A Unified Approach to Shear Design

by Robert J. Frosch, Qiang Yu, Gianluca Cusatis, and Zdeněk P. Bažant

The shear design expressions provided in the ACI 318-14 Building Code[1] were developed over 50 years ago. They were based on the test results available at the time[2,3] and with the goal of providing simplified design expressions that conservatively estimate shear strength. As time has passed and additional test results have become available, it is clear that there are limitations with the current approach. These limitations relate primarily to low percentages of flexural reinforcement, high-strength concrete, and member size. Furthermore, different design approaches are required for nonprestressed and prestressed members. As research has continued to progress, new perspectives of shear resistance have emerged, indicating that it is possible to eliminate limitations of past practice, unify design methods, and provide for improved safety of our structures.

Unified Shear Design

A unified approach is proposed for the shear design of structural concrete members. Consistent with current practice, the nominal shear strength V_n is the sum of the concrete contribution V_c and shear reinforcement contribution V_s: $V_n = V_c + V_s$. For both nonprestressed and prestressed concrete members, V_c is calculated using Eq. (1)

$$V_c = 5\lambda\sqrt{f_c'}\,b_w c \qquad (1)$$

where b_w is web width in in.; c is the cracked section neutral axis depth in in.; f_c' is concrete compressive strength in psi; and λ is the modification factor for lightweight concrete. The development of Eq. (1) is presented in References 4 and 5 for nonprestressed concrete, and the equation is further extended to prestressed concrete in References 6 and 7. The benefit of this approach is that this single design expression can be used to calculate the shear strength of any structural concrete member, including nonprestressed, prestressed, partially prestressed, and axially loaded (in tension or compression). Furthermore, this same expression can be used to calculate the shear strength of fiber-reinforced polymer (FRP) reinforced members. In fact, "Guide for the Design and Construction of Structural Concrete Reinforced with Fiber-Reinforced Polymer (FRP) Bars (ACI 440.1R-15)"[8] uses this design expression for the calculation of the concrete contribution to shear strength.

V_s is calculated using Eq. (2)

$$V_s = \frac{A_v f_{yt} d_t}{s} \qquad (2)$$

where d_t is the distance in inches from the extreme compression fiber of the member to the centroid of the reinforcement nearest the tension face. Equation (2) is similar to the current expression for V_s, with the slight modification of using d_t in place of the traditional value of effective depth d. This modification helps to avoid confusion if multiple layers of reinforcement are provided.

Equation (1) is similar to the current expression for V_c, but there is a major difference in that c is used rather than d, as illustrated in Fig. 1. In effect, the proposed expression considers the average shear stress over the uncracked depth of the cross section, rather than the average shear stress over the effective depth of the member. It is the use of c that unifies the shear strength expression, as c accounts for a number of parameters that affect shear strength.

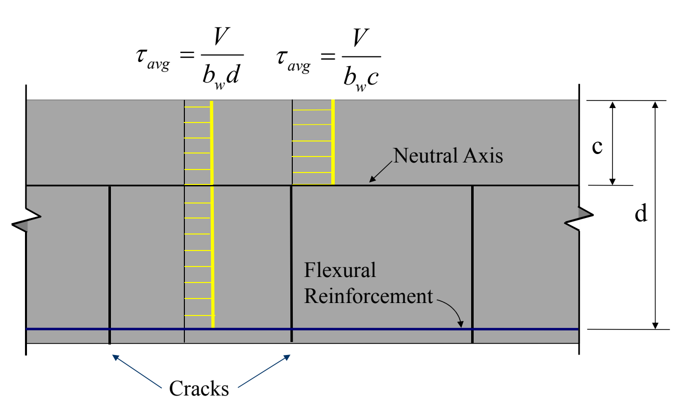

Fig. 1: Simplified illustration of distributions of average shear stress τ_{avg} over d and c

The neutral axis depth c is computed at the location where the shear strength is of interest. For nonprestressed members without axial force, c is computed as the elastic cracked section neutral axis depth. For a rectangular section with reinforcement concentrated near the extreme tension fiber, a cracked section analysis results in c presented by Eq. (3). For other members, such as prestressed and axially loaded members, c is calculated using strain-compatibility.

$$c = kd \quad (3)$$

where $k = \sqrt{2\rho n + (\rho n)^2} - \rho n$, with the reinforcing ratio given by $\rho = \frac{A_s}{b_w d}$ and the modular ratio given by $n = E_s / E_c$.

