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COMPRESSION FAILURE IN REINFORCED CONCRETE COLUMNS AND SIZE EFFECT

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Abstract

The size effect in the failure of columns or other reinforced concrete compression members is explained by energy release due to transverse propagation of a band of axial splitting cracks. According to their stress and strain states, three regions are distinguished in the column: cracking, unloading and invariable zones. The microslabs of the material between the axial splitting cracks are considered to buckle and undergo post-critical deflections. Based on the equality of the energy released from these regions and the energy consumed by formation of the axial splitting cracks in the band, the failure condition is formulated. It leads to a closed-form expression relating the characteristic size and the nominal strength of the structure. The results of laboratory tests of reduced-scale columns reported previously by Bažant and Kwon (1994) are analyzed according to the proposed formulation and are shown to be described by the proposed theory quite well. Although the present theory is formulated for a reinforced concrete column, it can be adapted to compression failures of other quasibrittle materials such as rocks, ice, ceramics or composites.

Introduction

Compression failure of quasibrittle materials has been studied extensively and important results have been achieved (Biot, 1965; Ashby and Hallam, 1986; Batto and Schulson, 1993; Horii and Nemat-Nasser, 1985, 1986; Kendall, 1978; Bažant,-1967; Bažant et al., 1991; Sammis and Ashby, 1986; Shetty et al., 1986). However, attention has been focused primarily on the microscopic

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mechanisms that initiate the compression failure rather than on the final global mode of failure and the size effect. Structures made of quasibrittle materials, such as concrete, rock, ice, ceramics and composites, are known to exhibit a significant size effect (Bažant, 1993a). The nominal strength of structure is not constant, as predicted for materials following yield or strength failure criteria, but decreases with an increasing size of structure. Previous studies have demonstrated that such a size effect exists in various tensile and shear failures, including the diagonal shear failure of reinforced concrete beams, torsional failure, punching shear failure, pullout of bars and anchors, etc.

The size effect on nominal strength of structure is due to the release of energy due to propagation of fracture or damage bands. Such a size effect must also be expected for compression failures in which the energy release depends on the structure size, for example, reinforced concrete columns. A general analysis of the size effect for compression failures has already been outlined in general terms (Bažant, 1993b). However, detailed formulas for the size effect have not been derived and comparisons with test results have not been made. This paper presents a more detailed theoretical analysis of the size effect in concrete columns and comparison with the test results. In full detail, the analysis of size effect in compression members will be presented in a forthcoming journal article.

Analysis of Energy Release

Consider a prismatic compression member (a column) loaded by axial compressive force P with eccentricity e (shown in Fig. 1). The column has length L, width D (taken as the characteristic dimension), and unit thickness $b = 1$. If one end cross section is subjected to axial displacement u and rotation θ and the other is fixed, linear elastic solution indicates that the initial normal stress in the cross sections before fracturing is

$$\sigma_0(x) = -\frac{E}{L}\left[u + \theta\left(\frac{D}{2} - x\right)\right], \tag{1}$$

in which E = Young's elastic modulus, and x = transverse coordinate measured from the compressed face (Fig. 1a).

Assume now that, at a certain moment of loading, axial cracks of spacing s and length h, forming a band as shown in Fig. 1a,b,c, suddenly appear, and that the microslabs of the material between the axial cracks, behaving as beams of depth s and length h, lose stability and buckle. This kind of buckling can happen in any of the three configurations shown in Fig. 1a,b,c. For all of them, the present type of approximate solution turns out to be identical if the length of the cracks in the pair of inclined bands in Fig. 1c is denoted as $h/2$. During the buckling of the microslabs, certain zones in the column undergo unloading. For the sake of simplified analysis, we assume that the column can be divided

