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INELASTIC ANALYSIS OF STRUCTURES

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Inelastic Analysis of Structures

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Contents

Preface . . . xv

Introduction . . . xix

PART I: PLASTIC ANALYSIS OF STRUCTURES UNDER UNIAXIAL STRESS – FUNDAMENTALS . . . 1

1 Uniaxial Stress-Strain Relations . . . 3
Problems . . . 7

2 Plastic Bars and Yield Hinges . . . 9
2.1 Plastic Bar . . . 9
2.2 Moment-Curvature Diagram . . . 10
2.3 Plastic Hinge . . . 13
2.4 Monosymmetric Sections . . . 15
Problems . . . 16

3 Incremental Analysis . . . 19
3.1 Trusses . . . 19
3.2 Beams . . . 26
3.3 Frames . . . 33
3.3.1 Standard Update of Structural Stiffness Matrix . . . 33
3.3.2 Indirect Update of Structural Stiffness Matrix . . . 36
Problems . . . 40

4 Elementary Limit Analysis . . . 43
4.1 Trusses . . . 43
4.2 Beams and Frames . . . 47
Problems . . . 52

5 Theorems of Limit Analysis . . . 53
Problems . . . 58

6 Methods of Limit Analysis . . . 59
6.1 Kinematic Approach . . . 59
6.2 Distributed Loads . . . 63
6.3 Combination of Mechanisms . . . 65
6.4 Nonrectangular Frames . . . 69

6.5 Static Approach – Method of Inequalities . . . 71
Problems . . . 74

7 Linear Programming in Limit Analysis . . . 77
7.1 Kinematic Approach . . . 77
7.2 Transformation to Standard Form . . . 78
7.3 Application to Trusses . . . 81
7.4 Application to Frames . . . 84
7.5 Static Approach . . . 91
7.6 Duality . . . 97
Problems . . . 101

8 Displacements at Incipient Collapse . . . 103
8.1 Trusses . . . 103
8.2 Beams and Frames . . . 107
Problems . . . 110

PART II: PLASTIC ANALYSIS OF STRUCTURES UNDER UNIAXIAL STRESS – FURTHER TOPICS . . . 113

9 Nonproportional and Cyclic Loads . . . 115
9.1 Nonproportionally Varying Loads . . . 115
9.2 Cyclic Loading and Shakedown . . . 119
Problems . . . 125

10 Theorems of Shakedown Analysis . . . 127
10.1 Basic Concepts . . . 127
10.2 Static Approach . . . 127
10.2.1 Melan's Theorem . . . 127
10.2.2 Alternating Plasticity . . . 130
10.2.3 Lower Bound Theorem . . . 132
10.3 Kinematic Approach . . . 133
10.3.1 Koiter's Theorem . . . 133
10.3.2 Upper Bound Theorem . . . 135
Problems . . . 137

11 Methods of Shakedown Analysis . . . 139
11.1 Static Approach . . . 139
11.2 Design Implications . . . 144
11.3 Kinematic Approach . . . 145
Problems . . . 151

12 Optimum Design . . . 153
12.1 Cost Function . . . 153
12.2 Limit Design . . . 157
12.3 Shakedown Design . . . 170
Problems . . . 181

13 Combined Plastic Bending and Compression or Tension . . . 183
13.1 Generalized Plastic Hinge . . . 183

13.2 Basic Theorems . . .
13.3 Simple Estimates of Collapse Load . . .
13.4 Application of Linear Programming . . .
Problems . . .

14 Plasticity Aspects of Reinforced Concrete . . .
14.1 Plastic Bending Capacity of Under-Reinforced Beams without Axia
Force . . .
14.1.1 Singly Reinforced Rectangular Cross Section . . .
14.1.2 Doubly Reinforced Cross Section . . .
14.1.3 T and L Beams . . .
14.1.4 Moment-Curvature Diagram . . .
14.1.5 Specifications of CEB Model Code . . .
14.2 Load Capacity of Reinforced Concrete Columns and Prestressec
Beams . . .
14.2.1 Columns . . .
14.2.2 Prestressed Concrete Beams . . .
14.2.3 Size Effect . . .
Problems . . .

Part III: PLASTIC ANALYSIS OF STRUCTURES UNDEF MULTIAXIAL STRESS . . .