The elastic cracked section neutral axis is appropriate for nonprestressed members without axial force for several reasons. First, for shear failures to be considered valid, the flexural reinforcement must remain below yield. Therefore, sections across the shear span remain elastic. However, even when considering beams that have resulted in shear failures post-yield of the flexural reinforcement, shear failure occurs outside of the yielded region, near locations where the moment is just above the cracking moment and where the elastic neutral axis depth remains applicable. For prestressed and axially loaded members, however, the failure location does not necessarily remain in the elastic region, and strain-compatibility is used to compute the neutral axis depth. For these members, c is a function of the flexural moment and the axial load. In general, as the moment is increased, c decreases. Furthermore, axial compression increases the neutral axis depth while axial tension decreases the depth.

For prestressed members, thin webs with flanges are often used, and web-shear failures are possible. While the current Code approach for V_{cw} can be used, it is possible to provide a design equation in the same format as that proposed for V_c but including the effects of axial stress f_{pc} and the vertical component of the effective prestressing force at the section V_p (Eq. (4)). This expression can be conservatively simplified by neglecting the axial stress contribution (Eq. (5)), making the equation similar to Eq. (1), but with the full member depth h replacing c because the section is uncracked. The relationships for a cracked section (Eq. (1)) and an uncracked section (Eq. (5)) thus correspond to the current Code's relationships for flexure-shear strength V_{ci} and web-shear strength V_{cw}, respectively.

$$V_{cw} = (5\lambda\sqrt{f'_c} + 0.2 f_{pc}) b_w h + V_p \quad (4)$$

$$V_{cw} = (5\lambda\sqrt{f'_c}) b_w h + V_p \quad (5)$$

Reinforcement Percentage

One of the primary limitations of the current ACI calculation procedure for shear strength is related to its accuracy and conservatism as the reinforcement percentage ρ is varied. As shown in Fig. 2 (test data consistent with those provided in Reference 4), there is a significant trend toward increasing conservatism with increasing reinforcement ratio. This plot includes data from beams with fiber-reinforced polymer (FRP) reinforcing bars; therefore, the data is plotted using the effective reinforcement ratio (ρ_{eff}), which is ρ multiplied by the modular ratio (E_r/E_s). While a wide range of ρ_{eff} is included, the practical range is from 0 to 2%. In focusing on this range, especially below 1%, it is observed that significantly unconservative results develop. It is for this reason that ACI Committee 440, Fiber-Reinforced Polymer Reinforcement, adopted a new expression for the design of such members.[8] The lack of conservatism for these very practical reinforcement ratios is also of concern for the continued use of this design expression for members with steel reinforcement.

The influence of the reinforcement ratio is essentially eliminated using the proposed design expression (Fig. 3). As evident across the range of reinforcement ratios, the calculated

