Creep, Shrinkage and Durability Mechanics of Concrete and other Quasi-Brittle Materials
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Continuous Relaxation Spectrum of Concrete Creep and Its Incorporation into Microplane Model M4

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Efficient numerical finite element analysis of creeping concrete structures requires the use Kelvin or Maxwell chain model, which is most conveniently identified from a continuous retardation or relaxation spectrum, the spectrum in turn being determined from the given compliance or relaxation function. The method of doing that within the context of solidification theory for creep with aging was previously worked out by Bažant and Xi, but only for the case of a continuous retardation spectrum based on Kelvin chain. The present paper is motivated by the need to incorporate concrete creep into the recently published microplane model M4 for nonlinear triaxial behavior of concrete, including tensile fracturing and behavior under compression. In that context, the Maxwell chain is more effective than Kelvin chain, because of the kinematic constraint of the microplanes used in M4. Determination of the continuous relaxation spectrum for Maxwell chain, based on the solidification theory, is outlined and numerical examples are presented.

1. INTRODUCTION

Within the service stress levels, concrete creep approximately follows the principle of superposition. The stress-strain relation has the form of a Volterra integral equation whose kernel is the compliance function of the material. Because the use of an integral equation in structural analysis is computationally inefficient, it is preferable to convert the integral-type creep law to a rate type form based on either the Kelvin or the Maxwell chain model, which can be most conveniently identified from a continuous retardation or relaxation spectrum.

Because Dirichlet expansion of a given compliance function directly leads to the Kelvin chain, the solidification theory was formulated with the retardation spectrum [3]. However, when a nonlinear triaxial constitutive model for concrete such as microplane model M4 is to be generalized for creep, the use of Maxwell chain is more convenient because of the kinematical constraint of the microplanes.

The purpose of this paper is to formulate a continuous relaxation spectrum corresponding to the Maxwell chain model for the solidification theory of aging creep of concrete. An additional purpose is show how the Maxwell chain can be incorporated into microplane model M4.

2. CONTINUOUS RELAXATION SPECTRUM

The Dirichlet series expansion of a nonaging relaxation function leads to the Maxwell chain model. Considering infinitely many Maxwell units with continuously distributed relaxation times, one may express the relaxation function as

$$R(\xi) = \sum_{\mu=1}^{N} E_\mu \exp\left[-\frac{\xi}{\tau_\mu}\right] \quad \Longrightarrow \quad R(\xi) = \int_0^\infty L^*(\tau) \exp\left[-\xi/\tau\right] d\tau \tag{1}$$

where the time lag, $\xi = t - t'$, is the only time variable in nonaging creep, and $L^*(\tau)$ is a continuous distribution of the elastic moduli corresponding to the Maxwell chain with infinitely many units.

In the case of a discrete spectrum, the determination of both τ_μ and E_μ from the test data is and ill-conditioned problem, τ_μ must in that case be properly chosen. A uniform distribution of τ_μ in the logarithmic scale of time is a good choice [2]. The values of E_μ corresponding to the chosen τ_μ may be be determined by minimizing the quadratic norm of the difference between the approximation and the given relaxation relaxation function. Because E_μ depends on the choice and spacing of τ_μ, the spectrum of E_μ as a function of relaxation time is not unique.

Setting $L^*(\tau) = L(\tau)/\tau$ and $\zeta = 1/\tau$, one can regard Eq. (1) as the Laplace transformation of function $\zeta^{-1}L(\zeta^{-1})$ [7].

$$R(\xi) = \int_0^\infty \zeta^{-1} L(\zeta^{-1}) \exp\left[-\xi\zeta\right] d\zeta \tag{2}$$

Therefore the function $L(\tau)$ can be determined by inverse Laplace transformation of Eq. (2). The same inversion procedure as used in the preceding development of retardation spectrum [3,8] is now adopted, although several other techniques to determine $L(\tau)$ exist. The inverse transformation of (2) is

$$L(\tau) = \lim_{k\to\infty} \frac{(-k\tau)^k}{(k-1)!} R^{(k)}(k\tau) \tag{3}$$

Thus, when the relaxation function $R(\xi)$ is given, the kth order approximate spectrum is obtained by using a finite value of k ($k \geq 1$). Same as for the retardation spectrum, the 3rd order $L(\tau)$ is found to suffice for good approximation.

