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MICROPLANE MODEL FOR TRIAXIAL DEFORMATION OF SOILS

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ABSTRACT

A general microplane model for triaxial behavior of soils is presented. The constitutive properties are characterized in terms of the stress and strain components on planes of various orientations in the material (microplanes), and the macroscopic response is obtained by integration of the contribution of the microplanes over all spatial directions. The microplane strains are assumed to be the resolved components of the macroscopic strain tensor, and the response of the microplane is assumed to depend on the volumetric strain.

INTRODUCTION

The development of computer technology during the past decades has made possible considerable advances in the numerical analysis and simulation of problems in geomechanics as well as in other related engineering fields. To fully benefit these new capabilities and further develop the computational techniques, it is necessary to acquire more accurate constitutive models.

In recent years, a number of such models for geotechnical materials based on microstructural considerations have been proposed [1–7]. Also, several models based on the microplane approach have recently been formulated for concrete and geomaterials (in many aspects concrete can be considered a geomaterial). These models were obtained for instantaneous as well as time–dependent response [8–10].

In this paper we present the basis for a general triaxial microplane model for soils. The general idea, originally suggested by G. I. Taylor [11] and first developed in the slip theory of plasticity [12], is to describe the material behavior on a plane of arbitrary

orientation in the material microstructure and then superimpose the contributions of the planes of all orientations, presently called the *microplanes*, to obtain the macroscopic response. The microplanes can be constrained either kinematically (strains on a microplane are the resolved components of the macroscopic strain tensor) or statically (stresses on a microplane are the resolved components of the macroscopic stress tensor). The election between these two possibilities depends on the characteristics of the particular material to be modeled.

It should be pointed out that the stress tensor and components which appear in the following derivation are to be interpreted as the *effective stresses*, in the sense of soil mechanics. At this stage in the development of the model, only drained tests are considered. A fully general model should, of course, include the pore pressure term, and work on this aspect is currently under way.

CONSTITUTIVE MODEL

We adopt the following basic hypotheses:

1. The strains on any microplane are the resolved components of the macroscopic strain tensor ε_{ij}, which represents a tensorial kinematic constraint.
2. The microplanes resist not only normal stress σ_N, but also shear stress, σ_{T_i}.
3. The response on each microplane depends explicitly on the volumetric strain ε_V.
4. The volumetric, deviatoric, and shear response on each microplane are mutually independent.
5. The vector of shear stress increments has the same direction as the vector of shear strain increments, i.e. $\Delta\sigma_{T_i} \sim \Delta\varepsilon_{T_i}$.

Hypothesis 1 has to be introduced because for overconsolidated clays, the response presents a peak and a softening branch asymptotically approaching a residual value. Thus, the strain for a given stress is not unique and a statically constrained model would produce constitutive model's instability in the post–peak range. According to this hypothesis, the components of the strain vector on a microplane whose direction cosines are n_i, are $\varepsilon_j^n = \varepsilon_{jk} n_k$. The normal strain component and its vector then are

$$\varepsilon_N = n_j \varepsilon_j^n = n_j n_k \varepsilon_{jk}, \qquad \varepsilon_{N_i} = n_i n_j n_k \varepsilon_{jk} \tag{1}$$

The vector of the shear strain component is $\vec{\varepsilon}_T = \vec{\varepsilon}^n - \vec{\varepsilon}_N$. Hence, the shear strain components and the shear strain magnitudes are:

$$\begin{aligned} \varepsilon_{T_i} &= (\delta_{ij} - n_i n_j) n_k \varepsilon_{jk} = \frac{1}{2}(n_j \delta_{ik} + n_k \delta_{ij} - 2 n_i n_j n_k)\varepsilon_{jk} \\ \varepsilon_T &= \sqrt{\varepsilon_{T_i}\varepsilon_{T_i}} = \sqrt{n_k \varepsilon_{jm} n_m (\varepsilon_{jk} - n_i n_j \varepsilon_{ik})} \end{aligned} \tag{2}$$

in which δ_{ij} is the Kronecker's unit delta tensor.