15 Simple Elastoplastic Constitutive Models . . .
15.1 Yield Conditions for Isotropic Materials . . .
15.1.1 Von Mises Criterion . . .
15.1.2 Tresca Criterion . . .
15.1.3 Gurson Criterion . . .
15.1.4 Burzyński Criterion and Drucker–Prager Criterion . . .
15.1.5 Mohr–Coulomb Criterion . . .
15.1.6 Rankine Criterion . . .
15.2 Flow Theory of Plasticity . . .
15.2.1 Flow Rule . . .
15.2.2 Fundamental Equations . . .
15.2.3 Loading Criteria and Elastoplastic Stiffness . . .
15.2.4 Karush–Kuhn–Tucker Conditions . . .
Problems . . .

16 Theorems of Plastic Analysis in Multiaxial Case . . .
16.1 Theorems of Limit Analysis . . .
16.2 Shakedown Theorems . . .
16.3 Global Uniqueness Theorem . . .
Problems . . .

17 Plastic Torsion and Shear . . .
17.1 Yield Condition . . .
17.2 Pure Torsion . . .
17.3 Combined Torsion and Bending . . .
17.3.1 Lower Bound . . .

17.3.2 Upper Bound . . . 263
17.4 Combined Bending and Shear . . . 265
17.4.1 Lower Bound . . . 265
17.4.2 Upper Bound . . . 269
Problems . . . 271

18 Limit Loads of Plates . . . 273
18.1 Basic Relations . . . 273
18.2 Kinematic Approach (Yield Line Theory) . . . 276
18.3 Static Approach and Strip Method . . . 285
18.4 Reinforced Concrete Plates . . . 289
Problems . . . 293

19 Plane Problems . . . 295
19.1 Yielding under Plane Strain . . . 295
19.2 Kinematic Approach . . . 297
19.2.1 Indentation Problem . . . 297
19.2.2 Applications to Soil Mechanics . . . 300
19.2.3 Applications to Reinforced Concrete Structures . . . 300
19.3 Application of Linear Programming . . . 301
19.4 Static Approach . . . 303
19.5 Slip Line Theory . . . 307
Problems . . . 314

PART IV: ADVANCED TOPICS IN PLASTICITY . . . 315

20 General Elastoplastic Constitutive Models . . . 317
20.1 Hardening . . . 317
20.1.1 Isotropic Hardening . . . 317
20.1.2 Linear Kinematic Hardening . . . 323
20.1.3 Mixed Hardening . . . 327
20.1.4 Nonlinear Kinematic Hardening . . . 327
20.1.5 General Hardening . . . 330
20.2 Drucker's Postulate and Uniqueness . . . 331
20.3 Nonassociated Flow . . . 335
20.4 Non-Smooth and Multi-Surface Plasticity . . . 339
20.4.1 Perfect Plasticity . . . 339
20.4.2 Hardening Plasticity . . . 346
20.5 Anisotropic Yield Criteria . . . 351
20.5.1 Hill Criterion . . . 351
20.5.2 Hoffman Criterion . . . 352
20.5.3 Tsai–Wu Criterion . . . 352
20.5.4 Comparison . . . 353
Problems . . . 354

21 Plastic Material Models for Concrete and Soils . . . 357
21.1 Concrete . . . 357

21.1.1 Material Behavior . . .
21.1.2 Failure Criteria . . .
21.1.3 Hardening and Softening . . .
21.1.4 Orthotropic Models and Form Invariance . . .
21.2 Soils . . .
21.2.1 Material Behavior . . .
21.2.2 Hydrostatic Compression . . .
21.2.3 Three-Dimensional Model (Cam-Clay) . . .
21.2.4 Other Models . . .
Problems . . .

22 Numerical Methods in Plasticity . . .
22.1 Finite Element Method . . .
22.1.1 Linear Elasticity . . .
22.1.2 Nonlinear Material Models . . .
22.1.3 Linearization . . .
22.1.4 Finite Elements for Elastoplastic Analysis . . .
22.2 Solution Strategies in Nonlinear Analysis . . .
22.2.1 Newton–Raphson Iteration under Load Control . . .
22.2.2 Direct Displacement Control . . .
22.2.3 Arc Length Control and Indirect Displacement Control .
22.3 Stress Return Algorithms . . .
22.3.1 Radial Return Mapping . . .
22.3.2 Closest-Point Projection . . .
22.3.3 Generalized Trapezoidal and Midpoint Rules . . .
22.3.4 Generalized Cutting Plane Algorithm . . .
22.3.5 Algorithms for Plane Stress Problems . . .
22.4 Algorithmic Stiffness . . .
Problems . . .

