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Comminution of concrete due to kinetic energy of high shear strain rate

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ABSTRACT: This paper outlines the basic idea of a macroscopic model on the dynamic comminution or fragmentation of rocks, concrete, metals, and ceramics. The key idea is that the driving force of comminution under high-rate shear and compression with shear is the release of the local kinetic energy of shear strain rate. The spatial derivative of the energy dissipated by comminution gives a force resisting the penetration, which is superposed on the nodal forces obtained from the static constitutive model in a finite element program. The present theory is inspired partly by Grady's model for comminution due to explosion inside a hollow sphere, and partly by analogy with turbulence. In high velocity turbulent flow, the energy dissipation rate gets enhanced by the formation of micro-vortices (eddies) which dissipate energy by viscous shear stress. Similarly, here it is assumed that the energy dissipation at fast deformation of a confined solid gets enhanced by the release of kinetic energy of the motion associated with a high-rate shear strain of particles. For simplicity, the shape of these particles in the plane of maximum shear rate is considered to be regular space-filling hexagons. The particle sizes are considered to be distributed according to the Schuhmann power law, but the formulation for any other suitable distribution can easily be obtained by replacing the Schuhmann distribution by the desired distribution. The condition that the rate of release of the local kinetic energy must be equal to the interface fracture energy yields a relation between the particle size, the shear strain rate, the fracture energy and the mass density. The density of this energy at strain rates >1,000/s is found to exceed the maximum possible strain energy density by orders of magnitude, making the strain energy irrelevant. It is shown that particle size is proportional to the −2/3 power of the shear strain rate and the 2/3 power of the interface fracture energy or interface shear stress, and that the comminution process is macroscopically equivalent to an apparent shear viscosity that is proportional (at constant interface friction) to the −1/3 power of this rate. A dimensionless indicator of the comminution intensity is formulated. After comminution, the interface fracture energy takes the role of interface friction, and it is pointed out that if the friction depends on the slip rate, the aforementioned exponents would change. The effect of dynamic comminution can simply be taken into account by introducing the apparent viscosity into the material constitutive model. The theory was inspired by noting that the local kinetic energy of shear strain rate plays a role analogous to the local kinetic energy of eddies in turbulent flow.

1 INTRODUCTION

The previous studies of high-rate dynamic fracture of rocks, concretes, ceramics, composites and metals have dealt mainly with the nucleation, propagation and branching of a dynamically propagating crack, their interference with elastic or shock waves, and the mechanism of development of the zones of densely distributed fractures, called the Mescall zones (Mescall and Weiss 1984, Freund 1990, Doyoyo 2002, Deshpande and Evans 2008, Wei et al. 2009). However, a comminution model in the form of a macroscopic constitutive equation that could be used in large dynamic finite element programs for global response of structures had been lacking until a fundamentally new kind of approach, based on the release of kinetic energy of shearing, was proposed in 2013 (Bažant and Caner 2013). The theory proposed here is partly inspired by analogy with turbulence. In high velocity turbulent flow, the rate at which energy is dissipated increased significantly by viscous shear stress. Similarly, energy dissipation in rapidly deforming confined solids must be enhanced by the release of the kinetic energy of high shear strain rate of fragmenting solid. Another inspiration for the present model is Grady's model for explosion in a hollow sphere (Grady, 1982). In that model, the kinetic energy of volumetric strain is considered to cause comminution. Grady calculated and experimentally justified that the average particle size is given by $\bar{s} = \left(48\Gamma/\rho\dot{\epsilon}_V^2\right)^{1/3}$ where Γ = interface fracture energy, ρ = mass density and $\dot{\epsilon}_V$ = volumetric strain rate. However, application of this equation to impact was not justified theoretically. This paper reviews the new and rigorous theoretical justification of the energy dissipation caused by comminution and discusses its application to fracturing of concretes and rocks. Application to missile impact will appear in detail in Bažant and Caner (2013b).