For practical implementation, the continuous spectrum must be approximated by discrete values corresponding to a discrete Maxwell chain model. It must be noted that, if a constant is added to the given relaxation function $R(\xi)$, $L(\tau)$ is not affected by the operation because $k \geq 1$. This means that $L(\tau)$ obtained by the foregoing procedure corresponds to infinitely many relaxation functions. Therefore a constant, E_∞, should be added to (1). The physical meaning of E_∞ is a spring modulus with an infinite relaxation time. This additional constant can be easily determined by the standard least square method.

The moduli of Maxwell units in (1) for chosen relaxation times are unambiguously determined by

$$E_\mu = L(\tau_\mu) \ln 10 \Delta(\log \tau_\mu) \tag{4}$$

$$= 2.303 L(\tau_u) \quad \text{when } \Delta(\log \tau_\mu) = 1.0 \tag{5}$$

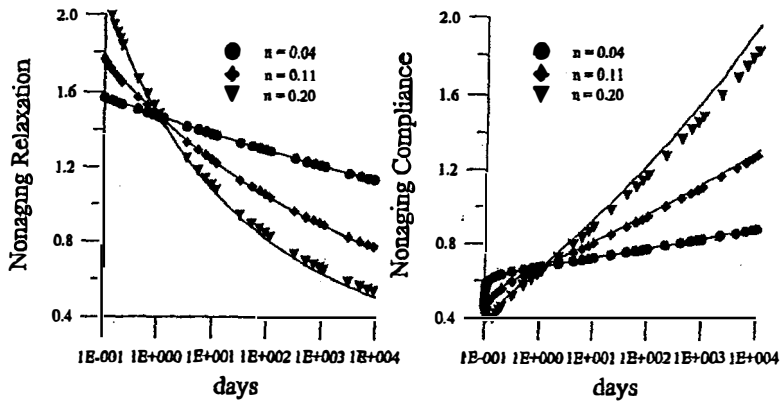

Figure 1. Comparison of snalytical relaxation and compliance functions (symbols) to their Maxwell chain approximation (solid lines) which was determined from the relaxation function (7); (a) nonaging relaxation function and (b) nonaging compliance function.

The E_μ values corresponding given τ_μ spacing are unique, but they of course depend on the spacing of τ_μ while being independent of the choice of τ_1. The final form of approximation by Maxwell chain is

$$R(\xi) = \sum_{\mu=1}^{N} E_\mu \exp\left[-\frac{\xi}{\tau_\mu}\right] + E_\infty \tag{6}$$

3. AMALGAMATION WITH B3 MODEL

A good approximation of a nonaging relaxation function can be obtained from the nonaging compliance function simply by algebraic inversion [1,6,7]. For the case of the log-power creep law [2],

$$R(\xi) = \frac{1}{C(\xi)} = \frac{1}{q_2} \ln^{-1}\left[1 + \left(\frac{\xi}{\lambda_0}\right)^n\right] \tag{7}$$

where q_2, n, and λ_0 are empirical material parameters. The log-power creep law and the corresponding exact and approximate relaxation functions are plotted in Fig. 1. As is expected from the experience with Kelvin chain approximations [3], the approximation is seen to be very good for small n, which is always the case for concrete.

For basic creep, the total strain rate in the B3 model consists of an elastic strain rate, aging viscoelastic strain rate, aging viscous flow rate, and inelastic strain rate. The aging viscoelastic strain rate is obtained by dividing the nonaging viscoelastic strain rate with

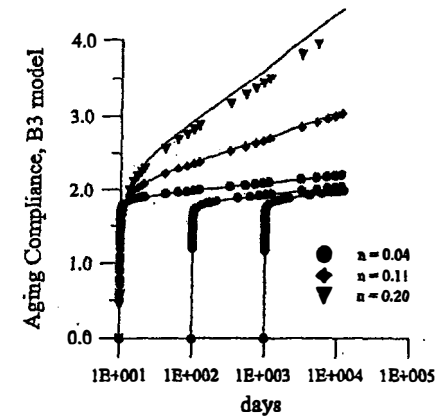

Figure 2. Analytical aging compliance function B3 (symbols) and its Maxwell chain approximation

the current load-bearing volume fraction, v.