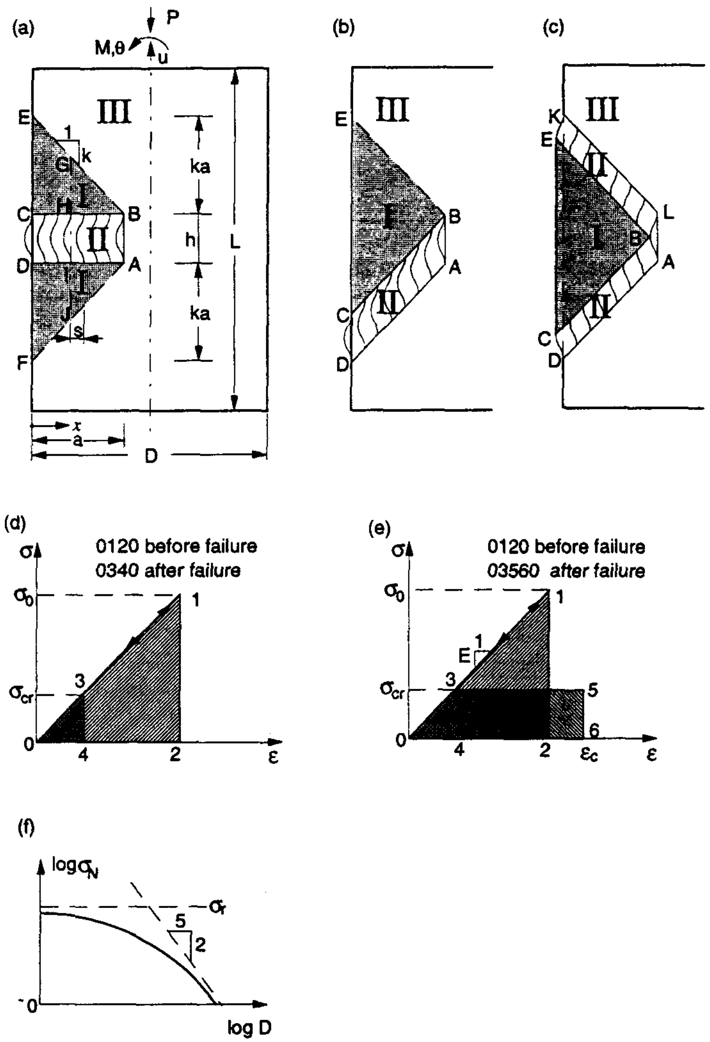

Figure 1: (a-c) Compression splitting cracks, (d,e) energy release, (f) size effect

into three regions (Fig.1): Region I—the elastic unloading region, shown as the shaded triangular area; region II—the slab buckling band, in which the microslabs are undergo post-critical deflections; region III—the region with no cracking nor unloading.

The key idea for failure analysis of the column is to calculate the change of stored strain energy caused by buckling (Bažant, 1993b). For region III, this calculation is not needed because no energy loss takes place. We need to calculate the energy losses only in the first two regions. The critical stress for buckling of the microslabs is

$$\sigma_{cr} = -\frac{\pi^2 E s^2}{3h^2}. \tag{2}$$

Region I is bounded by the so-called "stress diffusion lines" of slope k, whose magnitude is close to 1. The precise value of k could be obtained from the full elastic solution, but is not needed for the present purposes. For the analysis of size effect the important fact is that k is a constant if geometrically similar columns are considered. Under the assumption that the stress in the shaded triangular stress-relief zones is reduced all the way from the initial stress $\sigma_0(x)$ to σ_{cr}, the strain energy density before and after fracture is given by the areas of the triangles 0120 and 0340 in Fig. 1d. So the loss of strain energy density along a vertical line of horizontal coordinate x (Fig. 1a) is

$$\Delta\bar{\Pi}_r = \frac{\sigma_0^2(x)}{2E} - \frac{\sigma_{cr}^2}{2E}. \tag{3}$$

The situation is more complicated for the crack bands. The microslabs buckle, and the energy associated with the postbuckling behavior must be taken into account. Before the buckling of microslabs, the strain energy density is given by the area 0120 in Fig. 1a,e. After the buckling, the stored elastic strain energy is the area 03560. The analysis of postbuckling behavior of columns (Bažant and Cedolin, 1991, Sec. 1.9 and 5.9) indicates that the stress in the axis of the microslabs follows, after the attainment of the critical load, the straight line 35 which has a very small positive slope (precisely equal to $\sigma_{cr}/2$), compared to the slope E before buckling, and can therefore be neglected. Thus, line 35 can be considered to be horizontal. The triangular area 0340 in Fig. 1e represents the axial strain energy density of the microslabs, and the rectangular area 35643 represents the bending energy density. Because the microslabs remain elastic during buckling, the stress-strain diagram 035 is fully reversible and the energy under this diagram is the stored elastic strain energy. From the foregoing analysis, the change in strain energy density in Region II is the difference of areas 0120 and 03560 in Fig. 1e, that is,

