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Model B4: Multi-decade creep and shrinkage prediction of traditional and modern concretes

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ABSTRACT: To improve the sustainability of concrete infrastructure, engineers face the challenge of incorporating new concrete materials while pushing the expected design life beyond 100 years. The time-dependent creep and shrinkage response of concrete governs the serviceability and durability in this multi-decade time frame. It has been shown that current prediction equations for creep and shrinkage underestimate material deformations observed in structures outside of a laboratory environment. A new prediction model for creep and shrinkage is presented that can overcome some of the shortcomings of the current equations. The model represents an extension and systematic recalibration of model B3, a 1995 RILEM Recommendation, which derives its functional form from the phenomena of diffusion, chemical hydration, moisture sorption, and the evolution of micro-stresses in the cement structure. The model is calibrated through a joint optimization of a new enlarged laboratory test database and a new database of bridge deflection records to overcome the bias towards short-term behavior. A framework for considering effects of aggregates, admixtures, additives, and higher temperatures is also incorporated.

1 INTRODUCTION

The design of new, and assessment of existing concrete structures, requires accurate prediction of structural response during construction as well as operation until the end of the intended service life. Most structures of significance, such as bridges, are designed for a service life of at least 50 and often 100 years, which stresses the importance of accurate multi-decade prediction models. In particular, creep and shrinkage are significant for long-span bridges, high-rise buildings, and other statically indeterminate creep sensitive structures. Yet, precise laboratory data for model development, calibration, and validation is mostly limited to less than 6 years. The only source of information, exceeding the scope of laboratory tests, is structural measurements. This situation is further aggravated by the rapid development of new concrete materials while the multi-decade monitoring information is available only for historically used materials. Those, however, are of little direct significance for the current and future construction industry. Nonetheless, researchers have shown that multi-decade structural observations can provide insight into unanswered problems. Bridge deflection data, for example, clearly reveal a non-zero terminal slope of the creep compliance function, a feature that was previously disputed. This paper presents a new model, labeled B4, which overcomes the main shortcomings of the CEB-fib, ACI, JSCE and GL prediction models for concrete creep and shrinkage. Model B4 represents an extension and systematic recalibration of the theoretically founded model B3, which is a 1995 RILEM recommendation. In addition to introducing the so far missing separation of autogenous and drying shrinkage, model B4 takes into account the effects of different cement types, admixtures, and aggregate types. Admixtures such as fly ash, silica fume, water reducer, superplasticizer, retarder, accelerator, viscosity agent, and air entraining agent are known to affect creep and shrinkage and are inseparably linked to modern construction. Two sets of predictors for the major parameters of the creep and shrinkage model are presented—a composition based formulation for detailed analyses, and a strength based formulation for simple design.

The model is calibrated through a joint optimization of a new significantly enlarged database of laboratory creep and shrinkage tests and a new database of relative bridge deflection records. Statistical biases towards certain time periods, compositions, or environmental conditions are counteracted through a suitable weighting scheme. The quality of the derived model is ultimately documented by the model ability to (a) accurately predict single tests, (b) capture the effects of the major input parameters, and (c) optimally fit the full database of laboratory tests in a statistical sense.

2 DATABASE

For the development and calibration of model B4 an extended database (Hubler et al. 2013b), with roughly 1,350 creep and 1,800 shrinkage laboratory tests was created. This new database is about three-times larger than the previous RILEM database used to calibrate model B3. The creep database encompasses roughly 730 total creep curves and 640 basic creep curves. The shrinkage database encompasses 1220 total, 420 autogenous, and 180 drying shrinkage curves. The majority of data sets concerns concretes made from the R type cement, followed by RS type and SL type concrete with 15% each.

3 MODEL DEVELOPMENT

For the development of long-term prediction models, a sufficiently large and reliable dataset is required. Recently a database of both, short term tests providing insight into the influence of intrinsic composition parameters and environmental conditions as well as multi-decade bridge observations has been completed (Hubler et al. 2013b). With this new dataset it is possible to develop improved long term prediction formulations such as model B4 (Bazant et al. 2013). Further information regarding the development, calibration, and statistical verification are given in (Wendner et al. 2013a).

