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stress-strain curves and creep curves can be analytically determined. The increase in ice stiffness, strength and strain-softening effects with increased strain rate is reflected by the model; the post-peak decrease in stress is a direct consequence of brittle fracture. The model is also able to predict a limiting, constant strain rate for the tertiary creep phase.

Ice engineering has gained considerable importance in recent years, particularly in light of increased Arctic activity. The importance of the understanding of the mechanical properties of ice has correspondingly increased as a requirement for solutions to engineering problems in ice infested waters. The appeal of such a physically motivated model might then be enhanced when regarded as an effort to help describe, explain and predict the observed deformation response of ice.

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FRACTURE THEORY FOR NONHOMOGENEOUS BRITTLE MATERIALS WITH APPLICATION TO ICE

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ABSTRACT. - Brittle heterogeneous materials generally fracture with a dispersed zone of microcracking at the fracture front. The deformation and failure of these materials can be described by a nonlocal continuum theory, the special case of which is the blunt crack band model, in which a band of continuously distributed (smeared) cracks of a certain fixed width is assumed to exist at the fracture front. This model is easily implemented in finite element codes, and a dimensional analysis leads to a simple size effect law for the nominal stress at failure of geometrically similar specimens. The current state of this theory, which has been shown to apply to concrete and rocks, is briefly outlined and the possibility of application to ice is discussed. Comparison with a large series of test data by Butiagin suggests that nonlinear fracture mechanics based on the crack band model may indeed be applicable to ice.

1. Introduction

The failure of ice, whether river and lake ice or sea ice, seems to follow neither the strength concept (plasticity) nor the linear fracture mechanics. The tensile strength and failure loss have been studied in many works [1-25]; however, no systematic picture has yet emerged. It is clear that in absence of high hydrostatic pressures the failure is brittle rather than ductile, and the size effect on the nominal stress at failure is very pronounced. Although some recent works report that linear fracture mechanics might be applicable to some extent, this concept appears generally inapplicable because comparisons with linear elastic fracture mechanics did not involve specimens of sufficiently different sizes, and because the microscopic picture of fracture propagation shows the existence of microcracked zones at the fracture zone instead of a well defined sharp crack tip. Microstructurally, ice generally exhibits a coarse grain structure, which is particularly pronounced for sea ice, in which relatively large grains (of columnar arrangement) contain pockets of brine. In view of the heterogeneity of the microstructure, brittleness of failure, and the existance of dispersed zones of microcracking at the fracture front,

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it is logical to expect that the failure of ice should be well described by the recently developed crack band theory [26-36], which directly takes these material properties into account. This theory has been shown to work quite well for other brittle heterogeneous materials such as concrete [27-28, 30, 32] and various rocks [27, 29]. The purpose of this paper is to point out possible applications of the blunt crack band theory to ice and make some qualitative comparisons with the test data existing in the literature. This comparison will, however, be limited since funding for a systematic investigation has not yet been obtained.

Nonlocal Continuum

Progressive distributed microcracking, which is characteristic of a brittle heterogeneous material in its fracture process zone, may be described by a stress-strain relation which exhibits strain-softening, i.e., a gradual decline of stress at increasing strain. Such stress-strain relations are, however, inadmissible in the classical, local continuum mechanics since they always lead to material instability and localization of strain into a point, line, or surface, precluding the existence of strain-softening zones of a finite size in the local continuum. This dilemma may be resolved by introducing a nonlocal continuum concept, and in fact a statistical theory of heterogeneous materials shows that their continuum description is properly nonlocal. In the nonlocal continuum, the stress at a point is not only a function of the strain at that same point but also a function of the strain field in a certain neighborhood of that point. This neighborhood involves the so-called representative volume, the size of which may be obtained empirically and is typically several times the size of the inhomogeneity in the microstructure. Although the nonlocal continuum concept has been around for about twenty years, a form of nonlocal continuum theory that is applicable to strain-softening has been developed only recently [33-35]. According to this theory, called the imbricate continuum because it may be obtained as a continuum limit of an imbricated arrangement of finite elements, the following continuum relations apply;

$$S_{ij,j} = 0, \qquad S_{ij} = (1-c)\,\bar{\sigma}_{ij} + c\,\tau_{ij} \tag{1}$$

$$\sigma_{ij} = \bar{C}_{ijkm}(\underset{\sim}{\varepsilon})\,\bar{\varepsilon}_{km}, \qquad \tau_{ij} = C_{ijkm}(\underset{\sim}{\varepsilon})\,\varepsilon_{km} \tag{2}$$

$$\bar{\sigma}_{ij} = H\,\sigma_{ij}, \qquad \bar{\varepsilon}_{km} = H\,\varepsilon_{km} \tag{3}$$

$$H\,[\varepsilon(\underset{\sim}{x})] = \frac{1}{V}\int_{V(\underset{\sim}{x})} \varepsilon(\underset{\sim}{x}')\,\alpha(\underset{\sim}{x}')\,dV'(\underset{\sim}{x}') \tag{4}$$