23 Thermodynamic Approach to Constitutive Modeling . . .
23.1 Basic Concepts and Laws of Thermodynamics . . .
23.1.1 Internal Energy and Entropy . . .
23.1.2 First Law of Thermodynamics . . .
23.1.3 Second Law of Thermodynamics . . .
23.1.4 Free Energy and Dissipation . . .
23.1.5 Historical Remarks . . .
23.2 Linear Thermoelasticity . . .
23.3 Dissipation and Evolution Laws . . .
23.3.1 Maxwell Viscoelasticity . . .
23.3.2 Dissipative and Quasiconservative Thermodynamic Forces
23.4 Dissipation Potential . . .
23.5 Dual Potential . . .
23.5.1 Legendre–Fenchel Transform . . .
23.5.2 Uniaxial Perfect Plasticity . . .
23.5.3 Multiaxial Perfect Plasticity . . .
23.5.4 Hardening Plasticity . . .
Problems . . .

24 Elastoplastic Constitutive Models for Large Strain . . . 459
24.1 Kinematics of Large Transformations . . . 460
24.1.1 Deformation Gradient and Deformation Tensor . . . 460
24.1.2 Polar Decomposition . . . 463
24.1.3 Strain Tensors . . . 465
24.2 Equilibrium and Constitutive Equations . . . 467
24.2.1 Equilibrium Equations . . . 467
24.2.2 Internal Power and Dissipation . . . 468
24.2.3 Hyperelasticity . . . 469
24.2.4 Elasticity in Rate Form (Hypoelasticity) . . . 474
24.3 Large-Strain Plasticity . . . 478
24.3.1 Rate Formulation . . . 479
24.3.2 Formulation Based on Thermodynamic Potentials . . . 480
24.3.3 Comment on Plastic Buckling and Bifurcation . . . 487
Problems . . . 487

25 Crystal Plasticity and Microplane Constitutive Models . . . 489
25.1 Micromechanically Based Models for Crystalline Materials . . . 489
25.1.1 Crystallographic Models . . . 489
25.1.2 Slip Theory of Plasticity . . . 493
25.2 Microplane Models for Concrete . . . 494
25.2.1 History of Microplane Models . . . 494
25.2.2 Kinematic Constraint . . . 495
25.2.3 Constitutive Relations for a Microplane . . . 498
25.2.4 Microplane Model M4 for Concrete . . . 499
25.2.5 Advantages of the Microplane Model . . . 502
25.3 Microplane Models for Metal Plasticity . . . 505
25.3.1 Microplane Model Version of J_2-plasticity . . . 505
25.3.2 Microplane Model Version of Slip Theory of Plasticity . . . 507
25.3.3 Microplane Model for Plasticity with Kinematic Constraint . . 508
25.4 Vertex Effect . . . 509
25.4.1 Physical Source and Modeling of Vertex Effect . . . 509
25.4.2 Experimental Observations and Numerical Results for Metals . . . 510
25.4.3 Experimental Observations and Numerical Results for Concrete . . . 513
Problems . . . 514

PART V: TIME-DEPENDENT INELASTIC BEHAVIOR OF METALS AND CONCRETE . . . 515

26 Models for Localization of Softening and Size Effect . . . 517
26.1 Lack of Objectivity of Constitutive Models with Softening . . . 517
26.2 Attaining Objectivity with Mesh-Adjusted Softening Modulus . . . 520
26.3 Localization Limiters . . . 523
26.3.1 Nonlocal Continuum . . . 524
26.4 Plasticity at Continuum Limit on Approach to Nano-Scale . . . 535
Problems . . . 538

27 Viscoplasticity . . .
27.1 Bingham Model . . .
27.2 Perzyna Model . . .
27.3 Models of Duvaut–Lions Type . . .
Problems . . .