All available experimental data is biased due to the experimenter's preferences towards certain testing conditions and material compositions as well as the practical relevance of certain concrete mixes and environmental conditions. Furthermore, the influence of sampling density in time needs to be removed. In order to counteract these types of bias a hyperbox weighting scheme based on (Bažant et al. 2008) was introduced as depicted in Figure 1 for the simple two dimensional case. Herein the weights w_{ij} are determined in such a way that each curve i enters with the same weight as well as all half-decades j are represented equally.

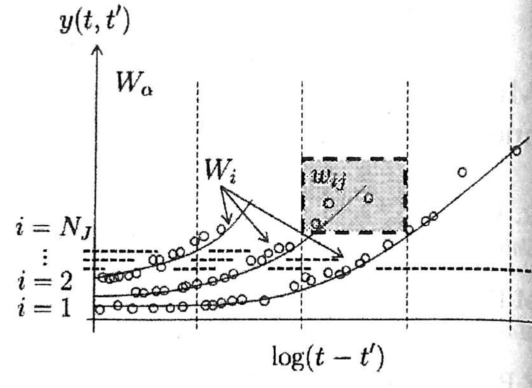

Figure 1. Hyperbox weighting scheme, applied to creep compliance, J, shrinkage strain, ε, and bridge deflections, δ.

4 DEALING WITH ERRORS

Unfortunately, many datasets are polluted measurement errors or at least missing input information that has to be identified and compensated for. The obstacle of errors in time or measurement value can be overcome by exploiting well established principles such as the asymptotic shape of drying shrinkage—a square-root time function—for short times after exposure to the environment (Bažant et al. 1995b). Consequently, linear extrapolation in the appropriate power scale allows for the extraction of errors in exposure time t_0 of shrinkage tests, and similarly in load application time t' of creep tests. This approach is well justified and robust but requires sufficient data for very short measurement times that are typically not available. Considering the total amount of data (literally hundreds of different curves, reported by different experiments with different setups) an automatic and robust must be applied, even if the accuracy for single curves is slightly compromised. Under the assumption of an overall low number of questionable data sets, an iterative optimization procedure for example can lead to good results. This procedure entails the alternating optimization of the parameters of the actual prediction equation and shifts of the individual curves, both in time and space. Convergence studies revealed a required minimum of three iterations until satisfactory convergence of more than 95% is reached. All shifts are considered only during the optimization phase to exploit the information that is contained in the relative shape; they do not enter the validation phase that is performed directly on the reported data. Figure 2 exemplarily shows the identified distribution of shifts in creep compliance which can be attributed to (a) real measurement errors, more

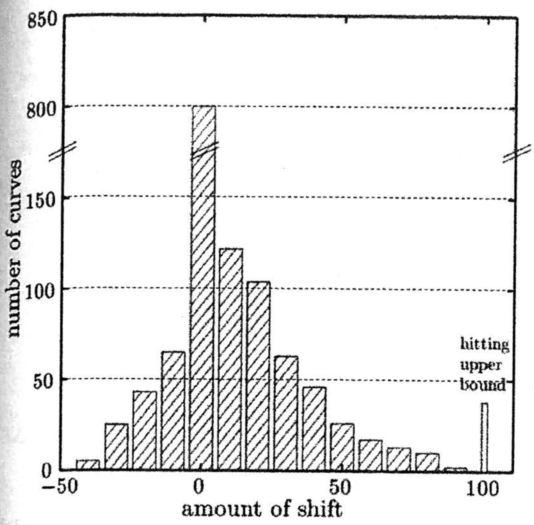

Figure 2. Distribution of shifts in creep compliance, ΔJ.

likely (b) missing initial deformations, or (c) solely a deviation from the mean fit.