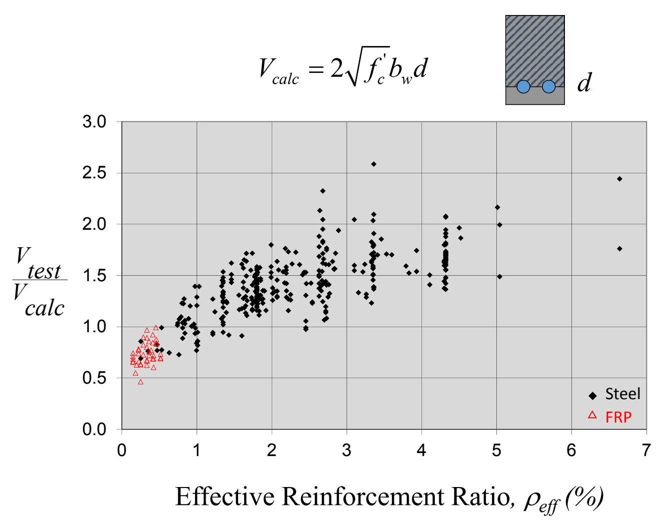

Fig. 2: Influence of reinforcement percentage—ACI 318

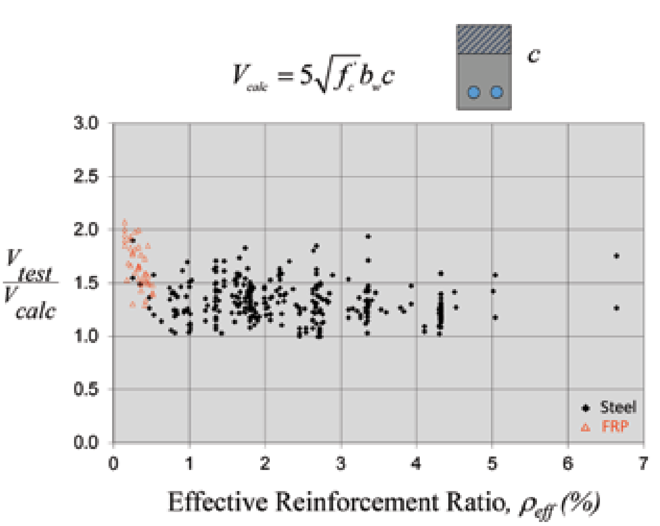

Fig. 3: Influence of reinforcement percentage—proposed expression

shear strengths are conservative, with the lowest results being consistently around 1.0. For very lightly reinforced members (particularly FRP members), the calculated shear strengths increase slightly in conservatism. Considering the smaller number of tests, especially for steel reinforcement in this range, this level of conservatism is reasonable.

Concrete Compressive Strength

The current design approach for shear strength limits the contribution of high-strength concrete. While concrete strengths greater than 10,000 psi can be used in a member, ACI 318 limits $\sqrt{f_c'}$ for calculation to 100 psi unless minimum web reinforcement is provided. For the proposed approach, a limit on the contribution of high-strength concrete is not required. As the concrete strength increases, c decreases while $\sqrt{f_c'}$ increases. The former results in a slightly lower shear strength, while the latter results in a higher shear strength. The net increase in the value of $\sqrt{f_c'}$ is greater than the decrease in c, resulting in slightly greater shear strengths for high-strength concrete. As shown in Fig. 4, the use of the proposed design expression provides consistent results across the wide range of concrete compressive strengths tested.

Size Effect

The current design approach for shear strength provides a constant average shear strength ($2\sqrt{f_c'}$) on a shear area of $b_w d$, regardless of the size of the member. Experimental results, however, indicate that shear strength does not increase in proportion to d. As shown in Fig. 5, if the reinforcement area is held constant while d is doubled (Fig. 5(a)), the current expression indicates a doubling of the shear strength while the proposed design expression indicates only a 50% increase in shear strength. Alternatively, if the reinforcement percentage is held constant while d is doubled, both equations result in a doubling of the shear strength. The proposed expression properly accounts for the reinforcement percentage through a reduction in the shear area ($b_w c$), and it provides a result that is not proportional with beam depth. This example also illustrates the importance of holding appropriate parameters (such as ρ) constant when evaluating the influence of individual variables for a given design expression.

Another limitation of the current design approach is the use of a constant shear strength ($2\sqrt{f_c'}$). Even when the shear area is considered as $b_w c$ (which accounts for the reinforcement percentage), a decrease in the average shear strength coefficient $\frac{V}{b_w c\sqrt{f_c'}}$ is observed with increasing d (Fig. 6). While a coefficient of 5 appears to be satisfactory over the range of depths shown, the average levels of safety decrease as the effective depth is increased. Therefore, extrapolation of test results beyond the range of the data should be considered carefully and based on the size effect theory. Consequently, Eq. (1) is modified by a size effect factor γ_d, resulting in Eq. (6). The factor is based on a proposal by ACI Committee 446, Fracture Mechanics,[9] which is reviewed in detail by Yu et al.[10]

$$V_c = \left(5\lambda\sqrt{f_c'}\,b_w c\right)\gamma_d \quad (6)$$

where $\gamma_d = 1.0$ if $d_t < 10$ in. or if $d_t < 100$ in. and $A_v \geq A_{v,min}$. Otherwise

$$\gamma_d = \frac{1.4}{\sqrt{1 + d_t / d_o}} \quad (7)$$

where $d_0 = 10$ in. if $A_v < A_{v,min}$ or $d_0 = 100$ in. if $A_v \geq A_{v,min}$