Here $\varepsilon_{km} = (u_{k,m} + u_{m,k})/2$ = small strain tensor in cartesian coordinates x_i (i = 1,2,3), latin subscripts referring to the coordinates, u_i = displacements, $\bar{\varepsilon}_{km}$ = mean strain tensor, τ_{ij} = local stress tensor, σ_{ij} = broad-range stress tensor, $\bar{\sigma}_{ij}$ = mean stress tensor, S_{ij} = total stress tensor, C_{ijkm} and $\bar{C}_{ijkm}$ = tensors of secant elastic moduli, local and broad-range, c = empirical coefficient between 0 and 1 but closer to 0, H = averaging operator, V $(\underset{\sim}{x})$ = a representative volume = sphere of diameter ℓ where ℓ = characteristic length of the material (typically several times the grain size), $\alpha(\underset{\sim}{x})$ = empirical weighting function. Instead of an integral averaging operator (Eq. 4), the averaging operator may be also approximated by a differential expression;

$$H \approx 1 + \lambda^2\,\nabla^2 \tag{5}$$

in which ∇^2 = Laplacian and λ = length constant proportional to ℓ.

It has been shown that finite element discretization of the foregoing continuum relations leads to a stable model even in the presence of strain-softening, and that stable strain-softening zones of finite size can be obtained. Thus, this nonlocal approach can be used for modeling materials in which the fracture process zone is not negligible in size. The solutions have been found to converge as the finite element mesh was refined [34].

The difference from the original nonlocal continuum theory consists in the fact that the stress, σ_{ij}, must be processed through the same averaging operator H as the strain, and that both local and nonlocal stresses must be used simultaneously.

Blunt Crack Band Theory

Except for certain special applications, particularly in dynamics and wave propagation, the aforementioned nonlocal continuum model appears unnecessarily sophisticated and its reduction to an ordinary finite element model appears possible. If one does not need resolution of stresses and strains throughout the fracture process zone (cracking zone), the cracking front may be assumed to be single-element wide, and according to the nonlocal continuum concept the width of the cracking front must be considered identical with the characteristic length of the material and must be assumed as fixed, in a constant relation to the size of the inhomogeneity. It has been shown that consistent and properly convergent results, which are objective with regard to the analyst's choice of the mesh size, can be obtained only with meshes in which the right size of the finite element compared to the grain size is used [27, 28, 26, 29].

The crack band model is defined by a triaxial constitutive relation based on the strain-softening diagram such as that shown in Fig. 1. It has been shown the fracture energy G_f is then expressed as

$$G_f = W\,w_c, \qquad W = \frac{1}{2}\left(\frac{1}{E} - \frac{1}{E_t}\right) f_t'^{\,2} \tag{6}$$

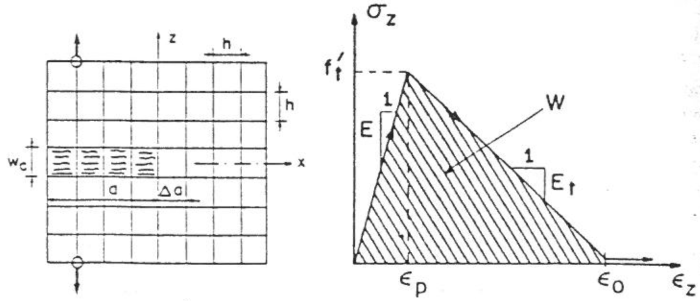

Fig. 1 - Finite Element Crack Band Model and the Tensile Strain-Softening Stress-Strain Relation Used in the Model.