5 MODEL B4

The new B4 prediction model is the successor of the well-established model and RILEM recommendation B3 and extends the range of applicability to modern concretes with admixtures. B4 captures the behavior of Portland cement concretes based on model parameters that are derived from the composition of the concrete and the environmental conditions. Further improvements include a recalibration for multi-decade behavior, and the introduction of a separation between autogenous shrinkage and drying shrinkage. In the service stress range (up to $0.40f_c'$) a linear dependence of creep strain on stress may be assumed as an acceptable approximation. This means that, for constant stress σ applied at age t', the strain evolution is given by

$$\varepsilon(t) = J(t,t')\sigma + \varepsilon_{sh}(t,t_0) + \varepsilon_{au}(t) \tag{1}$$

in which σ = uniaxial stress.

The stress-independent strain is split into the drying shrinkage, $\varepsilon_{sh}(t,t_0)$, and autogenous shrinkage, $\varepsilon_{au}(t)$. The compliance function, $J(t,t')$, as introduced in (Bažant 1995), is adopted without change. $1/E_0 \approx q_1/E_{28}$ represents the asymptotic (truly instantaneous) compliance estimated from the 28-day Young's modulus, C_0 denotes the basic creep compliance, and C_d the drying creep compliance (containing the scaling parameter q_5).

$$J(t,tc) = \frac{1}{E_0} + C_0(t,t') + C_d(t,t',t_0) \tag{2}$$

The basic creep compliance is the sum of an ageing viscoelastic term (subsequently referred to as q_2 term), a non-ageing viscoelastic contribution (q_3 term), and a flow term (q_4 term) (Bažant et al. 2013), where $Q(t,t')$ = binomial integral;

$$C_0(t,t') = q_2 Q(t,t') + q_3 \ln\left(1 + (t-t')^{0.1}\right) + q_4 \ln\left(\frac{t}{t'}\right) \tag{3}$$

Integral $Q(t,t')$ cannot be evaluated in a closed form, but a very good closed form approximation exists (Bažant et al. 1989).

While drying creep, like shrinkage, is bounded by a horizontal asymptote, the basic creep compliance is unbounded and can be characterized by an asymptotic terminal slope, $0.1q_3 + q_4$ in the logarithmic time scale.

The time functions of drying shrinkage and autogenous shrinkage are S-shaped curves bounded by final shrinkage values $\varepsilon_{sh\infty}$ and $\varepsilon_{au\infty}$ respectively. The evolution of drying shrinkage is described by

$$\varepsilon_{sh}(t,t_0) = -\varepsilon_{sh,\infty}(t_0) k_h \tanh\sqrt{\frac{t-t_0}{\tau_{sh}}} \tag{4}$$

where $\varepsilon_{sh}(t,t_0)$ is the evolution of drying shrinkage strains, $\varepsilon_{sh,\infty}(t_0)$ is the final drying shrinkage as a function of the curing time t_0, k_h is a factor describing the dependence on environmental humidity, as published in (Bažant 1995). The shrinkage halftime, τ_{sh} is predicted based on the cement type and admixture dependent basic value τ_0, modified for composition and effective diffusivity parameter $k_s D$, where k_s is a shape parameter of the cross-section (Bažant 1995) and D is the effective thickness of cross section. The autogenous shrinkage evolution is given by

$$\varepsilon_{au}(\tilde{t},\tilde{t}_0) = \varepsilon_{au\infty}\left[1 + \left(\tau_{au}/(\tilde{t}+\tilde{t}_0)\right)^{\alpha}\right]^{r_t} \tag{5}$$

where α, r_t are composition and cement type dependent parameters controlling the shape of the function and τ_{au} is the autogenous halftime. The influence of ageing and thus of the gain in stiffness, is accounted for (as originally proposed in (Bažant 1995), by the ratio between the 607-day modulus and the modulus at the end t_0 of curing.

The full model formulation including the time functions and parameters is given in Bažant et al. (Bažant et al. 2013). In this paper two sets of predictor equations for the parameters q_i, the final shrinkage parameters and the half-time parameters are given; one based solely on the strength, the other one based on the composition parameters w/c, c, a/c, cement type and admixture content. A further discussion of the creep and shrinkage model development, calibration and validation as well as an uncertainty quantification is given in (Wendner et al. 2013b, Hubler et al. 2013a).

