

## SAFE SHEAR DESIGN OF LARGE, WIDE BEAMS

It is very welcome that the authors Adam Lubell, Ted Sherwood, Evan Bentz, and Michael P. Collins of "Safe Shear Design of Large, Wide Beams" (Jan. 2004, pp. 67-78) try to take into account the size effect. However, the proposed reliance on stirrups is still unsafe and, in other words, may lead to safety margins much lower than for normal size beams. Although stirrups do mitigate the size effect in beam shear, they do not eliminate it;<sup>1,3</sup> refer to Fig. 1, in which the data points showing the average shear stress carried by concrete  $v_c = V_c/b_w d$  are plotted from the ACI small-beam database by which the current code formula  $v_c = 2\sqrt{f'_c}$  was justified ( $f'_c$  = specified compression strength of concrete;  $V_c$  = shear force carried by concrete;  $b_w$  = beam width; and  $d$  = beam depth). The Gaussian distribution fitted to the  $v_c$  values from the database is also shown. The mean of the database is the horizontal line at  $v_c = 3.1\sqrt{f'_{cr}}$  where  $f'_{cr}$  = average compressive strength of concrete from tests. The design requirement was set at the margin of the data cloud, at  $v_c = 2\sqrt{f'_{cr}}$ . But ACI requires the design to be based on the specified compressive strength  $f'_c$ , which is in this case related to  $f'_{cr}$  as  $f'_c \approx 0.75f'_{cr}$ . So the design formula lies (as shown in Fig. 1) at  $v_c = 2\sqrt{f'_c} = 2\sqrt{0.75f'_{cr}} = (2/3.1)\sqrt{0.75}(3.1\sqrt{f'_{cr}}) = 54\%$  of the mean  $v_c$  of the ACI small-beam database.

Obviously, for the large beams, the safety margin must be at least the same as for the small ones (if not larger). Any tests of large beams should give, on the average, at least  $v_c = 3.1\sqrt{f'_{cr}}$ . However, the Toronto large-beam tests without and with stirrups, shown in Fig. 1(a), gave  $1.02\sqrt{f'_{cr}}$  for no stirrups and  $1.84\sqrt{f'_{cr}}$  for the proposed minimum stirrups. This is, respectively, only 33 and 59% of the mean of the database. These reduction percentages represent a strong size effect—not only for beams without stirrups, but also for beams with stirrups. It is statistically incorrect to ignore scatter and assume that a test result for  $v_c$  exceeding  $2\sqrt{f'_c}$  is a proof of safety.

There now exist realistic commercial finite element codes based on the nonlocal concept of cracking damage, such as ATENA, SBETA, and DIANA. Figure 1(b) shows ATENA predictions of  $v_c$  for beams with stirrups, scaled up and down from the Toronto test shown in Fig. 1(a). Although size effect data for large beams with stirrups are still lacking, the ATENA results confirm again that, although the size effect for beams with stirrups is weaker, it still exists and is significant. The formulas derived by simplified fracture analysis<sup>3</sup> support this fact.

Based on the size effect theory,<sup>4,5</sup> experimental database,<sup>1,2</sup> and computer simulations,<sup>1,5</sup> we have proposed the following revision of ACI 318 Article 11.5.5.2:

"The factored shear force  $V_u$  must not exceed  $\phi(V_c + V_s)$  where  $V_s$  is the yield shear force carried by shear

reinforcement, and the shear force  $V_c$  carried by concrete is given by the smaller of the following two expressions:  $V_c = 2\sqrt{f'_c} b_w d$  and  $6.61\rho^{1/3}\sqrt{f'_c} db_w$  (where  $\rho = 100 \times$  steel ratio;  $V_c$  - lb;  $f'_c$  - psi;  $b_w, d$  - in.). Alternatively, one may use the smooth formula  $V_c = 3.8\sqrt{f'_c}(1 + d/d_0)^{-1/2} b_w d$  with  $d_0 = 1120(\rho/f'_c)^{2/3}$  (in the case of reliability analysis, it may be assumed that the formula with factor 2 corresponds to a 1% cutoff of the Gaussian distribution based on data regression errors with a coefficient of variation [COV] of 21.5%, the formula with factor 6.61 to a 5% cutoff with a COV of 21.5%, and the smooth formula with factor 3.8 to a 5% cutoff with a COV of 16.2%). These formulas for  $v_c$  apply whether or not there is shear reinforcement."