$$\Delta\bar{\Pi}_c = \frac{\sigma_0^2(x)}{2E} - \left[\sigma_{cr}\epsilon_c(x) - \frac{\sigma_{cr}^2}{2E}\right] \tag{4}$$

where $\epsilon_c(x)$ is the axial strain of the microslabs in the crack band after buckling. It is important to note that $\epsilon_c(x)$ is generally not equal to 04 nor 02 in Fig. 1e, but can be determined from the compatibility condition. The stress in region III of the column, as an approximation, may be assumed to be unaffected by introduction of the crack and thus constant during failure, and so region III behaves as a rigid body. Consequently, the line segment GJ in Fig. 1a at any x does not change length. Expressing the change of length of this segment on the basis of σ_{cr}, ϵ_c and $\sigma_0(x)$, and setting this change equal to zero, one obtains

$$\epsilon_c(x) = \frac{\sigma_0(x)}{Eh}\left[h + 2k(a-x)\right] - \frac{2k}{h}(a-x)\frac{\sigma_{cr}(x)}{E} \tag{5}$$

Integrating (3) and (4) over their specific area yields the total loss of potential energy at constant u and θ:

$$\Delta\Pi = \int_0^a \left(\frac{\sigma_0^2(x)}{2E} - \frac{\sigma_{cr}^2}{2E}\right) 2k(a-x)dx + \int_0^a \left\{\frac{\sigma_0^2(x)}{2E}\left[\sigma_{cr}\epsilon_c(x) - \frac{\sigma_{cr}^2}{2E}\right]\right\} h dx \tag{6}$$

where a = horizontal length of the crack band (Fig. 1a,b,c). When the column is failing, $\Delta\Pi$ must be equal to the energy consumed by the formation of the surfaces of all the axial splitting cracks. Assuming that there is no other energy dissipation but fracturing, we may write the energy balance criterion of fracture mechanics as:

$$-\left[\frac{\partial\Delta\Pi}{\partial a}\right]_{u,\theta} = \frac{\partial}{\partial a}\left(G_f h \frac{a}{s}\right) = \frac{h}{s} G_f \tag{7}$$

where G_f is the fracture energy of the axial splitting cracks, assumed to be a material property.

It can be shown (Bažant, 1993b) that the foregoing equations yield a failure criterion of the form

$$F(k, a, s, h, G_f, \sigma_N) = 0, \qquad \sigma_N = \frac{P}{bD}\left(1 + \frac{6e}{D}\right) \qquad (b = 1) \tag{8}$$

in which P = maximum load, and σ_N = nominal strength of the compression member. The value of the diffusion slope k can be approximately estimated as the k−value which gives the exact energy release rate for an edge-cracked tensile fracture specimen according to linear elastic fracture mechanics. The unknown spacing of the axial cracks, s, can be determined from the condition that load P be minimized, which requires that $\partial(\delta F)/\partial(\delta s) = 0$. It can further be shown that this condition indicates s to increase with size D as $D^{-1/5}$.

In the case of slender columns which undergo global buckling, the effect of slenderness on σ_N needs to be taken into account. This can be done by using

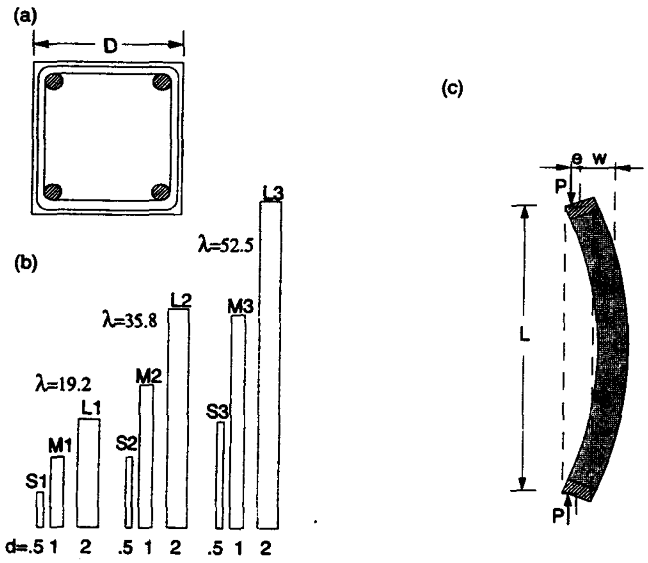

Figure 2: Reduced scale columns tested by Bažant and Kwon (1994)

one of the two concepts proposed in Bažant (1993; Eqs. 45–50). The simpler of these two concepts is based on modifying eccentricity e in Eq. 8 on the basis of the magnification factor for global buckling of columns (e.g., Bažant and Cedolin, 1991, Chapter 1).