Macroscopic stress–strain relation

We assume the existence of a functional relation between the strain and stress components on each microplane, i.e.

$$\sigma_V = \mathcal{F}_V(\varepsilon_V), \qquad \sigma_D = \mathcal{F}_D(\varepsilon_D), \qquad \sigma_T = \mathcal{F}_T(\varepsilon_T) \tag{3}$$

where $\varepsilon_T = \sqrt{\varepsilon_{T_i}\varepsilon_{T_i}}$. Differentiation of eqs. (3) leads to

$$d\sigma_V = \mathcal{F}_V'(\varepsilon_V)d\varepsilon_V, \qquad d\sigma_D = \mathcal{F}_D'(\varepsilon_D)d\varepsilon_D, \qquad d\sigma_T = \mathcal{F}_T'(\varepsilon_T)d\varepsilon_T \qquad (\cdot$$

Following the same procedure as in the derivation of other models based on tl microplane concept, we use the principle of virtual work as a weak constraint to obta the relation between microscale and macroscale:

$$\frac{4\pi}{3}d\sigma_{ij}\delta\varepsilon_{ij} = 2\int_S \left(d\sigma_N\delta\varepsilon_N + d\sigma_{T_r}\delta\varepsilon_{T_r}\right)f(\vec{n})dS \qquad (\cdot$$

The macroscopic work on the left-hand side is taken over the volume of a unit spher and the integral extends over the surface of a unit hemisphere, S. Function $f(\vec{n})$ is weighting function of the normal direction $\vec{n}$, which in general can introduce anisotroy of the material. If the microplanes are associated with the clay platelets, this function the distribution function of the clay particles, which can be obtained experimentally b X-ray difractometry (Bažant and Prat [9]). Substituting the variations of strain within th integral, according to eqs. (1) and (2), and setting $\sigma_N = \sigma_V + \sigma_D$, we obtain

$$\frac{2\pi}{3}d\sigma_{ij}\delta\varepsilon_{ij} = \int_S \left[n_in_j(d\sigma_V + d\sigma_D) + \frac{1}{2}(n_i\delta_{rj} + n_j\delta_{ri} - 2n_in_jn_r)d\sigma_{T_r}\right]f(\vec{n})dS\delta\varepsilon_{ij} \qquad (6$$

This equation must hold for any variation $\delta\varepsilon_{ij}$; therefore,

$$d\sigma_{ij} = \frac{3}{2\pi}\int_S \left[n_in_j(d\sigma_V + d\sigma_D) + \frac{1}{2}(n_i\delta_{rj} + n_j\delta_{ri} - 2n_in_jn_r)d\sigma_{T_r}\right]f(\vec{n})dS \qquad (7$$

Now we can substitute eqs. (4) into eq. (7) and, since from hypothesis 5 $d\sigma_{T_r}$ $\mathcal{F}_T'(\varepsilon_T)d\varepsilon_{T_r}$, we finally obtain

$$d\sigma_{ij} = D_{ijkm}d\varepsilon_{km} \qquad (8$$

where D_{ijkm} denotes the incremental stiffness tensor:

$$D_{ijkm} = \frac{3}{2\pi}\int_S \Big[[\mathcal{F}_D'(\varepsilon_D) - \mathcal{F}_T'(\varepsilon_T)]n_in_jn_kn_m + \frac{1}{3}[\mathcal{F}_V'(\varepsilon_V) - \mathcal{F}_D'(\varepsilon_D)]n_in_j\delta_{km} + $$
$$+ \frac{1}{4}\mathcal{F}_T'(\varepsilon_T)(n_in_k\delta_{jm} + n_in_m\delta_{jk} + n_jn_k\delta_{im} + n_jn_m\delta_{ik})\Big]f(\vec{n})dS \qquad (9$$

Microplane material functions

The microplane material functions $\mathcal{F}_V(\varepsilon_V)$, $\mathcal{F}_D(\varepsilon_D)$, and $\mathcal{F}_T(\varepsilon_T)$ introduced in eqs. (are determined empirically. For the volumetric component, we assume a relationship simil to the known experimental curves obtained from oedometric tests (Fig. 1):

$$\sigma_V = \mathcal{F}_V(\varepsilon_V) = \sigma_V^* e^{k_V(\varepsilon_V - \varepsilon_V^*)} \qquad (10$$

where $(\varepsilon_V^*, \sigma_V^*)$ is the preconsolidation state (or the initial state previous to unloadin and k_V is a parameter proportional to the compressibility index (C_c) for virgin loadin and to the swelling index (C_s) for unloading–reloading.