28 Material Models for Concrete Creep and Shrinkage . . .
28.1 Concrete Creep and Shrinkage and its Properties . . .
28.2 Linearity of Creep and Principle of Superposition in Time . . .
28.3 Simple Models for Creep and Shrinkage . . .
28.3.1 Basic Creep . . .
28.3.2 Mean Cross Section Shrinkage . . .
28.3.3 Mean Drying Creep in the Cross Section . . .
28.4 Age-Adjusted Effective Modulus Method (AAEM) . . .

29 Creep and Shrinkage Effects in Concrete Structures . . .
29.1 Homogeneous and Nonhomogeneous Structures . . .
29.2 Structural Analysis According to AAEM . . .
29.2.1 Statically Indeterminate Beam Systems . . .
29.2.2 Typical Creep and Shrinkage Problems Solved via AAEM .
29.2.3 Structures with Distributed Cracking . . .
29.3 Computational Approaches . . .
29.3.1 Multiaxial Constitutive Equation for Creep and Shrinkage .
29.3.2 AAEM via Elastic Finite Element Program . . .
29.3.3 Step-by-Step Solution Based on Hereditary Integral . . .
29.3.4 Rate-Type Creep Law and Internal Variables . . .
29.3.5 Exponential Algorithm . . .
29.3.6 Solidifying Kelvin Chain . . .
Problems . . .

Appendix A Linear Elastic Trusses . . .
A.1 Fundamental Variables . . .
A.2 Fundamental Equations . . .
A.3 Matrix Notation . . .
A.4 Stiffness Method . . .
A.5 Assembly of Stiffness Matrix . . .

Appendix B Linear Elastic Beams and Frames . . .
B.1 Fundamental Variables . . .
B.2 Fundamental Equations . . .
B.2.1 Kinematic Equations . . .
B.2.2 Constitutive Equations . . .
B.2.3 Equilibrium Equations . . .
B.3 Stiffness Matrix . . .
B.4 Members Connected by Hinges . . .
B.5 Frames . . .

Appendix C Linear Programming . . .

Appendix D Cartesian Tensors and Elasticity 645
D.1 Cartesian Tensors 645
D.2 Linear Elastic Stress-Strain Relations 648
D.3 Stress Invariants 650
D.4 Octahedral Stress and Haigh–Westergaard Coordinates 654
D.5 Fundamental Equations of Linear Elasticity 656
D.6 Clapeyron Theorem 657
D.7 Engineering Notation in Matrix Form 659

Appendix E Model B3 for Predicting Concrete Creep and Shrinkage 665
E.1 Frameworks for Modeling of Drying Effects 665
E.2 Model B3 666
E.2.1 Basic Creep Compliance Function 666
E.2.2 Drying Creep Compliance Function 667
E.2.3 Prediction of Model Parameters 667
E.3 Material Law for Free Shrinkage and Thermal Strain 669

Appendix F Softening Inelastic Hinges: Deviations from Plasticity and Size Effect 671
F.1 Size Effect on Moment-Rotation Diagram 671
F.2 Size Effect in Beams and Frames Failing by Softening Hinges 674

References 681

Author Index 713

Subject Index 719

Preface

Our main objective in writing this book has been to provide a textbook for ɡ
courses on plasticity, with some ramifications to time-dependent inelastic b
In our selection of the topics and the sequence of their exposition, we put
emphasis on structural engineering applications. There is nevertheless p
material for using the book in postgraduate courses in geotechnical, me
aerospace, naval, petroleum and nuclear engineering. We assume the backgr
the level of a B.S. degree in civil or mechanical engineering.

Plasticity has already been the subject of many books. So why another
hope to provide a book that is unique in many respects. It has been our inte
fill many needs that are not quite met by other books. Being considerably lar
a textbook for a single course, our book provides both a systematic expositi
fundamentals of plasticity, and an up-to-date introduction to most of the a
subjects. The courses with the coverage specified below could not be tau
from some other existing book. We proceed from simple to complex, and i
examples before generalizing. We try to be systematic and mathematically
while striving, above all, for clarity. We avoid an artificially formalistic pres
that hardly achieves more than impressing by mathematical sophisticati
book features complete and rigorous mathematical derivations of all the
Some derivations are more simple and others more rigorous than those
the previous textbooks. Despite being mainly a textbook, in the advanced
our book also covers most of the 'hot' topics of current research, and conta
new research results. A set of problems for the student is included at th
most chapters. Both simple and hard problems are suggested, the hard on
marked by an asterisk. It is planned to make the solutions available on the
`http://www.wiley.co.uk/inelastic`, which will also contain some addition
information, such as a set of links to sites providing software for the solution
programming problems.