2 THEORETICAL FORMULATION

2.1 *Main assumptions and analogy with turbulence*

We begin with the analysis of a simple idealized process in which the solid is comminuted to identical particles (Fig. 1). In the plane of maximum shear, we assume a regular hexagonal subdivision because it is space-filling and gives the smallest surface-to-volume ratio (Fig. 1a) and thus requires the minimum energy to form. In the direction normal to the hexagons, we assume the particles to be prismatic.

Consider that, at a certain moment, the strain rate (shown in Fig. 1b as a displacement regarded as infinitesimal) becomes high enough for the kinetic energy of shear strain rate to suffice for creating the fractures and interface slips that separate the particles of as yet unknown size. As that happens, the particles release their local kinetic energy, slip against each other, and regain their original undeformed shape after comminution, while the particle centers conform to the same macroscopic velocity field (Fig. 1c).

Assuming particle symmetry with respect to axes x and y, the simplified drop of kinetic energy

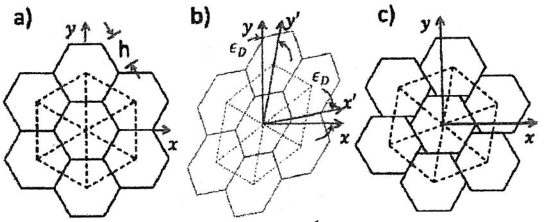

Figure 1. The comminution of material into prismatic hexagonal particles: (a) Undeformed material, (b) sheared material, and (c) comminuted material. The velocities are shown as infinitesimal displacements. The gaps at the hexagon corners are second-order small and thus negligible.

of the hexagonal prisms per unit volume due to comminution is:

$$\Delta K = -\frac{h}{V_p}\int_A \frac{\rho}{2}\left(\dot{u}^2 + \dot{v}^2 - \left(\dot{u}^+\right)^2 - \left(\dot{v}^+\right)^2\right)\mathrm{d}x\,\mathrm{d}y = -c_k \rho h^2 \dot{\epsilon}_D^2 \qquad (1)$$

where A = particle area, ρ = mass density, $c_k = I_p/(2hV_p)$, $V_p = (3\sqrt{3}/8)h^3$ = particle volume, and $I_p = (\sqrt{3}/32)h^4$ = polar moment of inertia of each hexagonal prism of side . It is interesting that the kinetic energy K_{shear} of a particle deforming by pure shear at rate $\dot{\epsilon}_D$ happens to be the same as the kinetic energy K_{eddy} of an eddy rotating as a rigid body of the same size at angular rate $\dot{\omega} = \dot{\epsilon}_D$. In both comminution and turbulence, the micro-level kinetic energy augments the kinetic energy of the macro-level motion.

2.2 *Comminuted particle size distribution*

In this work, particle sizes are considered to vary randomly, according to the following cumulative distribution (Schuhmann 1940, Charles 1957, Cunningham 1987, Ouchterlony 2005):

$$F(s) = \frac{s^k - h^k}{H^k - h^k}; \quad h \le s \le H, \quad 0 \le F(s) \le 1 \qquad (2)$$

where k = empirical constant ($k \approx 0.5$), s = variable particle size; and h, H = minimum and maximum particle sizes (usually H/h = 10 to 100). Since the macroscopic quasi-static constitutive law with non-localized strain softening includes the energy dissipation corresponding to material crushing into particles of the size d_a of the largest material inhomogeneities, H should be considered as one order of magnitude smaller, i.e., about 0.1 d_a. Then, the combined interface area per unit volume can be expressed as $S = C_s/h$. The loss of kinetic energy of the shear strain rate of the particles of all sizes per unit volume is given by $\Delta K = -\int_{s=h}^{H} c_k \rho s^2 \dot{\epsilon}_D^2 \mathrm{d}F(s) = -C_k \rho h^2 \dot{\epsilon}_D^2$ where $\bar{c}$, C_s and C_k are dimensionless constants.

Assuming that all of a kinetic energy decrement K is dissipated by an interface fracture energy increment $\Gamma\Delta S$, the interface fracture (or frictional slip) can occur when:

$$-\frac{\Delta K}{S} = -\frac{\mathrm{d}\Delta K/\mathrm{d}h}{\mathrm{d}S/\mathrm{d}h} = \Gamma \qquad (3)$$

After substitutions into Eq. (3) one gets:

$$h = \left(\frac{C_a \Gamma}{\rho \dot{\epsilon}_D^2}\right)^{1/3} \qquad (4)$$

where C_a is a dimensionless constant.

