FRACTURE
AND
SIZE EFFECT
in Concrete and Other Quasibrittle Materials

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Preface

Our book is intended to serve as both a textbook for graduate level courses in engineering and a reference volume for engineers and scientists. We assume that the reader has the background of the B.S. level mechanics courses in the departments of civil, mechanical, or aerospace engineering. Aside from synthesizing the main results already available in the literature, our book also contains some new research results not yet published and many original derivations.

The subject of our book is important to structural, geotechnical, mechanical, aerospace, nuclear, and petroleum engineering, as well as materials science and geophysics. In our exposition of this subject, we try to proceed from simple to complex, from special to general. We try to be as concise as possible and use the lowest level of mathematics necessary to treat the subject clearly and accurately. We include the derivations or proofs of all the important results, as well as their physical justifications. We also include a large number of fully worked out examples and an abundance of exercise problems, the harder ones with hints. Our hope is that the reader will gain from the book true understanding rather than mere knowledge of the facts.

A special feature of our book is the theory of scaling of the failure loads of structures, and particularly the size effect on the strength of structures. We present a systematic exposition of this currently hot subject, which has gained prominence in current research. It has been only two decades that the classical model of size effect, based on Weibull-type statistical theory of random material strength, was found to be inadequate in the case of quasibrittle materials. Since then, a large body of results has been accumulated and is scattered throughout many periodicals and proceedings. We attempt to bring it together in a single volume. In treating the size effect, we try to be comprehensive, dealing even with aspects such as statistical and fractal, which are not normally addressed in the books on fracture mechanics.

Another special feature of our book is the emphasis on quasibrittle materials. These include concrete, which is our primary concern, as well as rocks, toughened ceramics, composites of various types, ice, and other materials. Owing to our concern with the size effect and with quasibrittle fracture, much of the treatment of fracture mechanics in our book is different from the classical treatises, which were concerned primarily with metals.

In its scope, our book is considerably larger than the subject matter of a single semester-length course. A graduate level course on fracture of concrete, with proper treatment of the size effect and coverage relevant also to other quasibrittle materials, may have the following contents: Chapter 1, highlights of Chapters 2, 3, and 4, then a thorough presentation of the main parts of Chapters 5, 6, 7, and 8, parts of Chapters 9 and 12, and closing with mere comments on Chapters 10, 11, and 13. A quarter-length course obviously requires a more reduced coverage.

The book can also serve as a text for a basic course on fracture mechanics. In that case, the course consists of a thorough coverage of Section 1.1 and Chapters 2, 3, 4, 5, and 7.

Furthermore, the book can be used as a text for a course on the scaling of fracture (i.e., the size effect), as a follow-up to the aforementioned basic course on fracture mechanics (or to courses on fracture mechanics based on other books). In that case, the coverage of this second course may be as follows: the rest of Chapters 1 and 5, a thorough exposition of Chapter 6, the rest of Chapters 7 and 8, much of Chapter 9, followed by highlights only of Chapter 10, bits of Chapter 11, and a thorough coverage of Chapter 12.

Chapters 13 and 14, the detailed coverage of which is not included in the foregoing course outlines, represent extensions important for computational modeling of fracture and size effect in structures. They alone can represent a short course, or they can be appended to the course on fracture of concrete or the course on scaling of fracture, although at the expense of the depth of coverage of the preceding chapters.

We were stimulated to write this book by our teaching of various courses on fracture mechanics, damage, localization, material instabilities, and scaling.[1] Our collaboration on this book began already in 1990, but had to proceed with many interruptions, due to extensive other commitments and duties. Most of the book was written between 1992 and 1995.

Our book draws heavily from research projects at Northwestern University funded by the Office of Naval Research, National Science Foundation, Air Force Office of Scientific Research, Waterways Experiment Station of the U.S. Army Corps of Engineers, Argonne National Laboratory, Department of Energy, and Sandia National Laboratories, as well as from research projects at the Universidad Politécnica de Madrid, funded by Dirección General de Investigación Científica y Técnica (Spain) and Comisión Interministerial de Ciencia y Tecnología (Spain). We are grateful to these agencies for their support.

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[1] In the case of the first author: The course on Fracture of Concrete, introduced at Northwestern University in 1988, and intensive short courses on these subjects taught at Politecnico di Milano (1981, 1993, 1997), Swiss Federal Institute of Technology, Lausanne (1987, 1989, 1994), Ecole Normale Supérieure de Cachan, France (1992), and Lulea University, Sweden (1994). In the case of the second author: The undergraduate courses on Fracture Mechanics and Continuum Mechanics and the doctoral-level courses of Physics of Continuum Media and Advanced Fracture Mechanics at the Universidad Politécnica de Madrid, and intensive short courses on Fracture Mechanics taught at Universidad Politécnica and at Universidad Carlos III in Madrid (1994, 1995), and at Universidad de la Plata, Argentina (1995).