while the error of the creep and shrinkage law itself is small. In other serviceability problems such as freeze-thaw or corrosion cracking, the uncertainties are probably still larger. For example, regarding the permeabilities for chloride and ferrous oxides, or the shape of the sorption isotherms of frozen concretes and permeabilities for different pore fractions, we have at the present only order of magnitudes estimates. Nevertheless, even with this crude information, useful predictions which ensure adequate serviceability can be made if a probabilistic approach is used.

Even though the processes engendering serviceability loss are in principle always stochastic processes in time (as well as in space) [23,24], for many practical purposes the long-time structural response variables such as stress, strain or deflection can be considered to be functions of a certain set of random parameters $\theta_1,\ldots\theta_n$. These parameters can in general be divided into intrinsic and extrinsic ones. The intrinsic ones are the material characteristics fixed at casting, such as the cement fraction, water cement ratio, aggregate cement ratio, or for corrosion problems the permeabilities for oxygen and chloride ions, etc. The extrinsic ones are the environmental variables such as the relative humidity, temperature, or concentration of chloride ions. Even if the method of structural analysis is linear, the dependence of structural response, such as deflection, on these random parameters is in general highly nonlinear. This makes exact explicit calculation of the standard deviation of the response difficult, and one should preferably use approximate numerical probabilistic methods.

As recently shown by Bažant and Liu [32.33] a very effective method is the latin hypercube sampling, which is considerably more efficient, for a large number of material prameters, than the method of point estimates previously used by Madsen and Bažant [27]. In this method, the assumed or known cumulative distribution of each random parameter is subdivided into layers of equal thickness, and the problem is solved many times deterministically for parameter values randomly chosen at the centers of the intervals corresponding to these layers. The selections of parameter values are done in random in such a way that the number of layers for each parameter and the number of runs of the deterministic program for the structural response be equal and that the value of each parameter from each layer is used in one and only one run of the program. The collection of the results from all the computer runs represents, for each response variable such as deflection, a set of responses with equal probabilities. So the statistics such as the mean response and its standard deviation can be calculated in the usual manner and, assuming normal distribution, its parameters can be easily determined. This then makes it possible to also estimate the confidence limits for the structural response.

As an example, Fig. 6 shows the results from Bažant and Liu for the maximum shrinkage stress in a portal frame, obtained according to the BP model using

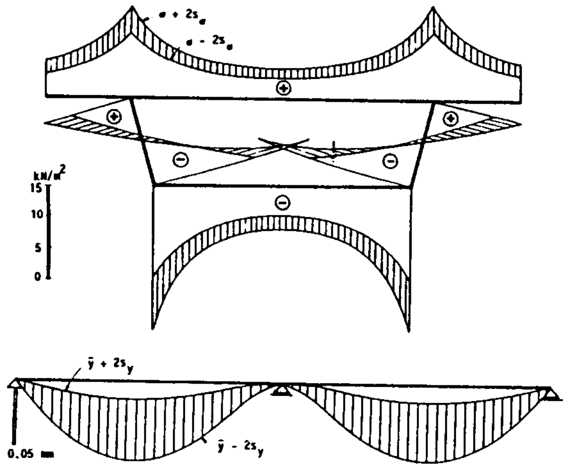


Fig. 7 - Example of the 95% confidence limits for longitudinal stresses at support cross section of a prestressed concrete box girder bridge (top) and deflections (bottom), obtained by latin hypercube sampling in Ref. 36.

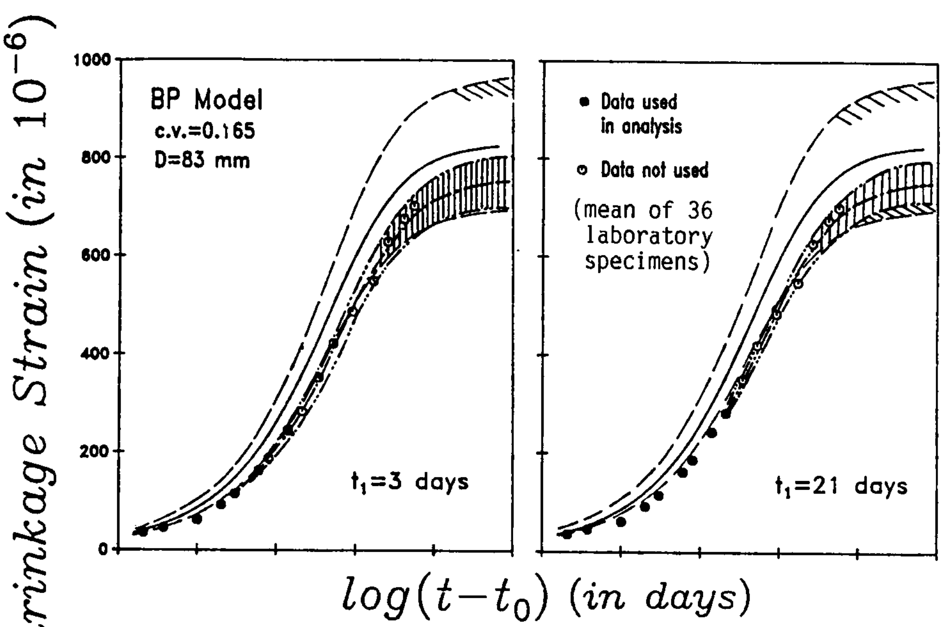


Fig. 8 - Example of long-time prediction of shrinkage strain, showing the initial band of 95% confidence limits and the posterior band after Bayesian updating, based on the solid data point only; after Ref. 35.