It is worth noting here that $h \propto \dot{\epsilon}_D^{-2/3}$. This result is in agreement with what Grady (1998) verified empirically (though not theoretically) for the impact of missiles and it serves as one experimental verification of the present theory (Eq. 4).

2.3 *Energy dissipation due to comminution and its implementation in a constitutive law*

Substitution of Eq. (4) into (1) further yields:

$$\Delta K = -\left(C_0 \Gamma^2 \rho\right)^{1/3} \dot{\epsilon}_D^{2/3} \tag{5}$$

where C_0 is a certain dimensionless constant. This expression suggests how to implement the energy sink due to comminution in a constitutive law for a macroscopic structural analysis. Note that ΔK has the dimension of a stress and can be interpreted as such.

To obtain a three-dimensional generalization, it is convenient to introduce an equivalent viscosity η_D such that the viscous stress strain relation $s_{ij} = \eta_D \dot{\epsilon}_{Dij}$ would give the same energy dissipation density as Eq. (5) for any deviatoric strain rate tensor in the variational sense; here s_{ij} is the additional deviatoric stress caused by the comminution.

Since 1) the energy density is the same as the stress, 2) s_{12} must be equal to ΔK when all other tensorial components vanish and 3) ΔK must be a tensorial invariant, it is necessary that $-\Delta K = \sqrt{s_{ij}s_{ij}/2}$. Accordingly, the energy sink due to the comminution process may be modeled by the equivalent viscosity:

$$\eta_D = \left(C_0 \Gamma^2 \rho\right)^{1/3} \dot{\epsilon}_D^{-1/3} \tag{6}$$

where $\dot{\epsilon}_D = \sqrt{\dot{\epsilon}_{Dij}\dot{\epsilon}_{Dij}/2}$.

Viscosity η_D can easily be implemented in the constitutive relation in a finite element program. It may be noted that the enhancement of dissipative viscous resistance to shearing is again a feature analogous to the enhancement of viscous resistance to flow due to turbulent eddies.

Finite element simulations indicate that, in practical applications such as impact, the rate of expansive volumetric strain rate, $\dot{\epsilon}_{Ex}$, plays no significant role. For explosions in a shale rock or in confined concrete, though, both of the rates of shear strain and of volumetric expansion may be important. It can be shown that, in that case, the foregoing theory can easily be generalized; e.g., $\dot{\epsilon}_D^{-1/3}$ in Eq. (6) needs to be replaced by $(\dot{\epsilon}_D^2 + \dot{\epsilon}_V^2)^{-1/6}$.

2.4 *The comminution intensity dimensionless indicator*

In view of the partial analogy with turbulence we introduce a dimensionless indicator of comminution intensity. The strain energy density U that is stored in the material may be expressed as $U = \tau^2/2G$ where G is the elastic shear modulus and τ is the shear stress. When $\Delta K \gg U$, then obviously the comminution cannot be caused by the release of strain energy and the release of kinetic energy is the only possible energy source for the comminution. Therefore, we may define the dimensionless number: $B_a = -\Delta K/U$ or:

$$B_a = \frac{G}{C_g \tau^2}\left(\Gamma^2 \rho \dot{\epsilon}_D^2\right)^{1/3} \tag{7}$$

which has the property that the comminution is:

kinetic energy driven if $B_a \gg 1$,
in transition if $B_a \approx 1$, (8)
absent if $B_a \ll 1$.

The equivalent viscosity may also be uniquely expressed in terms of B_a.