We also include a set of six appendices, four of which review, for the
convenience, the fundamentals of linear elastic analysis and the math
background of linear programming, and two give information on specialized
the code-type prediction model for creep of concrete and the size effect en
by softening in plastic hinges.

A special feature, which is not encountered in the basic texts on plasticit
found only in specialized treatises, is a thorough exposition of the plasticity a
concrete and reinforced concrete, including the basic principles of limit stat
Concrete, of course, is not a plastic material *per se*, but plasticity concepts
the yield surfaces and plastic potentials form a necessary part of models tha

plasticity to damage. Besides, the theory of plasticity is well suited to reinforced concrete structures that fail by the yielding of steel reinforcement. Similar comments can be made about our inclusion of plasticity models for soils.

To keep with the nature of most civil engineering applications, as well as to make the student's entry into the subject easier, the first two among five parts of our book are restricted to beam structures whose stress state may be simplified as uniaxial. Considerable attention is devoted to shakedown, another classical subject particularly important for structural engineering, but rarely treated consistently in textbooks. The classical topics at the margins of plasticity theory, such as the optimum design and linear programming, are included in our coverage. After digesting the basic concepts in the context of uniaxial stress, the students will find it easier to follow, in the third part, the extension of limit analysis to structures under multiaxial stress.

For the benefit of advanced doctoral students and postdoctoral researchers, we include in the last two parts of the book a number of advanced subjects normally not seen in basic textbooks – numerical algorithms, thermodynamic aspects, plasticity in finite strain, multisurface plasticity, anisotropic plasticity, nonlocal and gradient models for plasticity with strain softening and size effects, viscoplasticity and rate effects, microplane constitutive models, and vertex effects. We also provide a brief survey of polycrystal plasticity. With this scope, we hope to have covered a major part of what today constitutes the modern theory of plasticity. Expositions of the dislocation theory as the micromechanical basis of plasticity, dynamic plasticity, plastic buckling and bifurcations, plasticity of shells and constitutive properties of plastic composites could not be accommodated within the scope of this book.

Another special feature of our book is the inclusion of two chapters (among 29) on the creep of concrete and its effects in structures. Although this kind of inelastic behavior is very important for the durability of civil engineering infrastructure and sometimes affects the safety as well, most structural engineering curricula unfortunately do not have room for a full course devoted to this subject and, deplorably, graduate students leave the university without acquiring any knowledge of concrete creep. Our coverage of this subject provides a feasible compromise – an exposition brief enough not to lose the emphasis on plasticity yet sufficient to acquaint the student with the basic results needed for structural design. Due to space limitations, the treatment of creep is nevertheless much less systematic than that of plasticity, and most intricacies of this vast subject are inevitably skipped.

Our book can serve as a textbook for courses of several types:

- *A Quarter-Length First-Year Graduate Course* with a slight civil engineering emphasis may consist of the following chapters and sections: **1, 2, 4–6, 7.5, 8.2, 9**, essentials of **10** (without proofs), **11, 12.1, 12.2, 13.1–13.3, 14.1.1, 15.1, 15.2.1–15.2.3, 16.1, 17.1–17.3, 18, 19.1, 19.2.1–19.2.2**, only essential ideas and graphs from **19.5, 28.1–28.3, 28.4** without proof, and selections from **29.2.2**.
- *A Quarter-Length First-Year Graduate Course* with a slight mechanical engineering emphasis may consist of the following chapters and sections: **1–7, 13, 15, 16.1, 16.3, 17.1–17.3, 18.1–18.3, 19.1, 19.2.1, 19.5, 20.1, 25.4, 27.1**.
- *A Semester-Length First-Year Graduate Course* in structural engineering may fully cover chapters **1–19** and **28** and sections **29.1** and **29.2**. In mechanical engineering, one may omit **8–11, 12.3** and **14** and add **20.1–20.3, 22.1, 22.2.1, 22.3.1, 22.3.2, 25.4** and **27.1**.