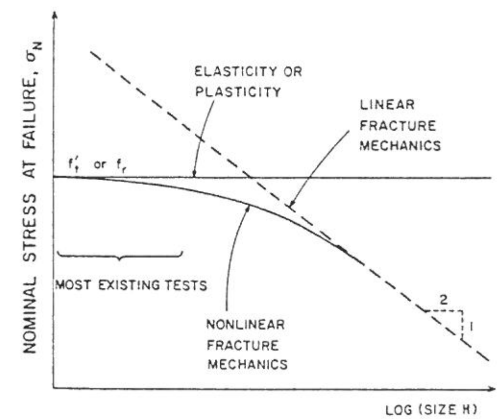

Fig. 2 - The Size Effect Law for Blunt Fracture [31].

in which w_c is the crack front width, which must be considered as a material constant, as already mentioned, and W = the area under the tensile stress-strain diagram = energy dissipated by cracking in the fracture process zone per unit volume of the zone; f'_t = direct tensile strength of the material, E = elastic Young's modulus, and E_t = the tangent modulus for strain-softening, which is negative. If the analyst needs to use in the fracture process zone elements that are larger than the characteristic size $w_c = \ell$, consistent results can still be obtained if the tensile strain-softening stress-strain relation is adjusted so as to yield the correct value of fracture energy G_f. Finite elements smaller than ℓ can be used only if the finite element form of nonlocal continuum is adopted; it is only for $w_c = \ell$ that the nonlocal continuum approximation by finite elements reduces to an ordinary finite element model.

A relatively large set of fracture test data exists in the literature on concrete. It has been clear from these data that neither the concept of strength or yield nor the linear elastic fracture mechanics is a satisfactory model. It has been found [27, 28] that the existing test data can be very well described (in fact with an overall coefficient of variation of only 6%) by the crack band model, in which the crack front is approximately $w_c \approx 3\ d_g$ where d_g = grain size (the maximum aggregate size in the case of concrete). This is true of both the maximum load data and the load data on the dependence of the energy required for crack growth on the crack extension from the notch tip (R-curves). It appears that the crack band theory is capable of a complete representation of the fracture behavior of concrete as known at present. A good agreement with the existing test data has been also achieved for rocks [27, 29]. In this light, the question whether the same model might be applicable to ice is rather tempting.

Size Effect Law

In the classical, linear elastic fracture mechanics, the basic phenomenon is the energy flow from the uncracked structure into the crack tip region as the crack propagates (the energy release rate). Accordingly, the total energy consumed by fracture is a function of the crack length, a. On the basis of this simple fact it is possible to show by dimensional analysis that the nominal stress at failure is, according to linear elastic fracture mechanics, proportional to $d^{-\frac{1}{2}}$ when geometrically similar structures of different sizes are compared; d = characteristic size of the structure, and the nominal stress at failure is defined as $\sigma_N = P/bd$ where P = maximum load, and b = thickness. Thus, when log σ_N is plotted versus log d, the size effect of linear elastic fracture mechanics is represented by the straight line of slope = - 1/2 in Fig. 2.

For the strength or yield criterion, by contrast, there is no size effect, since this criterion is represented in Fig. 2 by a horizontal straight line. What is now the size effect law for nonlinear fracture mechanics based on the crack band theory?

The typical property of the crack band theory, which comes from the nonlocal continuum concept, is the existence of a blunted crack front of width $w_c = \ell$. Accordingly, the total energy consumed by fracture of length a depends on:

1. The length of the fracture, a; and

2. The area that has undergone cracking, $A_c = a\ell$.

The dependence on fracture length a represets the energy which flows into the fracture front from the remaining uncracked part of the structure, and the dependence on area $a\ell$ represents the energy which was stored within the area that cracked.

Assuming the aforementioned two dependencies of the total energy release as the basic hypothesis, one can show by dimensional analysis and similitude arguments that the nominal stress at failure for geometrically similar specimens (which fail during fracture propagation rather than fracture initiation — the majority of situations) is governed by the following size effect law [31];

$$\sigma_N = B f'_t \left(1 + \frac{d}{nd_g}\right)^{-\frac{1}{2}} \tag{7}$$

in which B and n are two empirical constants (for concrete, $n \approx 25$; Ref. 32). This law is represented in Fig. 2 by the solid curve. It represents a gradual transition from failures governed primarily by the strength criterion, which applies for very small structures or specimens, to failures governed by linear elastic fracture mechanics, which applies for very large structures or specimens.

The size effect law for blunt fracture (Eq. 7) has been verified experimentally by tests of concrete fracture specimens of different sizes [32, 40], and also by failure data for the diagonal shear failure of longitudinally reinforced beams [32]. Since this law results from a simple dimensional analysis from a rather general hypothesis which should be applicable to brittle nonhomogeneous materials in general, it is logical to expect that this size effect law should also be applicable to ice.