This code proposal allows a closer prediction of the selected test data in the authors' Table 1, but the improvement is only minor (the COV of errors being 14.0% instead of 14.7%) because the size range is insufficient to distinguish among different theories.

The design of the Bahen Center transfer beam in the article (Fig. 1, bottom left) happens to satisfy the foregoing code proposal (which is documented by the fact that  $V_u$  in that figure far exceeds  $V_u$ ). The reason is that the stirrups are about twice as heavy as required. If the required stirrups were used, the design according to the authors' proposal would be unsafe.

ATENA simulations, however, reveal that the design in the authors' Fig. 1 is unsafe for a different reason: For such a large beam with very heavy stirrups, concrete fails at small stirrup stresses, much before the plastic yield capacity of stirrups can be mobilized, and the failure load is only  $V_u \approx 1440$  kip (6400 kN), which is less than the required force  $V_u$  due to factored loads (to calculate this value, the fracture energy and tensile strength of the concrete had to be estimated because they were not reported). So we run into a problem different from the size effect on  $v_c$ —the contribution of stirrups to shear strength of very large beams can be much less than  $A_v f_y d/s$ . This problem will need to be addressed in the code also.

The crack spacing theory of size effect based on MCFT, expounded in the authors' Eq. (2) to (5), can be scientifically disproved by three arguments:<sup>1</sup> 1) At maximum load, the stresses transmitted across the inclined shear crack are negligible, and the shear force  $V_c$  is transmitted almost totally by compressive stresses parallel to this crack (this fact, conforming to strut-and-tie models and predicted by the fracturing truss model,<sup>3</sup> is verified by ATENA simulations);<sup>1</sup> 2) The crack spacing  $s$  is only partly related to size effect—although, like size effect,  $s$  is determined by the stability of energy release, it also depends on other extraneous factors;<sup>6</sup> and 3) The large-size asymptote of Eq. (3) or (5) is  $v_c \alpha d^{-1}$ , while the strongest thermodynamically admissible size effect is  $v_c \alpha d^{-1/2}$  (dictated by the maximum possible energy flux