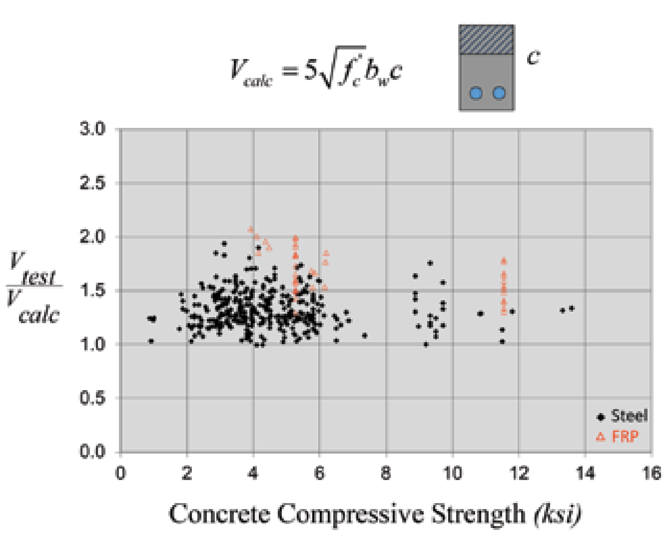

Fig. 4: Influence of concrete strength—proposed expression

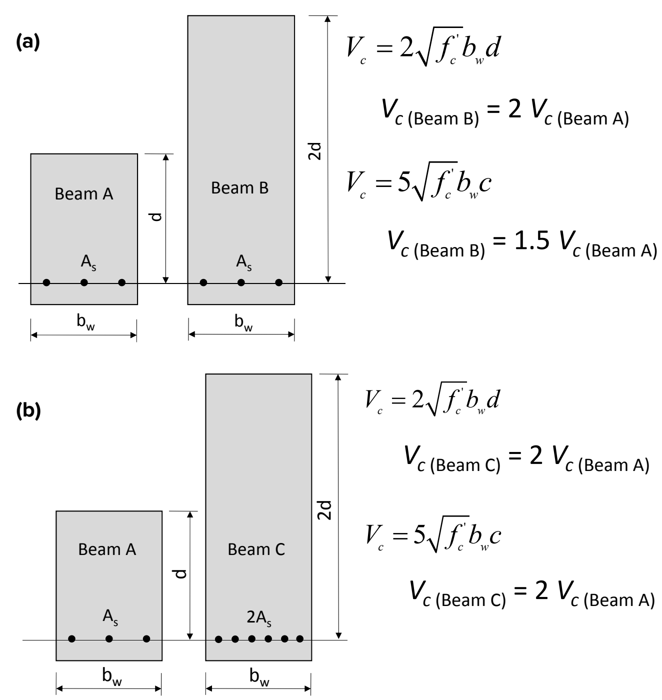

Fig. 5: Influence of doubling beam depth: (a) constant reinforcement area A_s; and (b) constant reinforcement percentage ρ

For beams without shear reinforcement, the size effect does not need to be considered if the depth of the extreme tension reinforcement is less than 10 in. Similarly, if minimum shear reinforcement is provided, the size effect does not need to be considered for depths less than 100 in. Therefore, for most practical members, a size modification is not needed. By simply providing minimum transverse reinforcement, a beam depth must exceed 8 ft before the size effect must be included in calculations. For members of greater depths, a reduction in shear strength is accounted for by multiplying the base concrete shear strength (Eq. (1)) by the fractional value determined from Eq. (7).