- *A Second Course on Plasticity for Doctoral Students* in structural engineeri cover chapters and sections **20, 21, 22.1, 22.2.1, 22.2.2, 22.3.1, 22.3.2 25.2** and **26.1–26.3**. In mechanical engineering one may omit **21, 25.2 25.2.5** and add **25.3, 25.4** and **27.1**. In a computationally oriented cou entire chapter **22** can be covered.
- *A Short Course for Post-Doctoral Researchers and Advanced Doctoral S* may start with the three-dimensional formulation of plasticity in chapters **20** and include the advanced topics in chapters **22–24**. A course with empl structural engineering may also cover chapters and sections **21, 25.2, 26.1 28.1, 28.2** and **29.3**, while a course with emphasis on mechanical engineeri instead cover chapters **26, 27** and **25** without sections **25.2.4–25.2.5** and

The first course outline listed above has been used by the second author course on Inelastic Structural Analysis, which he has been regularly teacl Northwestern University since 1970. The lecture notes that he had prepa this course during the 1970s served as the point of departure for writi book – an arduous effort that began in earnest in 1993, right after tl author completed his doctoral study at Northwestern University. The preser of various advanced subjects in this book have been tried in a number c courses or advanced graduate courses taught at various institutions[1]. The bc completed during the first author's Visiting Scholar appointment at North University in the summer of 2000, and the second author's Visiting Pı appointment at the Swiss Federal Institute of Technology at Lausanne (EF March 2001.

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Introduction

The roots of some elementary ideas of the theory of plasticity can be trace over three and half centuries. In Galileo's (1638) calculation of the collapse lo cantilever, one may discern the assumption of a uniform distribution of tensile s over the cross section, even though the assumption of a concentrated compressic at the compression face was far from realistic. About a century later, Giovanni discussed the safety of Michelangelo's dome of Saint Peter's cathedral in a ma which one could detect the ideas of the static approach to limit analysis (Ben 1991). In the debates of the stability of masonry arches, vaults and domes La Hire, Boscovich, Lamé, Clapeyron, Fourier and Pauker, continuing into th nineteenth century, one could also perceive various elementary ideas of plastic a (Benvenuto, 1991). The first realistic and almost complete static analysis of along with the concept of plastic slip and yield condition, is found in Coulomb's study of earth-retaining walls of military fortifications.

Various elementary ideas of plastic deformation and failure, and the reduc buckling loads gradually emerged throughout the nineteenth century in the of pioneers such as Lüders (1854), Tresca (1868), de St. Vénant (1870), Lévy Rankine (1876), Bauschinger (1881), Considère (1891), Engesser (1895), Ha (1896) and Mohr (1900). The static theorem of limit analysis was anticipated Carvelli and Cocchetti, 2000) in the work of Rankine in 1859 and Kötter i and its intuitive enunciations can be found in the work of Kazinczy (1914) a inaugural lecture of Kist (1917). During the first quarter of the twentieth cent basic concepts, such as the yield surfaces, flow rules, slip lines, and plastic appeared, principally in the works of von Kármán (1909), von Mises (1913), (1924) and Reuß (1930). An important milestone was the resolution of the t problem (Nádai, 1923) and indentation problem (Hencky, 1923; Prandtl, 192 materials science foundation of metal plasticity in the dislocation theory was Taylor (1934) and others.

The static and kinematic theorems of limit analysis were in general first in a Russian conference proceedings article by Gvozdev (1938), long unknowr West. At about the same time, the static shakedown theorem was first pr Melan (1936), being anticipated a few years earlier by himself and Bleich (193 fact that Melan's theorem implies the static theorem of limit analysis was rec much later.

The general concepts of plasticity, which are expounded in Parts I–III of th and comprise the general multiaxial stress-strain relations, normality and co maximization of plastic energy dissipation, limit state theorems, shakedown, o design, plastic hinges, yield line theory of plates and slip line theory, were esta shortly after World War II by Shanley (1947), Hill (1950), Drucker (1950), Greenberg and Prager (1951), Prager and Hodge (1951), Symonds and Neal (1951), Koiter (1953b), etc.; see Nádai (1950a) and Prager (1959) for additional references.

