

# EFFECT OF COMPOSITION ON BASIC CREEP OF CONCRETE AND CEMENT PASTE

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**ABSTRACT:** A two-level composite model for predicting the basic creep of aging concrete from its composition and the properties of its constituents is proposed. On the macroscale, concrete is treated as a composite of elastic aggregate embedded in the matrix of creeping hardened cement paste. The composite action is described by a combined series-parallel model in which a portion of the paste acts in parallel with the aggregate and the remaining portion in series with this parallel coupling. The portion of the paste coupled in parallel is determined as the amount of paste needed to fill the voids when the aggregate is at its maximum possible compactness, and the remaining portion of the paste then corresponds to the series coupling. On the microscale, the hardened cement paste is considered as a composite of elastic anhydrous cement grains embedded in a matrix of cement gel with voids filled by water and air. The aging is considered by an extension of the previously proposed solidification theory, in which the creeping constituent, the gel, is considered to have nonaging viscoelastic properties, and the aging caused by the chemical reaction of cement hydration is totally ascribed to the volume growth of the load-bearing (bonded) portion of hardened cement gel. The model is calibrated and verified by means of a comprehensive data set reported by Ward, Neville and Singh.

## INTRODUCTION

Aside from environmental effects, the error in predicting the creep properties of concrete is the greatest source of error in the estimates of creep effects in concrete structures. The reason for this large error is no doubt the fact that the existing formulas used in codes and design recommendations [CEB-FIP (1990); ACI-ASCE (1958); BP-model: Bažant and Panula (1978, 1979); BP-KX-model: Bažant et al. (1991, 1992), Bažant and Kim (1991, 1992a-c); French Code BPEL: Béton (1991); and model B3: Bažant and Baweja (1994)] for predicting the creep of a given concrete are at least to a large extent empirical. They have been formulated without any use of the mechanics of composite materials. A few attempts to relate the creep of concrete to its composition by some rational calculations have been made [e.g., Neville (1984) and Popovics (1986)]; however, they have not used composite material models that would describe the complex influence of aging due to hydration and would treat realistically the variation and redistribution of stress between the aggregate and the cement paste caused by creep.

The objective of this paper is to propose a new model that treats the creep interaction of cement paste and aggregate as well as the phenomenon of solidification due to hydration of cement in a realistic way, and at the same time agrees with the comprehensive test data existing in the literature. However, from the viewpoint of mechanics of composite materials, the model will be conceptually simple. It will be a uniaxial model involving series and parallel couplings, which do not capture the triaxial aspects of interactions in a composite. Treatment of such triaxial aspects together with the complexities of creep with aging appears to present formidable difficulties and preclude development of closed-form solutions. The analysis will be restricted to the basic creep; that is, creep at no moisture movement through the material. The

creep at drying as well as drying shrinkage are beyond the scope of this paper.

A composite model for predicting the creep of concrete is needed not only for structural analysis. Its development is even more important in order to gain basic understanding of the composition factors that influence the magnitude of creep. Such understanding should allow development of better concretes exhibiting lower creep, which are needed for many practical applications, such as bridges, tall buildings, or nuclear-reactor containments and vessels (Granger et al. 1993; de Larrard et al. 1990).

## CONCRETE AND CEMENT PASTE AS COMPOSITE MATERIALS

Concrete can be regarded as a composite that consists of mineral aggregate and hardened cement paste, corresponding to the macroscale and microscale, respectively. In more detail, one can further distinguish coarse aggregate (gravel) and fine aggregate (sand), and correspondingly regard concrete as a composite consisting of gravel in a matrix of mortar, and mortar as a composite consisting of sand in a matrix of hardened cement paste. The corresponding scales are then called the macroscale and mesoscale. The hardened cement paste, too, can be regarded as a composite consisting of hardened cement gel with unhydrated cement particles, and pores filled by capillary water and air with water vapor, which could be imagined to correspond to the microscale and submicroscale. On a still smaller scale, the cement gel itself contains far smaller pores, as small as only a few water molecules in thickness. These are filled with adsorbed water or other types of strongly held but evaporable water.

Obviously, modeling the microstructure at all these scales would be extremely complex, and so the problem must be simplified. In this study we consider composite models at only two scales: the concrete as a composite of aggregate and cement paste; and the cement paste as a composite of cement gel, anhydrous cement, and evaporable water in capillary pores.

The determination of mechanical properties of composites from the properties of their constituents has been studied extensively and much has been learned [e.g., Hashin (1983), Hashin and Shtrickman (1963), Christensen (1979), Aboudi (1991), Mura (1982), and Nemat-Nasser and Horii (1993)]. However, the previous works have dealt with elastic and plastic properties. Composites exhibiting aging creep, which are of concern here, have apparently not been investigated. The

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aging characteristic of viscoelastic creep brings about serious complication. They cause the creep problems to lead to Volterra integral equations in time with nonconvolution kernels, which cannot be solved analytically. Their numerical solution is not difficult, but does not help understanding of the behavior. We strive for a solution described by closed-form expressions rather than computational algorithms.