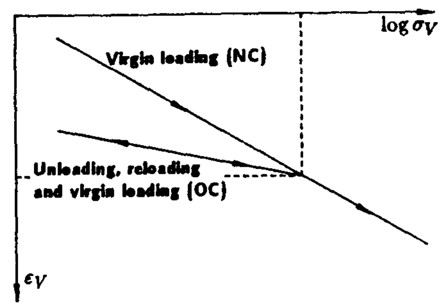
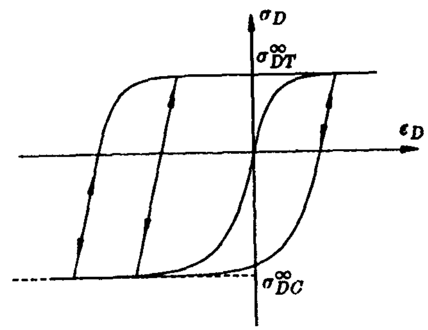


Figure 1 *Volumetric stress–strain curve.*

For the deviatoric component, we need to distinguish between tension and compression. Thus, we assume the following relations with a horizontal plateau:

$$\begin{aligned} \sigma_D &= \mathcal{F}_D(\varepsilon_D) = \sigma_{DC}^{\infty}[1 - e^{-k_{DC}|\varepsilon_D|}] \qquad &\text{if } \sigma_D < 0 \\ \sigma_D &= \mathcal{F}_D(\varepsilon_D) = \sigma_{DT}^{\infty}[1 - e^{-k_{DT}|\varepsilon_D|}] \qquad &\text{if } \sigma_D \geq 0 \end{aligned} \qquad (11)$$

where the positive sign of σ_D corresponds to tension and σ_{DC}^{∞}, σ_{DT}^{∞}, k_{DC}, and k_{DT} are empirical material constants not entirely independent, because $\sigma_{DC}^{\infty}k_{DC} = \sigma_{DT}^{\infty}k_{DT} = C_D^0$; C_D^0 is the initial elastic modulus. Eqs. (11) apply only for loading on the microplane; for unloading, we assume on each microplane linear elastic behavior with elastic modulus C_D^0 (Fig. 2).



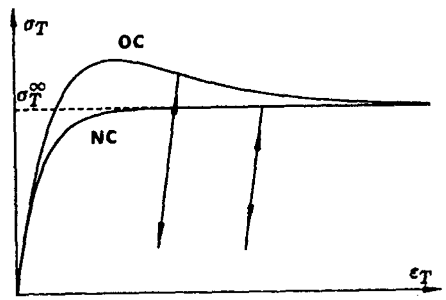


Figure 2 *Deviatoric stress–strain curve.*

Figure 3 *Shear stress–strain curve.*

Finally, for the shear component, the stress–strain relation should depend on the overconsolidation ratio, to account for its effect on the macroscopic response. Hence,

$$\sigma_T = \mathcal{F}_T(\varepsilon_T) = \sigma_T^{\infty}[1 + (a\varepsilon_T - 1)e^{-k_T\varepsilon_T}] \qquad (12)$$

where $a = a_0(\mathrm{OCR}-1)$. σ_T^∞, k_T and a_0 are empirical parameters. Parameter a depends on the overconsolidation ratio OCR: $a = 0$ for normally consolidated clays, and $a > 0$ for increasing values of OCR. For $a = 0$, the function in eq. (12) is identical to those in eqs. (11) for the deviatoric component, while for $a > 0$ and a sufficiently large value of a_0, eq. (12) presents a tension peak, followed by a descending branch down to a residual value σ_T^∞ (Fig. 3).

VERIFICATION WITH EXPERIMENTAL RESULTS

Figures 4–6 show fits of several drained test data from the literature. Figure 4 exhibits data reported by Henkel [13] for drained tests of overconsolidated clay. The model reproduces with good accuracy the stress peak, as well as the volume change response. Figures 5 and 6 show data reported by Nakai et al. [14] for standard and true triaxial, respectively, tests on normally consolidated clay. The results obtained from the model for the stress–strain as well as the volume change response are also in good agreement with the experimental data.

