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**A Simple Method for Determining Material Fracture Parameters from Peak Loads.** Paper by Tianxi Tang, Chengsheng Ouyang, and Surendra P. Shah

Discussion by Zdenek P. Bazant

**Note:** The Authors' Closure was inadvertently omitted from the following Discussion in the January-February *ACI Materials Journal*. We reprint here the Discussion and Authors' Closure in their entirety.

**By ZDENEK P. BAZANT**

*Fellow American Concrete Institute, Walter P. Murphy Professor of Civil Engineering and Materials Science, Northwestern University, Evanston, Ill.*

The cylindrical specimen with a central slit proposed by the authors is a valuable addition to the family of fracture specimens for concrete.

The determination of material fracture parameters from measured peak loads of different specimens as proposed by the authors is a considerably simpler approach than the compliance method previously used for the two-parameter model. However, it might be of interest to mention that the proposed concept is related to the generalized size effect method proposed by Bazant and Kazemi<sup>2</sup> that is used in the RILEM recommendation.<sup>8</sup>

In view of the authors' comment, it should be noted that neither the formulation of Bazant and Kazemi nor the RILEM recommendation requires the test specimens to be geometrically similar. The RILEM recommendation states that the specimens "should be geometrically similar," as the authors say, but obviously this is only recommended from the viewpoint of statistical errors. The size effect and shape effect are known only approximately and thus the variation of both the size and shape introduces a greater random scatter than does the variation of the size only. As stated in the last paragraph on p. 119 of the paper by Bazant and Kazemi, the so-called size effect method, based on the size effect law, is applicable also for a set of specimens of different geometries and in particular different notch lengths.

This fact is of interest because there is a one-to-one relationship between the parameters of the two-parameter model and those of the generalized size effect law presented by Bazant and Kazemi. As presented in Bazant, Gettu, and Kazemi (1991), Eq. (11), as well as in Eq. (19) of the RILEM recommendation, the critical crack tip opening displacement is obtained as

$$CTOD_c = (32G_f c_f / \tau E_c)^{1/2} \quad (1)$$

in which  $G_f$  = fracture energy,  $E_c$  = Young's modulus of concrete, and  $c_f$  = effective length of the fracture process zone which is one of the two parameters in the generalized size effect law). In view of this relationship, the procedure proposed by the authors is equivalent to the identification of the

fracture parameters from the maximum load according to the generalized size effect law. This equivalence is discussed in more detail in Bazant (1996).

Another point inviting further comment is the statistical approach. The authors consider only the statistical variation of  $CTOD_c$  but disregard the statistical variation of fracture toughness  $K_{Ic}^{IC}$ . However, both parameters exhibit significant random scatter. The random scatter of the fracture toughness is smaller, but is still quite significant. It is about the same as the scatter of the tensile strength of concrete. Properly, the statistical regression should be conducted considering both parameters as statistical variables. This approach might be inconvenient in conjunction with the two-parameter model; however, it is quite simple with the generalized size effect law. For that law, the statistical evaluation of the test data can be reduced to a linear regression yielding two statistical variables: the slope of the regression line and the coordinate of the centroid of the data points. From their statistical characteristics, the statistics of the material fracture parameters, including the fracture energy, fracture toughness, and  $CTOD_c$ , can be easily evaluated.

For practical purposes, it is important for users to be aware of the uncertainty of the fracture parameters that they use. Unless a method of statistical analysis of test results with two random material parameters is formulated for the two-parameter fracture model, it thus appears that the statistical regression of the authors' data on the basis of the generalized size effect law might be preferable (and would not be any more complicated). The results could then be easily converted to the parameters of the two-parameter model and provide their coefficients of variation.

**REFERENCES**

- Bazant (1996), "Size Effect Aspects of Measurement of Fracture Characteristics of Quasibrittle Material," *Proceedings, Second International Conference on Fracture Mechanics of Concrete and Concrete Structures, V. 3, Fracture Mechanics of Concrete Structures*, ETH, Zurich, 1995, F. H. Wittmann, ed., Aedificatio, Freiburg, Germany, pp. 1749-1770.
- Bazant, Z. P., and Cedolin, L. (1991), *Stability of Structures: Elastic, Inelastic, Fracture, and Damage Theories*, Chapters 12 and 13, Oxford University Press, New York, 1991.

**AUTHORS' CLOSURE**

The authors wish to thank Professor Bazant for his interest in the paper. Our comments in reply are as follows.

The proposed simple method for determining material fracture parameters  $K_{Ic}^S$  and  $CTOD_c$  from the peak loads, referred to as the peak-load method for brevity, is a test method based on the two parameter fracture model. This method makes it possible to obtain  $K_{Ic}^S$  and  $CTOD_c$  where a closed-