2.5 *Transition formulation and apparent viscosity*

For small shear strain rates, Eq. (6) cannot be applied directly not only because as $\dot{\epsilon}_D \to 0$, $\eta_D \to \infty$, but also at small shear strain rates, comminution should cease to happen. To deduce the form of the transition formula, the viscosity given in Eq. (6) can be expressed in terms of B_a:

$$\eta_D = \frac{\Gamma}{\tau}\left(\frac{C_0^{2/3} G \rho}{C_g B_a}\right)^{1/2} \tag{9}$$

The transitional apparent viscosity must approach to zero for $B_a \ll 1$ and at the other extreme it should approach to Eq. (9) for $B_a \gg 1$. Thus the transitional apparent viscosity can be expressed as

$$\eta_D = \frac{\Gamma}{\tau}\left(\frac{C_0^{2/3} G \rho}{C_g} \frac{B_a^{n-1}}{1 + B_a^n}\right)^{1/2} \tag{10}$$

where n = an empirical constant controlling how sharp the transition is; $n \geq 1$.

3 RESULTS AND DISCUSSION

Comminution takes place in high speed perforation of concrete structures for which several data are available in the literature. One such data (Adley et al. 2012) is shown in Fig. (2), which shows as a function of the thickness of the wall. The entry velocity of the missile is 310 m/s. The values are computed with an explicit dynamic finite element

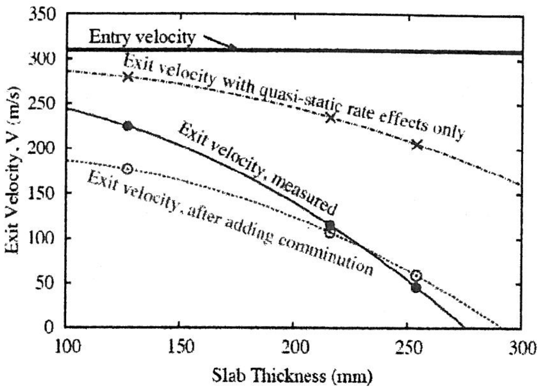

Figure 2. The comparison of measured exit velocities and predictions by model M7 using only quasi-static strain rate effect and both quasi-static strain rate effect and effect of comminution of concrete

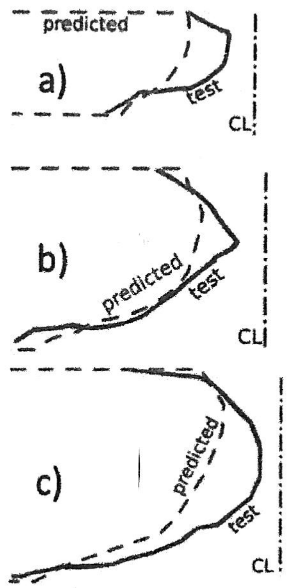

Figure 3. Experimental (solid lines) and predicted (dashed line) crater shapes in the perforation of a) 127 mm, b) 216 mm and c) 254 mm plain concrete walls.

program using the microplane model, first under the assumption that the only rate effects are the quasistatically calibrated rate effects, which consist of viscoelasticity of intact concrete between the cracks and of the rate of bond breakage at the fracture front controlled by activation energy (Bažant and Caner 2013, Caner and Bažant 2013). As seen, this simple assumption leads to a gross overestimation of the exit velocities. However, when the presently formulated equivalent viscosity due to kinetic comminution is included, the data for the two thicker walls are fitted perfectly.

For the thinnest wall, the exit velocity is still overestimated (Fig. 2). However, this is likely explained by differences in the specific moisture contents in the nanopores of concrete. For a lower moisture content, Hopkinson bar experiments have shown a lower strength in high-rate shear, and this is the case for the thinner wall since it dries faster.

In Fig. (3), the experimental and predicted crater shapes are shown. The predictions are for a projectile guided so that it impacts the wall at a right angle and exits the wall at a right angle too, because the experimental crater shapes were available only for that case, although in general the projectiles must be exiting at a different angle than the impact angle. Nevertheless, the predicted crater shapes are in good agreement with the experimental shapes for all three slabs. In obtaining these predicted crater shapes, elements distorted excessively have been eroded, and as the erosion criteria, a max. principle strain of 0.005 and 0.01 have been employed with little difference in the results but significantly longer runtimes when the larger threshold is used.

Another intriguing application of this dynamic comminution theory may be the fracturing of gas or oil shale by electro-hydraulic pulsed arc (Maurel et al. 2010) or by chemical explosion in the pipe of a horizontal borehole. However, since no such data exist in the public domain, the application of the present comminution theory in fracture of shale rock is yet to be carried out.