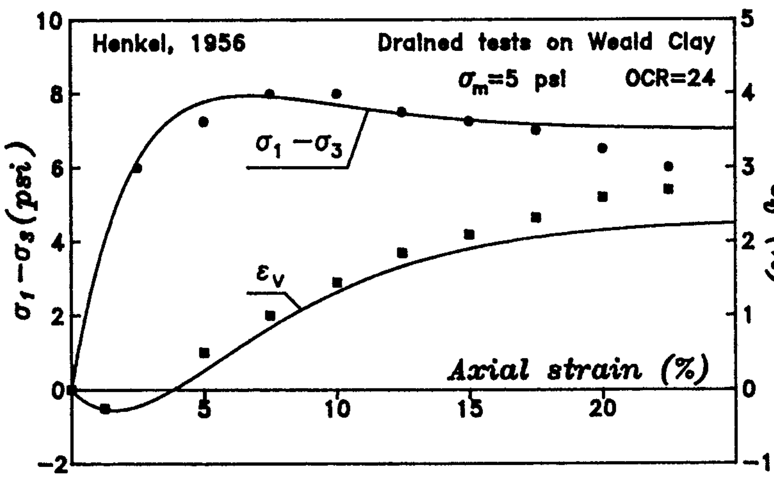


Figure 4 *Comparison with Henkel's data for overconsolidated clay.*

Remark.– A theoretically correct use of the constitutive relations with strain-softening, such as the present one, requires introduction of some localization limiter, as exhibited by a nonlocal continuum model with local strain [15]. This nonlocal approach has already been succesfully implemented on the microplane model for concrete (Bažant and Ožbolt, paper in preparation). But this development is beyond the scope of the present short contribution.

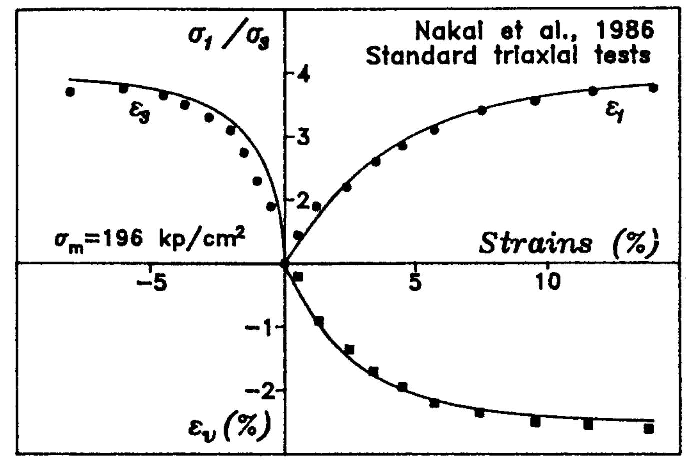


Figure 5 *Comparison with Nakai's data from standard triaxial tests.*

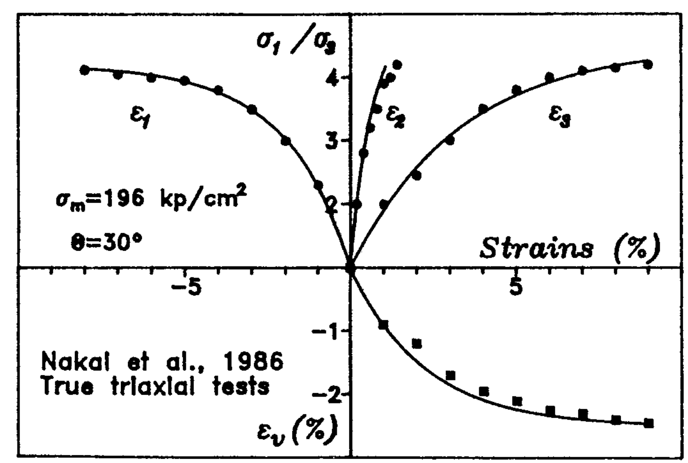


Figure 6 *Comparison with Nakai's data from true triaxial tests.*

SUMMARY AND CONCLUSIONS

A new constitutive model for the triaxial behavior of soils based on the microplane concept has been presented. A kinematically constrained structure is assumed, and normal and shear inelastic strains are taken into account. The model seems to represent well the drained response of normally consolidated as well as overconsolidated clays.

The constitutive properties are specified independently on planes of various orientations (*microplanes*). On each microplane the stress–strain relation is specified by means of three functions chosen empirically to be an image of the expected macroscopic response.

On each microplane the response is explicitly dependent on the volumetric strain (ε_V), which is the resolved component on the microplane of the macroscopic hydrostatic strain tensor.

The model can be applied not only to isotropic but also to anisotropic materials without any additional complexity. The anisotropy is simply taken into account by means of a weighting function, $f(\vec{n})$, of the unit normal vector $\vec{n}$ to the microplane, which may represent a general distribution function of the orientation of the clay particles.

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