eight random parameters, each subdivided into thirty-two layers, while the number of computer program runs is also thirty-two. The solid lines in the figure represent the 90% confidence limits obtained by the latin hypecube sampling, while the dashed and dash-dot lines represent results of other approximate calculations. For the current code recommendatios of ACI or CEB-FIP, the scatter band would be appoximately twice as wide.

The method of latin hypercube sampling has been applied in calculations of creep effects of various structures such as prestressed concrte box girder bridges. Fig. 7, taken from the work of Křístek and Bažant [36], illustrates such an application for the calculation of the 95% confidence limits for the longitudinal stresses at the support cross section of a three span box girder bridge (top) and the corresponding deflections (bottom). These results, obtained from sixteen latin hypercube samples (i.e., sixteen layers, sixteen runs) and with eight random parameters, reveal that especially the deflections are rather sensitive to the uncertainties in the material properties and environmental influences. It should be noted that the width of the scatter band in Fig. 7 would approximately double if the current ACI or CEB-FIP models were used instead of the BP model.

The large uncertainties in long-time serviceability predictions apparent from the preceding examples can be drastically reduced by adopting a Bayesian approach [32,35]. In this approach, one needs additional information on the behavior of the material of the structure, such as some limited short-time measurements of creep and shrinkage of test cylinders or of the structure itself. As shown in Ref. 32 and practically illustrated in Ref. 35, the Bayesian approach can be combined very effectively with the method of latin hypercube sampling. For this purpose, one needs to calculate by latin hypercube sampling from the uncertainties of the model parameters the statistics of response for those short-time variables which are measured. The probabilities of the individual latin hypecube samples, which are normally equal, are then in this method adjusted according to Bayes probability theorem as follows:

$$P''(\underset{\sim}{X}_m^{(k)}) = P(\underset{\sim}{\theta}^{(k)}|\underset{\sim}{X}) = c_1\ L(\underset{\sim}{X}|\underset{\sim}{\theta}^{(k)})\ P'(\underset{\sim}{\theta}^{(k)}) \tag{6}$$

in which P' = the prior probability of random parameter sample $\underset{\sim}{\theta}^{(k)}$, P'' is the posterior probability after Bayesian updating, $\underset{\sim}{X}_m^{(k)}$ is the column matrix of response values, c_1 is a normalizing factor ensuring that the sum of all posterior probabilities equals 1, and $L(\underset{\sim}{X}|\ \underset{\sim}{\theta}^{(k)})$ gives the likelihood function which implements the Bayesian updating; this function represents the prior probabilities of the measured values X given that the random parameter values are $\underset{\sim}{\theta}^{(k)}$. If all distributions are assumed to be normal and if all the short time measured values $\underset{\sim}{X}_m$ used for updating are assumed to be mutually independent, the likelihood function is easily calculated.

The results of Bayesian updating are illustrated in Fig. 8 (taken from [35]) for the problem of long-time shrinkage strains using the BP model. The wide scatter band gives the 95% confidence limits based on prior information, and the inner cross-hatched band gives the posterior, updated confidence limits based on the knowledge of the measurements up to either 3-days or 21-days (data shown by the solid points). The remaining measurements, shown by empty points, are not used for updating but are shown for comparison with the results, indicating that indeed the Bayesian updating contracts the predicted scatter band in a correct manner [35].

It must be emphasized that successful use of Bayesian updating requires the availability of a good and physically correct model. If the current ACI or

CEB-FIP prediction models are used in this type of Bayesian analysis [35], one finds that initial measurements do not lead to any improvement of predictions, principally because the basic trend of these models grossly deviates from the basic trend of measurements.

The Bayesian updating based on latin hypecube sampling is a general method [32] which can be applied to any structural problem of creep and shrinkage, as well as any long-time serviceability problem. The Bayesian approach offers the means of drastically reducing the uncertainties of long-time serviceability calculations and should be used in practice whenever possible, provided one deals with a creep-sensitive (or serviceability-sensitive) problem.

Treating the structural response as a mere function of random parameters is not quite realistic as far as the influence of environmental humidity and temperature is concerned. The environmental parameters represent stochastic processes in time, and the type of their fluctuation in time, as characterized by their autocorrelation function in time or its power spectrum, is important. Due to the nature of the diffusion equation, the humidty or temperature fluctuation components of low frequency (long period) penetrate their influence deep into the structure, while those of high frequency (short period) affect only a thin surface layer of the structure [28,30]. The attenuation of the amplitude of the fluctuations roughly follows the inverse square root of the depth below the surface, with the proportionality coefficient being larger for a longer fluctuation period.