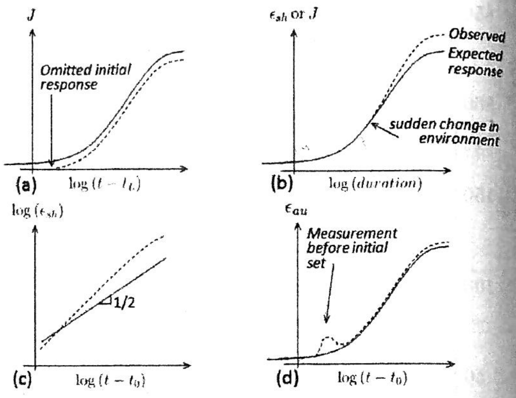

Figure 3. Distribution of shifts in creep compliance, ΔJ.

6 STATISTICS

Creep and shrinkage test data can be characterized as heteroscedastic which is a major source of complications for the formulation of statistical indicators and impairs the statistical tests of significance and regression analysis. For optimization and validation two statistical indicators are recommended. One is the coefficient of determination, $R^2 = 1 - S_{err}/S_{tot}$, which relates the sum of squares of residuals S_{err} to the total sum of squares S_{tot}; R^2 is proportional to the sample variance and can be seen as a measure of how well the trend in the data can be reproduced by the model. The second and more representative statistical indicator is the coefficient of variation of the root-mean-square error *CV*(*RMSE*) which is defined in analogy to the coefficient of variation. This dimensionless measure quantifies the expected normalized prediction error and is a good measure of accuracy. Naturally, it is a requirement that first all the data are checked for plausibility and the systematic errors are removed based on theoretical considerations. Among others, imprecisely reported concrete ages at time of load application, loading durations and, in particular, measurement errors need to be identified (Hubler et al. 2013b). In many cases the elastic deformation in creep tests is not included, the sensor position and gauge length are wrongly reported, or the environmental conditions are unclear. Figure 3 shows examples of compromised data and their respective sources.

After removing all questionable data sets and filtering the database for inputs that comply with the range of applicability of the major models that are currently endorsed by engineering societies a comparison is possible.

Some of the more widely used models in practice are the model of the American Concrete Institute ACI-92 (ACI 1992), the Eurocode model which is based on the *fib* model Code 1990 (CEB-FIP 1993) with its revision in 1999 (fib 1999), the new *fib* Model Code model 2010 (fib 2012), and the model of the Japanese Society of Civil Engineers ((JRA) 2002, (JSCE) 1996). Additionally, professional engineers and scientists have suggested models such as the Gardner-Lockman model GL2000 (Gardner et al. 2001), and the B3 model (Bažant et al. 1995b, Bažant et al. 1995a, Bažant et al. 1996, Bažant et al. 2000, Bažant 1995), developed by Bažant and his co-workers and approved in 1995 as RILEM Recommendation.

It is important to note that such a comparison has to go beyond pure database wide statistics in order to extract (a) the quality of the chosen time function, (b) the functional dependence on environmental conditions, size, the age at the time of loading, or even composition. Only after a model satisfactorily passes these tests, (c) global statistics can be performed to judge the quality of the overall calibration (of the constants in the semi-empirical and empirical equations). The total number of available shrinkage data points is given in Figure 4 according to half decades after exposure to the environment $t - t_0$.

A detailed discussion and comparison of different shrinkage models is given in (Hubler et al. 2013a) and of creep models in (Wendner et al. 2013b) where conclusions regarding the form of the time functions and the predictive qualities of the model are given. The overall statistics are plotted in Figure 6 for the creep model and Figure 7 for the shrinkage formulation. The creep statistics are based on a combination of the full experimental database and the bridge deflection data of 69 bridge spans. Only data points older than 10^3 days have been considered. Overall it can be noted that the composition based model B4 as well as the strength based model B4 s are superior. However, it is interesting that the strength based formulation provided better long term prediction than the more complex model B4.