The size effect law can be particularly simply identified from test data on maximum loads of specimens of geometrically similar shapes. Eq. 7 can be algebracially transformed to a linear equation $Y = AX + C$ where $Y = (f'_t/\sigma_N)^2$, $X = d$, $C = B^{-2}$ = Y-intercept, and $A = C/nd_g$ = slope of the straight regression line in the plot of Y versus X; see Fig. 3.

The fracture energy G_f, as defined by Eq. 6, can be obtained from the slope A of the size effect regression plot. As has been shown in Ref. 40,

$$G_f = \frac{g(\alpha_0)}{AE_c} \tag{8}$$

in which $g(\alpha_0)$ = nondimensional energy release rate for a sharp crack according to the linear fracture mechanics, which can be found in various handbooks for typical specimen shapes and can be always easily obtined by linear finite element analysis; $\alpha_0 = a_0/d$ where a_0 = notch length of fractüre specimen. Determination of the fracture energy from the size effect according to Eq. 8 seems the simplest method of its experimental identification.

The size effect law (Eq. 7) makes it also possible to determine the R-curve, i.e., the dependence of the energy required for crack growth on the crack extension $c = a - a_0$ from the notch tip. The R-curve is simply obtained by plotting the fracture equilibrium curves on the basis of the failure loads of the specimens of various sizes (after their smoothing with the size effect law), and then finding the envelope of all these fracture equilibrium curves. The fracture energy G_f is then the final asymptotic value of this envelope (see Ref. 40).

Applicability to Ice

Many aspects of tensile strength, failure and fracture of lake and river ice as well as sea ice have been studied in considerable detail [1-25]. Particular attention has been given to the effect of temperature, strain rate, salinity, and brine volume on strength [e.g. 18, 15, 16]. Recently it was observed by Urabe [19] that "the linear elastic fracture mechanics concept is effective for analyzing the fracture phenomena of sea ice". However, this conclusion is of limited applicability, because aspects which are typical of nonlinear fracture mechanics, especially the size effect,have not been explored. As far as the law governing fracture is concerned, the size effect is paramount.

The size effect has been discussed by Weeks and Assur [3], but within the context of tensile strength rather than fracture mechanics. No test data seems to be available for fracture of geometrically similar ice specimens of sufficiently different sizes. Conduct of such tests is highly desirable but could not be realized so far because of lack of funding. Therefore, we must content ourselves in the present analysis with what can be inferred from the size effect on strength. Within the context of linear elastic fracture mechanics, such information would be insufficient; however, within the context of nonlinear fracture mechanics based on the crack band theory, some informtion can be obtained because, due to the presence of the tensile strength f'_t in the model (Eq. 6), the presence of a notch is not required for the applicability of this theory. The only feature that is required is

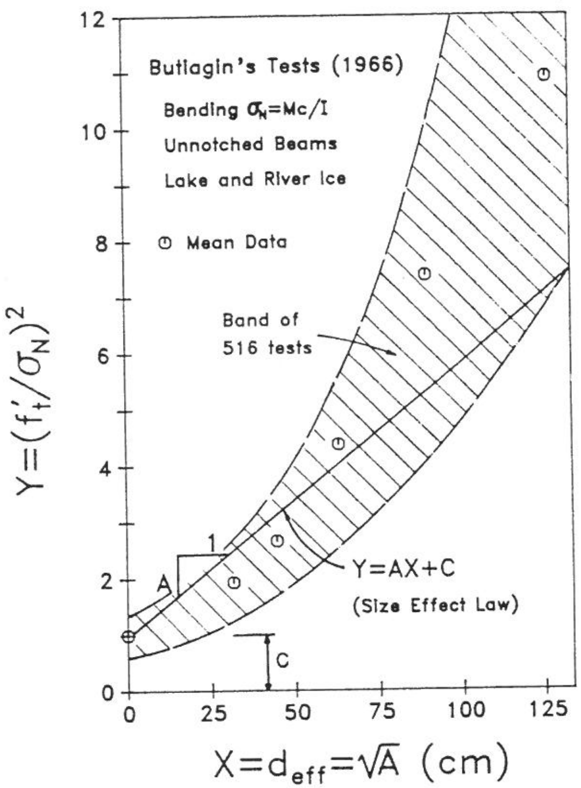

Fig. 3 - Linear Regression for Size Effect and Comparison with Butiagin's Test Data (1966).