4 CONCLUSIONS

In this paper, a macroscopic model on the dynamic comminution or fragmentation of rocks, concrete, metals, and ceramics is presented. The main assumption is that the driving force of comminution under high-rate compression is the release of the local kinetic energy of shear strain rate. The present theory indicates that the density of kinetic energy available for comminution is proportional to the (2/3) power of the shear strain rate, the particle size or crack spacing is proportional and the (−2/3) power of that rate, and the energy dissipation by comminution is equivalent to a shear viscosity decreasing as the (−1/3) power of that rate. Confirmation of the theory is provided by fitting the data on both the measured exit velocity of projectiles penetrating concrete walls of different thicknesses and the measured crater shapes in these walls.

The formulation, in principle, can be applied to the fracturing of gas or oil shale by electro-hydraulic pulsed arc or by chemical explosion in the pipe of a horizontal borehole as well.

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REFERENCES

Adley MD, Frank AO & Danielson KT (2012) "The high-rate brittle microplane concrete model: Part I: Bounding curves and quasi-static fit to material property data". *Comput Concr,* 9(4):293–310.

Bažant ZP & Caner FC (2013) "Comminution of solids caused by kinetic energy of high shear strain rate, with implications for impact, shock and shale fracturing." *Proc., National Academy of Sciences* 110 (48), 19291–19294.

Bažant ZP & Caner FC (2013b) "Impact comminution of solids due to local kinetic energy of high shear strain rate: I. Continuum theory and turbulence analogy" *J Mech Phys Solids*, doi: http://dx.doi.org/10.1016/j.jmps.2013.11.008. In press.

Charles RJ (1957) "Energy-size reduction relationships in comminution". *Min Eng* 9:80–88.

Cunningham CVB (1987) "Fragmentation estimation and the Kuz-Ram model—four years" on. *Proceedings of the 2nd International Symposium on Rock Fragmentation* by Blasting, eds Fourney WL, Dick RD (Society for Experimental Mechanics, Bethel, CT), pp 475–487.

Mescall J & Weiss V (1984). "Materials behavior under high stress and ultrahigh loading rates—Part II". *Proceedings of the 29th Sagamore Army Conference* (Army Materials and Mechanics Research Center, Watertown, MA).

Deshpande VS & Evans AG (2008) "Inelastic deformation and energy dissipation in ceramics: A mechanism-based constitutive model". *J Mech Phys Solids* 56:3077–3100.

Doyoyo, M. (2002). "A theory of the densification-induced fragmentation in glasses and ceramics under dynamic compression" *Int J Solids Structures,* 39, 1833–1843.

Caner, F.C. & Bažant Z.P. (2013) "Impact comminution of solids due to local kinetic energy of high shear strain rate: II. Microplane model and verification" *J Mech Phys Solids,* doi: http://dx.doi.org/10.1016/j.jmps.2013.11.009. In press.

Freund LB (1990) "Dynamic Fracture Mechanics" (Cambridge Univ Press, Cambridge, UK).

Grady DE (1982) "Local inertial effects in dynamic fragmentation". *J. Appl. Phys.* 53:322–325.

Grady DE (1998) "Shock-wave compression of brittle solids". *Mech Mater* 29:181–203.

Maurel O, et al. (2010) "Electrohydraulic shock wave generation as a means to increase intrinsic permeability of mortar". *Cement Concr Res* 40:1631–1638.

Ouchterlony F (2005) "The Swebrec function: Linking fragmentation by blasting and crushing". *Mining Technology* 114(March):A29–A44.

Schuhmann R, Jr. (1940) "Principles of comminution, I. Size distribution and surface calculation". *The American Institute of Mining, Metallurgical, and Petroleum Engineers (AIME) Technical Publication* 1189 (AIME, Englewood, CO).

Wei Z, Evans AG & Deshpande VS (2009) "The influence of material properties and confinement on the dynamic penetration of alumina by hard spheres". *J Appl Mech* 76: 051305-1–051305-8.