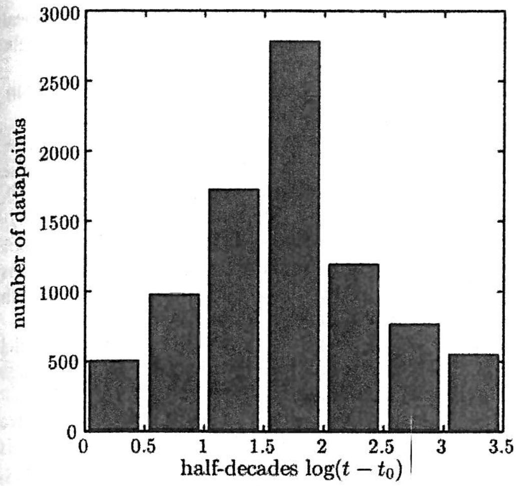

Figure 4. Number of shrinkage data points.

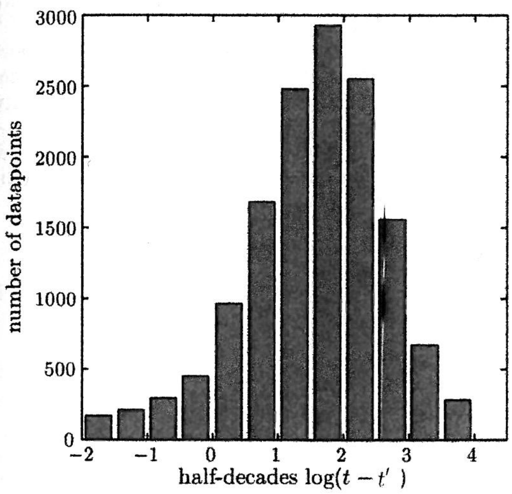

Figure 5. Creep laboratory data points.

If all shrinkage data are considered, including datasets with significant amounts of admixtures or reactive additives, model B4 performs better than model B4 s which is on a similar level as the models predecessor model B4 and GL2000. Surprisingly the fib Models 1999 and 2010 cannot catch up with the other models. In case concretes without admixtures are analyzed the performance of the fib models is comparable to their competitors. The detailed investigation provided in (Hubler et al. 2013a) changes the picture further and leads to a clear conclusion: only models with a split into autogenous and drying shrinkage can accurately reproduce the experimental data—these are model B4 and the fib models.

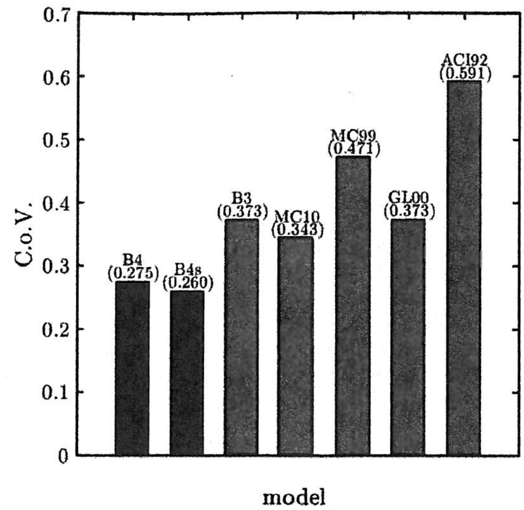

Figure 6. Comparison of creep models based on full laboratory database and bridge bridge deflection data.

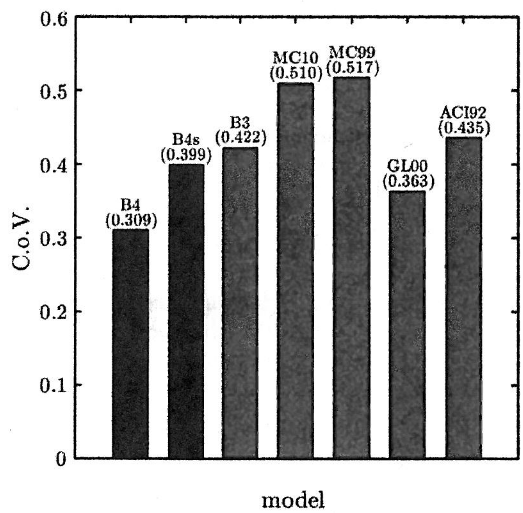

Figure 7. Comparison of shrinkage models based on full laboratory database.

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