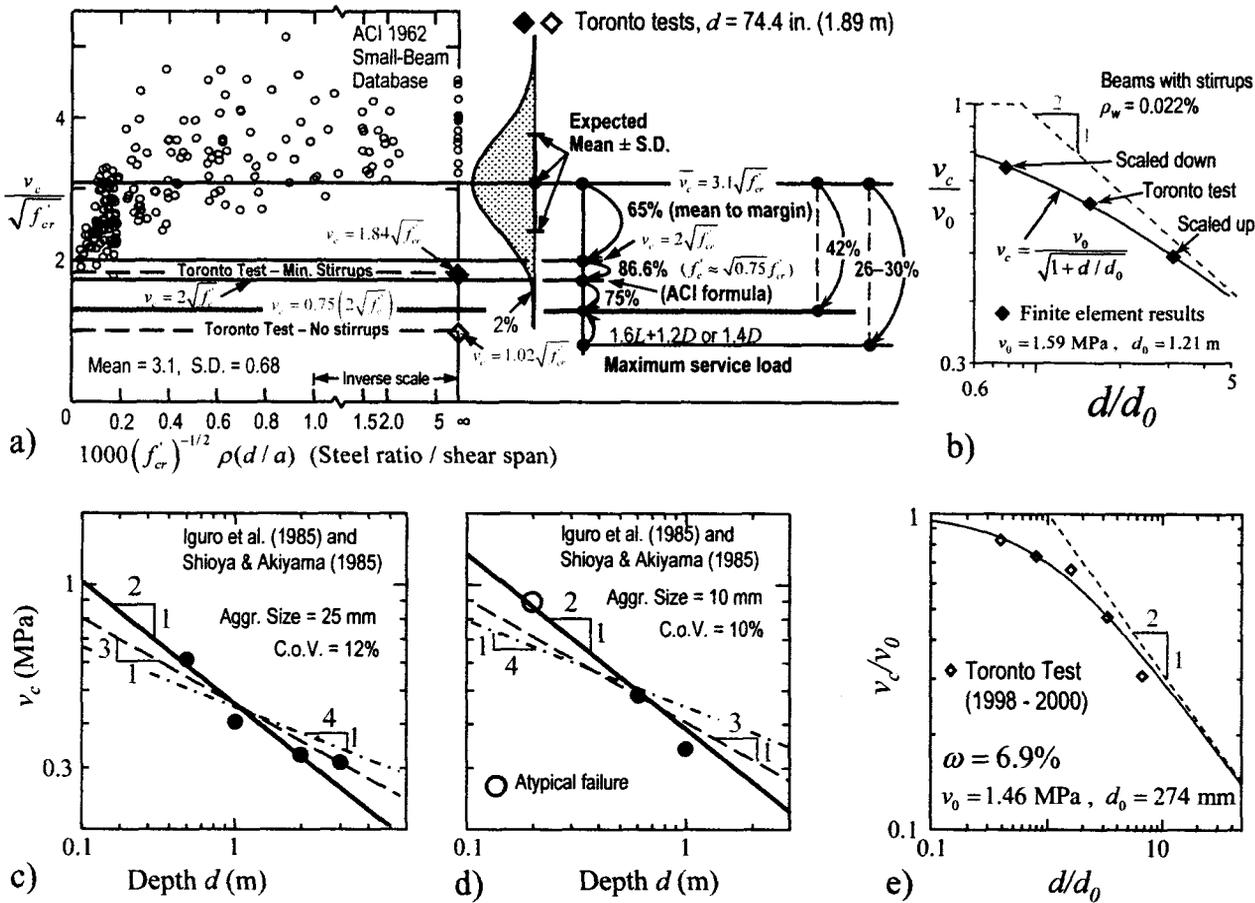


Fig. 1: (a) Comparison with ACI database; (b) FEM simulation of size effect; (c), (d), and (e) large beam test data

into the propagating fracture front) is  $-1/2$  (the asymptotic trend is not verifiable by the combined ACI 445 database because of large scatter due to mixing of dissimilar beams and different concretes and because of prevalence of tests in which contaminating factors other than the size were varied).

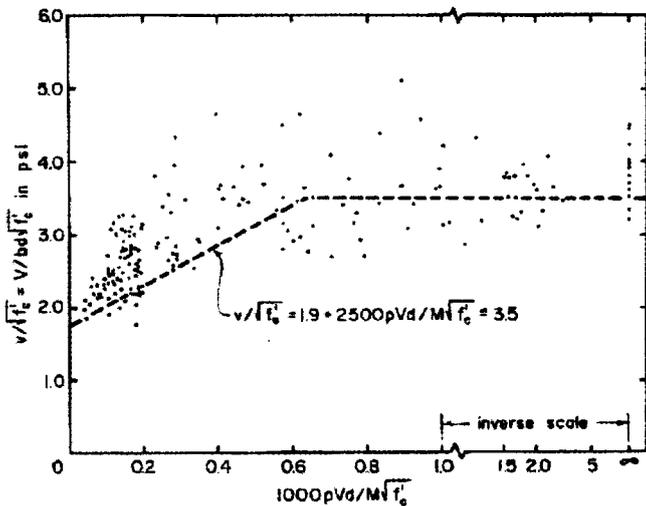
In their Fig. 5, the authors plot the previous Japanese tests of large beams without stirrups in a linear scale of size  $d$ . But such plots are known to be misleading. They obscure the large-size asymptotic trend, which is important for extrapolation to larger sizes of primary practical concern. These data and the Toronto data from tests without stirrups are replotted in Fig. 1(c) to (e) as  $\log v_c$  versus  $\log d$  (as is standard in size effect theory). This demonstrates that the data agree well with the terminal slope of  $-1/2$  predicted by the energetic size effect theory,<sup>4,5</sup> which corresponds to  $v_c \propto d^{-1/2}$  and contradicts the terminal slope of  $-1$  for the crack spacing theory. Correctness of the terminal slope is essential because the code provisions must inevitably rely on theoretically

based extrapolation; 86% of the 398 test data points in the ACI 445 database pertain to beam depths less than 20 in. (0.5 m), 99% less than 43 in. (1.1 m), and 100% less than 79 in. (1.89 m), while the size effect is of practical concern mainly for the range from 40 to 400 in. (1 to 10 m).