As an example of the influence of size, if a one-way slab has a d_t value of 15 in. and is not reinforced for shear, the shear strength from Eq. (1) would be multiplied by 0.89. Similarly, for a beam containing minimum shear reinforcement, a depth d_t of 120 in. would result in a multiplier of 0.94. Members must become very large for this factor to provide a significant influence. However, considering the limited number of large beam tests, reductions based on sound theoretical considerations are appropriate.

As shown in Fig. 7, the proposed approach (Eq. (6)) provides good agreement with test results provided in the Joint ACI-ASCE Committee 445 database for nonprestressed members. For comparison purposes, the results using ACI 318 are also provided. As is evident, the proposed design procedure significantly improves structural safety.

Although direct comparisons with raw databases as presented in Fig. 2 through 4 and 6 and 7 are simple and widely used, one must be aware of two hidden disadvantages:

- The mean values of secondary variables vary systematically over each variable's range; and
- The data are typically crowded in one part of the range while being sparse in another.

In the case of Fig. 6 and 7, the mean values of the steel ratio in successive size intervals decrease by about an order of magnitude, and the mean values of a/d vary almost as much. Therefore, the size effect plots shown herein actually represent the effect of a certain combined variation of beam size, reinforcement ratio, and shear span. Obviously, this is misleading if the size effect trend is to be evaluated. To separate the effects of steel ratio and shear span a, one would need to either conduct multivariate regression of the database or create filtered subbases from which these secondary effects are removed. To keep the approach presented herein at a simple uniform level, such refined comparisons are omitted.

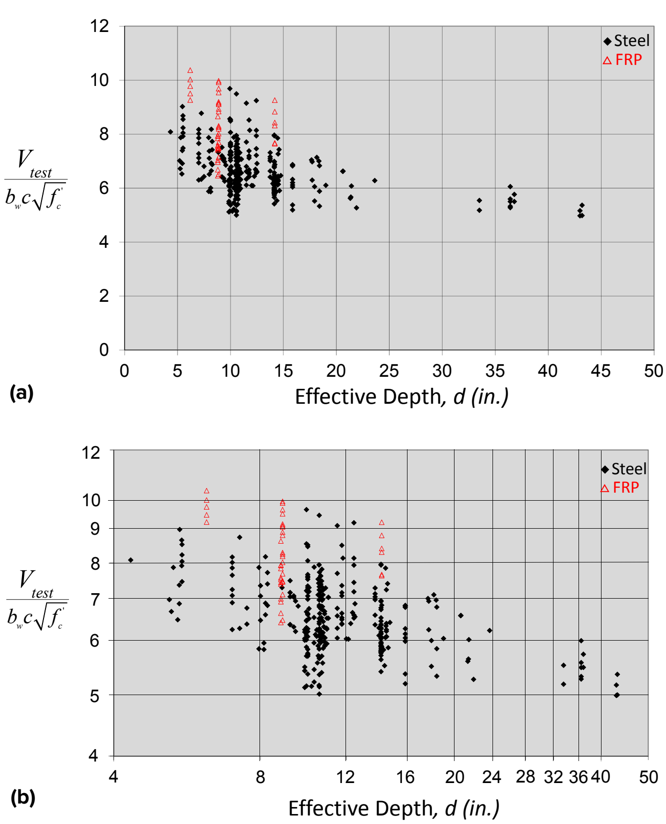

Fig. 6: Influence of member depth: (a) linear scale; and (b) log scale (Note: All data plotted without any filtering for the variation of ρ, a/d, and $\sqrt{f_c'}$ throughout the range)