The second half of the last century was a period of rapid refinement and extensive ramification, which continue at an unrelenting pace until today and are for the most part described in Parts IV and V of this book.

Plasticity concepts began to impact structural analysis and design at the beginning of the last century, although design codes based on limit states were not instituted until the middle of that century. When subjected to the service loads, structures must generally respond in an elastic manner. A century ago, the standard design approach was to calculate the maximum stress according to the theory of elasticity, and make sure that it would not exceed a certain allowable stress, which was set sufficiently smaller than the material strength or yield limit. Later it was recognized that in most design problems (fatigue of metals excepted), this approach often leads to designs that are wasteful to varying degrees. The reason is that only some structures fail at a load at which the material strength or yield limit is exhausted at one point of the structure. Many structures redistribute stresses in such a way that the structure fails at a higher load, sometimes only a little higher but often a much higher load, which is attained only after a large part of the structure has plasticized. Simply setting the allowable stress value higher is not a solution, since the safety of some designs would become inadequate. If the theory of elasticity with allowable stress were still used as the basis of design, many efficient modern structures distinguished by slenderness could not even be built.

A realistic approach to design is to calculate the collapse load of the structure from the minimum expected value of material strength or yield limit, and then make sure that this collapse load would not be exceeded by the actual loads multiplied by a suitable safety factor (which is determined from experience and probabilistic considerations). Depending on the type of material, two different kinds of theories, the first older and more mature than the second, are needed for calculating the collapse load:

- If the material is plastic, as typical of most metals (provided the metal has not been fatigued), then the right approach is the *theory of plasticity*.
- If the material is brittle, then the right approach is either *fracture mechanics*, if the failure is caused by propagation of one or several large cracks, or *damage mechanics*, if the failure is caused by the spread of a zone of cracking or other distributed damage confined to the microscale.

This book deals only with the former.

To help understanding, the first two parts of this book are restricted to structures such as beams, trusses and frames whose stress state may be simplified as uniaxial. The advantage is that the basic concepts and results, such as the limit design theorems, normality and convexity, maximum plastic dissipation and shakedown, are understood more easily. This facilitates understanding of the behavior under multiaxial stress, which is the subject of Part III.

Plastic design of structures requires resolving problems of two basic types:

- Formulation of a realistic material model.
- Calculation of the collapse load if the material model is available.

Both are very rich problems. Most of the first three parts of the book deal latter problem, most of the fourth part with the former, and the fifth equ both.

Although 'brute-force' computational approaches such as the finite elemen are nowadays capable of providing numerical answers to most problems of t type, much of the present exposition will dwell on analytical or semi-a solutions. It is, of course, these solutions that lead to an understanding of the convey insight into the structure behavior, are simple enough to be used optimization or probabilistic safety studies, and provide indispensable chec correctness of computational approaches.

The constitutive models for plastic materials have proven to be a fc problem, which has been tackled continuously from the emergence of plasti the present. Despite major advances in the past, this is still a very activ research. Some important phenomena, e.g. the vertex effect, are still ignor constitutive models typically used in the current computational practice.

While the first two parts of the book will rest on a simple material model a only to uniaxial stress, the third part of the book will expound the basic co models of multiaxial stress, and the fourth part will deal with advanced asp as the plastic hardening, anisotropic plasticity, restrictions on constituti stemming from thermodynamics, plasticity in large strain, microplane model based on the idea of polycrystal, and problems caused by material softening. part will extend the treatment to time-dependence of inelastic behavior.

As a special feature of this treatise that should be welcome by civil engi fourth part of the book will discuss the plasticity aspects of quasibrittle mate as concrete. The fifth part will briefly describe the time-dependent inelastic of concrete and its consequences in structures, which are completely diffe those of plasticity.

In the aftermath of the excessive enthusiasm of the 1970s which le development of plasticity models for all kinds of inelastic response of conc field in recent years has been in a sobering but fruitful period in which the li of plasticity have been properly recognized and modeled. Concrete, of cou a plastic material, but the theory of plasticity is useful for describing some its behavior, providing one pays proper attention to the inelastic strain lo engendered by post-peak strain softening, with the inherent size effects sensitivity of finite element solutions.