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## References

1. Bažant, Z. P., and Yu, Q., "Designing Against Size Effect on Shear Strength of Reinforced Concrete Beams without Stirrups," *Structural Engineering Report 03-02/A466s*, Northwestern University, Evanston, IL, 2003 (circulated to ACI Committee 445, submitted for publication), 40 pp.
2. Bažant, Z. P.; Yu, Q.; Gerstle, W.; and Hanson, J., "Justification of Proposed Update of ACI Code Provisions for Shear Design of Reinforced Concrete Beams," ACI Subcommittee 446B Report to ACI Committee 446, 2004, 36 pp.



**Fig. 2: Derivation of basic ACI shear strength expression**

3. Bažant, Z. P., "Fracturing Truss Model: Size Effect in Shear Failure of Reinforced Concrete," *Journal of Engineering Mechanics*, ASCE, V. 123, No. 12, 1997, pp. 1276-1288.  
 4. Bažant, Z. P., *Size Effect on Structural Strength*, Hermes Science Publications, London, 2002, 294 pp.  
 5. Bažant, Z. P., and Planas, J., *Fracture and Size Effect in Concrete and Other Quasibrittle Materials*, CRC Press, Boca Raton, FL, 1998, 640 pp.  
 6. Bažant, Z. P., and Cedolin, L., *Stability of Structures*, Sec. 12.5, Oxford University Press, New York, 1991, 1014 pp.

### Authors' response

The authors thank Bažant and Yu for their interest in the article and for their strong advocacy of the need to account for the size effect in evaluating the shear strength of large reinforced concrete members. They bring up a number of interesting points that deserve further explanation.

The discussers believe that a large beam with stirrups, such as that shown in Fig. 15 of the article, may have a much lower safety margin than that for "normal size beams." Based on their statistical interpretation of the ACI 1962 "small beam database" shown in their Fig. 1(a), they argue that tests on large beams should give an average  $v_c$  of at least  $3.1\sqrt{f'_c}$ . They conclude that as the Toronto large beam with stirrups had a  $v_c$  of  $1.84\sqrt{f'_c}$ , there was still a "strong size effect."

The original 1962 figure used to derive what the ACI 318-02 commentary calls "the basic expression for shear strength of members without shear reinforcement" is shown in Fig. 2. While this figure is similar to the discussers' Fig. 1(a), it is important to note that the "basic expression" is not  $v_c = 2\sqrt{f'_c}$  but is the more detailed equation shown in Fig. 2 here and now given in the code as Eq. (11-5). For

beams made from normal-strength concrete with a shear span-to-depth ratio of 3 and with about 1% of longitudinal reinforcement, this more detailed equation predicts that  $v_c$  will be about  $2.1\sqrt{f'_c}$ . For the 194 tests in the 1962 figure, the average ratio of observed to calculated shear strength was 1.097 with a coefficient of variation of 15.1%. Hence, the average "small beam" shear strength that we would expect for the type of beams discussed in the article would be  $1.097 \times 2.1\sqrt{f'_c} = 2.3\sqrt{f'_c}$  on average, not  $3.1\sqrt{f'_c}$  as suggested by the discussers. In this regard it is interesting to note that the average shear strength of the seven beams with depths less than 12 in. (300 mm) listed in Table 1 of the article was  $2.26\sqrt{f'_c}$ .

Figure 15 of the article demonstrated that providing just minimum stirrups in a large member can increase the shear capacity by a factor of nearly 3. As was explained in the article, the member with stirrups reached its flexural capacity prior to a final shear failure occurring. If the member had been constructed with longitudinal steel with a higher yield strength, it would have reached an even higher shear strength. Given that when this beam failed the shear near the support was 94% of the nominal shear capacity predicted by the ACI code, the test demonstrates that if there was a size effect for this very large member with minimum stirrups, it was not significant. In this regard, it is interesting that the "smooth" design equation proposed by the discussers would predict that the beam with stirrups shown in Fig. 15 would fail at an applied load of 160 kip (710 kN), which is only 55% of the observed failure load.