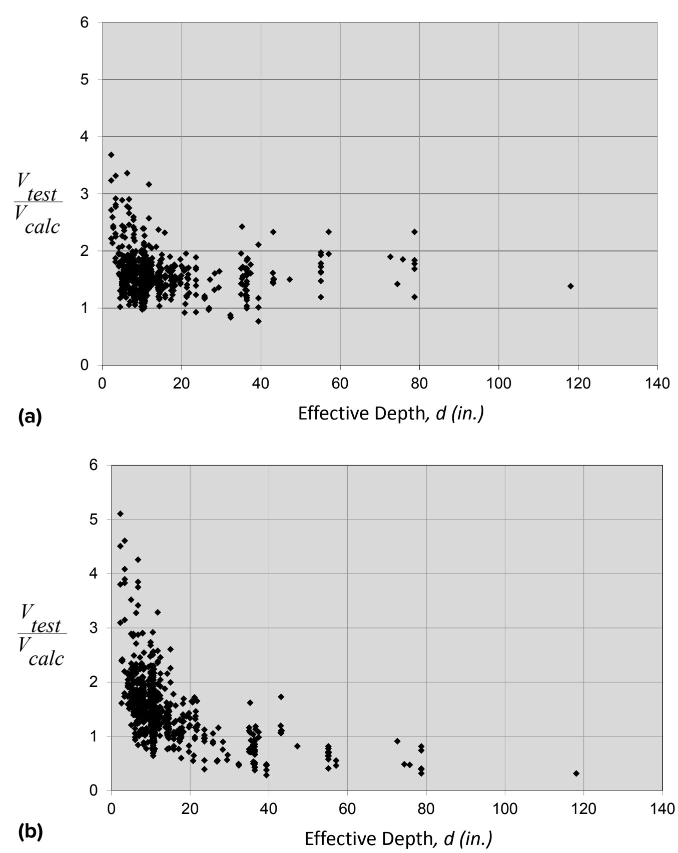

Fig. 7: Predictions made using design procedures are compared against data from the Joint ACI-ASCE Committee 445 database for nonprestressed members: (a) using proposed procedure (Eq. (6)); and (b) using current ACI 318 procedure

They can, however, be found in Bažant et al.[9] and Yu et al.[10]

Strength Limits

The ACI Code limits the maximum shear that can be resisted by a member (Eq. (8)). The intent of this limit, as outlined in Commentary Section R22.5.1.2 of ACI 318-14, is to "minimize the likelihood of diagonal compression failure in the concrete and limit the extent of cracking."

$$V_u \le \phi(V_c + 8\sqrt{f'_c}b_w d) \qquad (8)$$

It is proposed that a similar limit be maintained such that the maximum contribution of shear provided by the shear reinforcement be limited to $4V_c$, consistent with experience and current practice. Therefore, the maximum shear is proposed to be limited according to Eq. (9).

$$V_u \le \phi(5V_c) \qquad (9)$$

This simple limit, in which the maximum shear strength is limited to five times the concrete contribution of the shear strength, provides ease of calculation and clarity.

Conclusions

The proposed design procedure provides a simple approach consistent with both the current ACI design philosophy, which considers $V_n = V_c + V_s$, and with current ACI design assumptions such as critical sections. By providing a new expression for the concrete contribution of shear strength, $V_c = \left(5\lambda\sqrt{f'_c}b_w c\right)\gamma_d$, a unified design procedure is achieved which eliminates the numerous design expressions required in the current Code. This design expression can be used for the shear design of structural concrete members reinforced with either steel or FRP bars and is applicable to both nonprestressed and prestressed members, members including both nonprestressed and prestressed reinforcement, and members with axial forces. This approach has the further advantage of extending to the calculation of two-way shear strength,[11] where $V_c = 10\lambda\sqrt{f'_c}b_o c$, and is currently provided as the shear design method (one-way and two-way) for members with FRP reinforcement.[8] By providing a single, unified method for the shear design of structural concrete members, it is possible to both simplify the Code and enhance structural safety.

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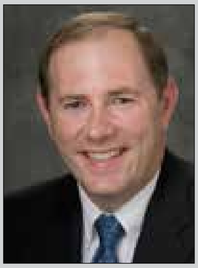

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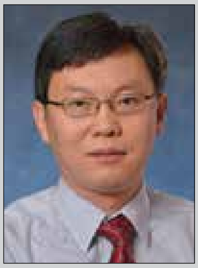

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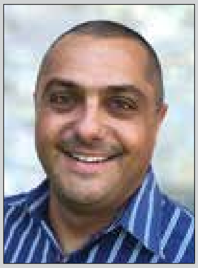

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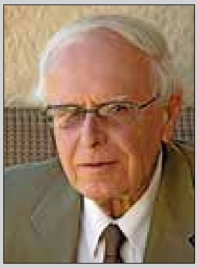

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