Assuming the ratio a/D for failures of compression members of various sizes to be the same, the following relationship between the characteristic dimension D and the nominal strength σ_N can be deduced from the foregoing formulation.

$$D = k\frac{(\sigma_0 - \sigma_{cr} + \sigma_N)(\sigma_0 + \sigma_{cr} - \sigma_N)}{(\sigma_N - \sigma_{cr})}, \qquad \sigma_N = \frac{P}{bD} \quad (b = 1) \tag{9}$$

in which σ_0, σ_{cr} = constants = critical normal compressive stress for the microslab buckling, which is the same as the intrinsic compression strength of the material.

Comparison to Test Results

The present formulation has been compared and calibrated with the test data for failure of reduced-scale reinforced concrete columns (Fig. 2) reported in

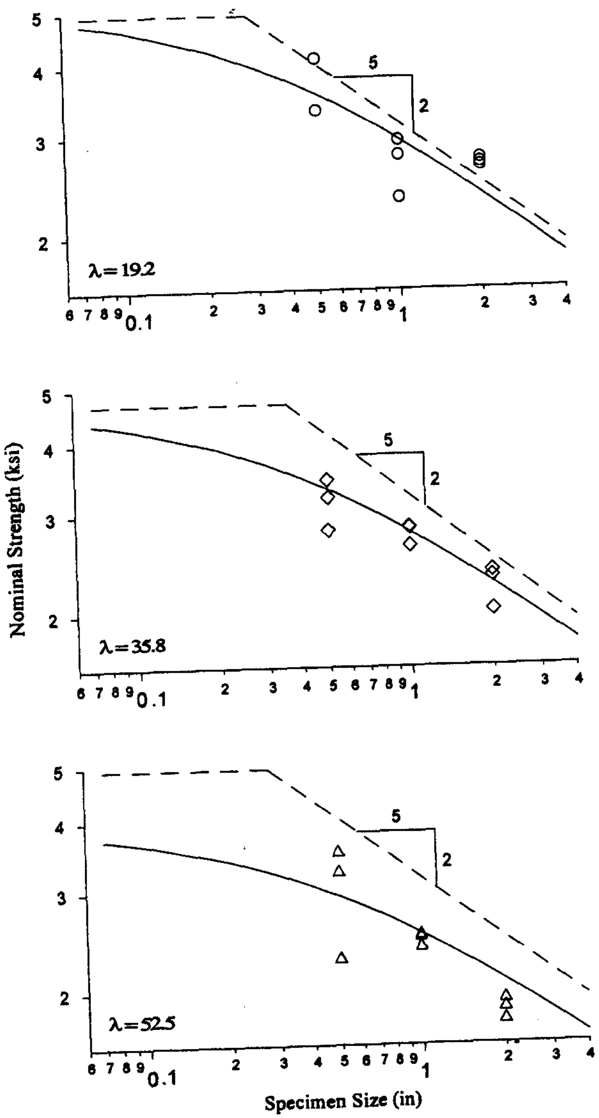

Figure 3: Size effect from experiment and from theory

Bažant and Kwon (1994). Fig. 3 shows the comparison between the experimental data and the theoretical results from Eq. 8. The data points are the experimental results for various column slendernesses $\lambda = \ell/r$; r = radius of gyration of the cross section and ℓ = effective length of the column. The predictions according to the present theory are indicated by the curves. The horizontal asymptote of the size effect curve according to the strength theory and the inclined asymptote according to linear elastic fracture mechanics (which has the slope $-2/5$) are also marked in the figures. As one can see, reasonable agreement with the test results has been achieved.

It should be noted that the existing code procedures for concrete structures give no size effect, which is contradicted by the present test results.

Conclusion

The mechanism of compression failure of quasibrittle materials can be described as transverse propagation of a band of axial microcracks. Assuming the axial stress transmitted by the band to be limited by buckling of the microslabs of the material between the axial splitting racks, the failure loads can be calculated on the basis of the energy release. This calculation predicts a size effect which is in reasonable agreement with available reduced-scale laboratory tests.

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