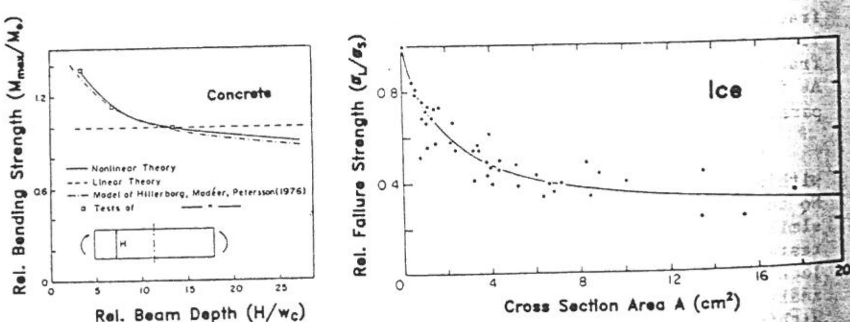

Fig. 4 - The Dependence of the Nominal Tensile Strength for Bending, σ_N, on Depth d or Effective Depth d_{eff} of Tests Beams; (a) for concrete (after Bažant and Oh, 1983), (b) For Ice (after Weeks and Assur (3, p. 959), with Test Data by Butiagin).

that the failure* must occur during crack band propagation rather than at crack band initiation, and this is probably true for many of the existing strength tests.

For concrete [28] it has been shown that the apparent tensile strength in bending, which can be identified with the nominal stress at failure σ_N, decreases as the depth of the test beam increases. This is simply a consequence of the strain-softening property. Fig. 4 (a) shows computation results from the crack band model in which σ_N is plotted as a function of the beam depth, d [28]. These computational results compared very well with the measurements by Hillerborg, Modeer and Petersson (1976) [28], shown as the data point in Fig. 4(a). Extensive data on the bending tensile strength of river and lake ice specimens were collected by Butiagin [1, 2] and organized by Weeks and Assur [3]. These data are shown in Fig. 4(b) taken from the work by Weeks and Assur [3, page 959], in a plot of σ_N versus the effective size of the cross section, d_{eff}. Comparison with Fig. 4(a) for concrete reveals a similar trend. This suggests that the crack band theory, nonlocal continuum, and the corresponding size effect law may be applicable to ice.

The question is further examined in Fig. 3, which represents the aforementioned linear regression plot for the size effect. The data points represent the means and the cross-hatched area represents the scatter band of 516 test results by Butiagin. The figure reveals a rising trend (if there were no size effect the data band would be horizontal), and it also shows that a straight line for the size effect law is totally confined within the scatter band of data. However, the trend of these data is significantly curved. Yet, this fact cannot be considered as a refutation of the size effect law and the crack band theory on which it is based. This is because Butiagin's tests have not been collected from specimens which would be precisely comparable according to the size effect law. First of all, the specimens were not geometrically similar, and especially for the deeper beams the length of the beam was shorter than that required for geometrical similiarity. This would cause the shear force near the failure crack to be higher for the larger specimens, which may cause an additional reduction of the failure load for the large specimens, thus causing the data band to curve upwards toward the right in Fig. 3. Furthermore, the cross sections in Butiagin's tests were not geometrically similar either. These test data were reported as a function of the effective size d_{eff} which represents the square root of the cross section area of the beam. This means that d_{eff} measures not only an increase in the beam depth but also an increase in the beam width, for which the size effect law does not apply at all. As far as the beam width is concerned, there is no doubt also some size effect, which is, however, totally a statistical phenomenon, due to the fact that over a larger width there is a higher chance of encountering a region of a smaller local strength. This statistical effect is probably much weaker than the size effect due to fracture aspects, however, it is no doubt present in the data in Fig. 3. This would, therefore, mean that

*i.e., the maximum load

for the larger specimens, which also have a larger width, the failure load would be smaller than that corresponding strictly to the size effect law, and this may be another reason for the upward curvature of the data band in Fig. 6.

In the light of the foregoing disturbing factors, the disagreement between the straight line for the size effect law and the trend of the data band cannot be considered serious. To the contrary, the overall rising trend, along with the fact that the straight line for the size effect law can be passed totally within the data band, suggests that the crack band theory with its size effect law may in fact be applicable to ice, as might be logically expected. However, this tentative conclusion will have to be confirmed by precisely controlled experiments.