In the article, two possible shear designs for the large transfer beams of the new engineering building at the University of Toronto were discussed. One beam had stirrups and was the original design that now holds up the building. The other wider beam without stirrups was an alternative that would be permitted by the ACI Code. It is not correct that the built beam had stirrups "twice as heavy as required." While the nominal shear capacity ( $V_n = 2240$  kip [9965 kN]) is significantly higher than the factored shear force ( $V_u = 1540$  kip [6850 kN]), the factored shear resistance is  $\phi V_n = 0.75 \times 2240 = 1680$  kip (7470 kN), which is only 9% in excess of that required. Furthermore, the amount of stirrups in this beam is not "very heavy." The amount of stirrups provided is only 39% of the maximum amount permitted by the ACI Code. Our analytical models predict that these stirrups will begin to yield at a shear force of about 1800 kip (8000 kN) and that a flexural failure at midspan will occur when the shear reaches 2020 kip (8985 kN). Hence, we do not agree that the actual Bahen Center transfer beams are unsafe.

The final comments of the discussers question the theoretical and behavioral validity of a model based on aggregate interlock stresses. They note that direct tensile

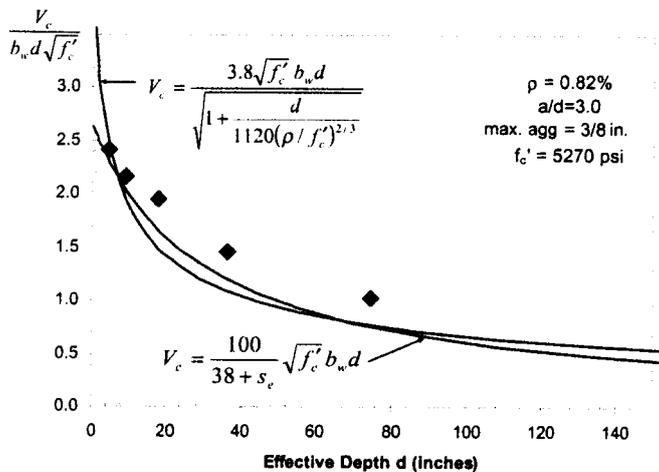


Fig. 3: Comparison of two theoretically based size effect proposals with Toronto test data. Note: 1 in. = 25.4 mm, 1 psi = 6.89 kPa

stresses across the diagonal crack are negligible at ultimate load. This is not disputed but, of course, shear stresses on the crack are not perpendicular to the crack but parallel to it.

Extensive studies in the past<sup>1,2</sup> clearly indicate that the majority of shear in cracked reinforced concrete members without stirrups is transferred by aggregate interlock in this fashion. Note that shear on the crack is apparently not a softening phenomenon and localization is unlikely as the entire shear crack must slide simultaneously to cause a shear failure. Thus, unlike the case of plain concrete, it is not clear whether the principles of fracture mechanics apply to the case of shear in reinforced concrete.

The "extraneous factors" that influence crack spacing but should not influence size effect according to fracture mechanics presumably include the presence of longitudinal reinforcing bars near mid-depth of the beam. As shown in Fig. 7 of the article, such bars control crack spacing and, hence, substantially mitigate the size effect. Another such crack spacing factor, which largely mitigates size effect, is the presence of stirrups.

While the authors do not agree with the discussers with respect to size effect for members containing stirrups, we are in agreement that for members without stirrups there is a significant size effect. Figure 3 compares the size effect proposals from the original article and from the discussion with the Toronto size effect data selected by the discussers. The Bažant and Yu proposal is based on fracture mechanics principles while the Lubell et al. proposal is based upon the modified compression field theory.

It is interesting that within the practical range of member depths, the two proposals are in very close agreement and both predict the trend of the experimental

results very well. These two experimentally verified theoretical models both indicate that wide beams or one-way slabs with an effective depth of, say, 30 in. (760 mm) may fail at a shear stress less than 65% of the nominal shear capacity given by the ACI Code. For a member 48 in. (1220 mm) deep, there is a possibility of failure at a shear stress of only 50% of the ACI Code value. The neglect of this size effect in the current ACI Code may pose a threat to public safety.

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### References

1. Fenwick, R. C., and Paulay, T., "Mechanisms of Shear Resistance of Concrete Beams," *Journal of the Structural Division*, ASCE, V. 94, No. ST10, Oct. 1968, pp. 2235-2250.
2. Taylor, H. P. J., "Investigation of Forces Carried Across Cracks in Reinforced Concrete Beams in Shear by Interlock of Aggregate," TRA 42.447, Cement and Concrete Association, London, 1970, 22 pp.

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