The precise calculation of this behavior can be based on the spectral method for random processes, an approach similar to that used in stochastic dynamics. However, for concrete there is an additional important complication in that the material undergoes aging, which requires a nonstationary stochastic process analysis. Under certain assumptions about ergodicity, the power spectral method for random processes has been generalized to linear boundary value problems with aging properties in time [43]. The basic result of this generalization is that the spectral densities of input, S_f (e.g., environmental humidity process), and of response, S_g (e.g., the stress or deflection process), are related by an equation of the same form as for nonstationary processes, however with additional parameters, and the same is true for the associated standard deviations s_f and s_g:

$$S_g(\underset{\sim}{x}, t, t_0, \omega) = |\Phi(\underset{\sim}{x}, t, t_0, \omega|^2 S_f(\omega) \tag{7}$$

$$s_g = |\Phi(\underset{\sim}{x}, t, t_0, \omega| s_f \tag{8}$$

in which Φ = complex valued frequency response function of the structure obtained by linear analysis, ω = circular frequency of the component of the input process in time, t = age of concrete, t_0 = age at the time of exposure to random environment, and $\underset{\sim}{x}$ = coordinate vector. The response statistics based on Eqs. 7 and 8 have been calculated by finite element method (with complex valued nodal displacements) for some selected simple problems, such as the effect of humidity fluctuation on the wall of a reactor containment shell [28].

A simple form of the effect of the frequency of fluctuations of environmental humidity on creep as a function of the depth of the point below the surface or the thickness of the cross section has been given by Diamantidis, Madsen and Rackwitz [30] a formula which neglects the aging aspect of concrete but might be adequate for many practical purposes.

The current state-of-the-art [1] may perhaps be succinctly characterized by the following observations:

1. The prediction of the overall creep and shrinkage in a cross section from concrete composition and design parameters such as strength is at present highly uncertain, characterized by 95% confidence limits which are of the order ± 85% for the current code recommendations and about ± 37% for the most sophisticated model presently available (BP model).

2. The only presently available means of substantially reducing these uncertainties is to measure the short-time values of either structural response or test specimens, and then extrapolate in time. The simplest way to extrapolate is statistical regression. However, a better way is the Bayesian approach, in which the measured information is combined with the prior knowledge of all concretes. With the Bayesian approach, the 95% confidence limits for long-time predictions can be reduced to about ± 12%.

3. With the use of contemporary standard recommendations, the prediction errors are largely model errors rather than the actual randomness of creep and shrinkage.

4. The standard design recommendations of engineering societies should specify not only the recommended creep and shrinkage prediction model but also its coefficient of variation or confidence limits. Adoption of a more sophisticated model would then be rewarded with a smaller error, and the engineer would be able to choose a model of proper simplicity and sophistication, depending on the error which he deems to be acceptable for design. The acceptable value of the error will depend on whether the designed structure is creep sensitive or creep insensitive.

5. The errors of the current creep and shrinkage models in code recommendations are so large that finite element analysis, and even statically indeterminate frame analysis, of a creep sensitive structure hardly makes any sense unless statistics are calculated. The error due to the use of these models is in fact much larger than the error caused by replacing finite element analysis with simple back-of-the-envelope solutions for frames such as the portal method.

6. The probabilistic method of structural creep and shrinkage analysis is now available and ready for use in practice [1]. Its simplest version treats creep and shrinkage as a function of several random material parameters and calculates the statistics by a sampling method.

5. CONCLUSION

As is clear from the preceding exposition (despite the fact that it puts emphasis on the works carried out at the author's home institution), the progress in computational methods for long-time serviceability problems, particularly creep and shrinkage, has been significant during the recent years. Computational mechanics opens a new avenue which should be fully exploited in serviceability problems. All these problems are characterized by the presence of numerous influencing factors and simultaneous action of many physical processes which determine the long-time response. Although in a given problem only some of the influencing prameters and some of the processes dominate, computations are required to determine these parameters and processes and predict the long-time response.

Inevitally, all long-time serviceability problems have a strong random character, and structural calculations should be done in a probabilistic manner. Although the probabilistic treatment is rather complex where a detailed physical description is attempted, some simple methods are now available. The approach which is presently available for serviceability computations simplifies the response to be a function of a set of random model parameters and the statistics are obtained by a sampling method, the latin hypercube sampling, which can be also combined with Bayesian updating and can be applied to finite element programs [44].

Structural calculations of damage which leads to serviceability loss must take into account various important physical phenomena such as the effects of simultaneous drying on creep, as manifested in the stress-induced shrinkage, and must involve calculations of cracking. Generally, diffusion calculations have to be coupled in these problems with stress and cracking analysis.

With the aforementioned ingredients, computational mechanics can greatly improve the serviceability design of concrete structures.

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