$$\dot{\varepsilon} = \dot{\varepsilon}_e + \dot{\varepsilon}_v + \dot{\varepsilon}_f + \dot{\varepsilon}_0 = q_1\dot{\sigma} + \frac{\dot{\gamma}}{v(t)} + \frac{q_4}{\eta(t)}\sigma(t) + \dot{\varepsilon}_0 \tag{8}$$

where $\dot{\varepsilon}_e$ = elastic strain rate, $\dot{\varepsilon}_v$ = aging viscoelastic strain rate, $\dot{\varepsilon}_f$ = flow strain rate, $\dot{\varepsilon}_0$ = inelastic strain rate, q_1 and q_4 = empirical material parameters, and $\sigma(t)$ = the current stress. The nonaging viscoelastic strain rate $\dot{\gamma}(t)$ is approximated by the Maxwell chain, and the approach of exponential algorithm is used to obtain a quasi-elastic incremental approximation of the constitutive law;

$$\Delta\sigma = E''(\Delta\varepsilon - \Delta\varepsilon'') \tag{9}$$

$$E'' = q_1 + \frac{1}{v_{i+1/2}D} \quad \text{and} \quad \Delta\varepsilon'' = \frac{\Delta\gamma''}{v_{i+1/2}} + q_4\frac{\Delta t}{\eta_{i+1/2}} + \Delta\varepsilon_0 \tag{10}$$

where

$$D = \sum_{\mu=1}^{N} E_\mu\lambda_\mu + E_\infty \quad \text{and} \quad \Delta\gamma'' = \left[\sum_{\mu=1}^{N} \sigma_\mu^i \Delta y_\mu \lambda_\mu\right]/D \tag{11}$$

with

$$\Delta y_\mu = \frac{E_\mu}{\eta_\mu}\Delta t = \frac{\Delta t}{\tau_\mu} \quad ; \quad \lambda_\mu = [1 - \exp(-\Delta y_\mu)]/\Delta y_\mu \tag{12}$$

4. INCORPORATION TO MICROPLANE MODEL

In microplane model M4 which has recently been developed in Northwestern University [4,5], the constitutive law is formulated in terms of stress and strain vectors on each microplane. The strain vectors are kinematically constrained to the strain tensor. Because of this constraint, the Maxwell chain is more convenient for implementing creep than the Kelvin chain (of course, if a static constraint were used, the Kelvin chain model would be more convenient). On the each microplane, the strains are decomposed into the volumetric strain, deviatoric strain, and shear strain. The trial stress of each component is computed by uniaxial elastic constitutive law. The Maxwell chain is implemented simply by replacing the elastic constitutive law on the microplane by the viscoelastic constitutive law given in Eq. (9). The increments of the volumetric, deviatoric and shear stress components may be written as

$$\Delta\sigma_V = E'_V(\Delta\varepsilon_V - \Delta\varepsilon''_V)\,,\ \Delta\sigma_D = E'_D(\Delta\varepsilon_D - \Delta\varepsilon''_D)\,,\ \text{and}\ \Delta\sigma_T = E'_T(\Delta\varepsilon_T - \Delta\varepsilon''_T) \qquad (13)$$

5. CONCLUSIONS

(1) For the nonaging constituent in the solidification theory, the Maxwell chain is equally convenient as the Kelvin chain model since the nonaging relaxation function can be easily obtained from the nonaging compliance function, with good accuracy.

(2) The continuous relaxation spectrum of the nonaging constituent in the solidification theory is unique and is easily determined on the basis of the inverse Laplace transform. The modulus of each unit in a discrete Maxwell model is easily and uniquely determined from the continuous relaxation spectrum when its retardation time is chosen. The Maxwell chain model may be generalized for aging according to the solidification theory to yield a complete incremental quasi-elastic constitutive law.

(3) The kinematic constraint in microplane model M4 makes the Maxwell chain more convenient then the Kelvin chain.

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