Concluding Remarks

Based on logical inferencies from the microstructure of ice, the observations of microcracking, and some qualitative comparisons for bending strength data, it seems likely that the fracture of ice may be modeled by the crack band theory, which is based on a nonlocal continuum concept and implies a size effect law which represents a smooth transition between the strength or yield criterion and the failure criterion of linear elastic fracture mechanics. This conclusion is, however, tentative and would have to be subjected to systematic experimental verification.

Fracture mechanics modeling of failure of ice is particularly important with regard to the size effect. A correct knowledge of the size effect is essential for extrapolation from small scale laboratory or field tests to the prediction of failure of real size ice structures. This is rather important, e.g., for oil drilling platforms in the arctic ocean, as well as for the technology of ice breaking, for the predictions of bearing capacity of artificially strengthened sea ice sheets serving, e.g., as landing strips for aircraft, for the determination of the pressure of river ice on bridge piers, etc.

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Appendix II. - Statistical Size Effect

The decrease of observed nominal strength with the size of specimen has usually been explained statistically. The strength value is assumed to vary randomly throughout the material, and so in a specimen of a larger size the chance of encountering a smaller minimum strength is higher. The size effect law obtained in this manner then depends on the assumed statistics of strength distribution, or the statistics of the size of microscopic flaws which cause the local strength reduction. This classical, statistical approach has, however, several significant differences from the present size effect law.

One difference is due to the fact that for volumes larger than a certain characteristic volume the chance of encountering a still smaller strength (or a still larger flaw) does not increase. In such a case the statistically based size effect reaches an asymptotic value at a certain specimen size, and does not decrease any more for larger specimen sizes. On the other hand, the present size effect law continues decreasing.

Another, more significant difference is due to the fact that the amount of energy released from the uncracked part of the structure into the fracturing region is very important. Thus, the same specimen supported rigidly or elastically would have the same strength according to the statistical theory, while according to the present energy-based size effect law the elastically supported specimen can have a much smaller strength. Obviously, a similar phenomenon arises from the geometry of the structure, in the various possible manners the geometry of the uncracked regions of the structure can significantly affect the failure.

Some statistical size effect surely exists, but it may be much less significant than the present energy-based size effect. At least for concrete it seems at the present that all the known size effect can be adequately explained by the energy-based size effect law, although statistically-based theories have previously been offered to provide an explanation.

SEA ICE INDENTATION ACCOUNTING FOR STRAIN-RATE VARIATION

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ABSTRACT: Global and local indentation pressures in the creeping mode of sea ice deformation are obtained, accounting for the spatial variation of strain-rates. Two approximate methods of analysis are considered; the upper bound and strain path methods. Theoretically postulated velocity fields required in the analysis are calibrated with field measurements. Sea ice behavior is described by a multi-axial power-law creep model and by the multi-axial extension of a new uniaxial model which accounts for both hardening and softening behavior. Results are compared with previously published indentation formulas.

INTRODUCTION

Two levels of ice loading are typically considered in the design of drilling and production platforms for the Arctic. Global ice pressures govern the overall structural geometry and dimensions as well as the foundation design, while local pressures are likely to dictate wall thicknesses and local framing, and may well govern structural cost. Most of the emphasis on ice force research has been on predicting global forces. Only during recent years, as the focus changed from overall feasibility to preliminary and detailed design, has the importance of local pressures emerged. Peak local pressures may be as high as three times the average global pressure. It is widely recognized that uncertainties exist in ice load prediction models in use today and that in some cases design loads may be overestimated by an order of magnitude.

Uncertainties in existing ice load models arise primarily from four sources: (i) incomplete modeling of the thermomechanical behavior of sea ice, (ii) use of semi-empirical formulations, calibrated without adequate regard for similitude modeling and scale effects, (iii) failure to realistically model the contact forces at the ice-structure interface and the presence of macrocracks, and (iv) not accounting for the finiteness of the environmental forces driving the ice features. Both approximate analytical methods and more rigorous numerical models based on the finite and boundary element methods of analysis can be used to study ice-structure interaction at full scale with realistic models for material and interface behavior.

This paper employs two approximate methods of analysis, the upper bound and strain path methods, to study the problem of sea ice indentation in the creeping mode of deformation, accounting for the spatial variation of strain-rates. This is a problem of concern for artificial islands in the Arctic nearshore region, where "break-out" and/or steady indentation conditions occurring in the winter form a basis for select-

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