At first one might desire to adapt to the problem some of the powerful methods for composites, such as the Hashin-Shtrickman variational bounds (Hashin and Shtrickman 1963), the method of composite spheres (Hashin 1962), the self-consistent model (Hill 1965), or the Mori-Tanaka method [e.g., Benveniste (1987)]. However, in the context of aging creep, these methods appear to be too complicated. They try to capture the triaxial behavior, but for concrete creep the more important issue is the time effect. The stresses in the paste and aggregate vary strongly in time, as the load is gradually transferred from the creeping component (the matrix of hardened cement paste) to the elastic component (the mineral aggregate). This causes a significant redistribution of stresses between the aggregate and cement paste, and within the paste between the recently and earlier solidified hydration products. This redistribution is greatly affected by aging; that is, the progress of the chemical reaction of cement hydration. These reasons preclude the use of the simple effective modulus method known from classical viscoelasticity.

Therefore, we will use for the composite action a simple parallel-series model, which is uniaxial. This is not completely unrealistic, because it is known from measurements that the Poisson ratio for basic creep of concrete is nearly constant in time and nearly equal to its elastic value  $\nu = 0.18$ . Because of material isotropy, the volumetric and deviatoric compliance functions are simply obtained as  $J_V = 3(1 - 2\nu)J$  and  $J_D = 2(1 + \nu)J$ , where  $J =$  uniaxial compliance function, which we will try to predict from the composition of concrete;  $J_V$  and  $J_D$  then follow automatically.

The parallel and series couplings of two constituents are known to provide the upper and lower bounds on the stiffness of an elastic composite, called the Reuss and Voigt bounds, respectively (Christensen 1979). Often, however, these bounds are too far apart to be useful for prediction of the actual stiffness. Any value between these bounds can nevertheless be obtained by a combination of parallel and series couplings of the constituents in the model [Fig. 1(a)]. This is what we choose to pursue here. The drawback is that, compared to the aforementioned more sophisticated composite models, the ratio of subdivision of the constituents between the series and parallel couplings, characterized in Fig. 1(a) by cross-section parameter  $\alpha$  and length parameter  $\beta$ , is not determined by the model and must be deduced by other considerations or experiments.

First the model of Fig. 1(a) will be applied to concrete as a composite of aggregate and cement paste, in which case the shaded area  $A_g = \alpha\beta$  is chosen to represent all of the aggregate, and the unshaded areas coupled in parallel and in series

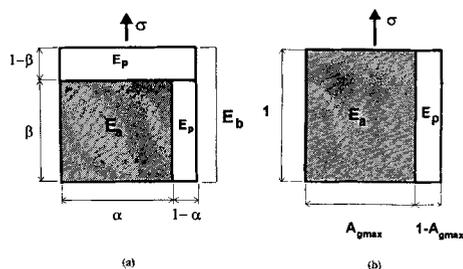


FIG. 1. (a) Combined Parallel Series Coupling Model for Concrete; (b) Series Coupling Model

represent portions of the hardened cement paste. The portion in series can be imagined to correspond to the thin layers in the narrow regions of near contact between adjacent aggregate pieces, and the portion coupled in parallel to the rest of the hardened cement paste. This also reflects the fact that the hardened cement paste forms a contiguous matrix, but the aggregate is not contiguous.

For the hardened cement paste we will consider a similar parallel-series model but with variable volumes of the constituents, in order to model the aging.

## CONCRETE AS COMPOSITE OF AGGREGATE AND CEMENT PASTE

According to the model in Fig. 1(a),  $\alpha\beta = A_g =$  volume of aggregate grains per unit volume of concrete (thus, if  $A_g$  is given and  $\beta$  is chosen,  $\alpha$  can be determined). Obviously, according to the parallel-series model, the Young's elastic modulus of concrete,  $E_c$ , is given by

$$\frac{1}{E_c} = \frac{1 - \beta}{E_p} + \frac{\beta}{\alpha E_a + (1 - \alpha)E_p}, \quad \alpha = \frac{A_g}{\beta} \quad (1)$$

in which  $E_a$  and  $E_p =$  elastic moduli of the aggregate and the paste.

The aging creep, as mentioned, will be treated by the age-adjusted effective modulus method proposed by Bažant (1972), which reduces the problem to quasielastic analysis. The use of this approximate but quite accurate method is one basic idea of the present approach for the composite. The method is applicable only to problems with steady loads, in which the stress and strain rates decay in time.

The aging creep of concrete is characterized by the compliance function  $J(t, t_0)$  representing the strain at age  $t$  caused by a unit uniaxial stress applied at age  $t_0$ . Alternatively, the creep properties can be characterized by the relaxation function  $R(t, t_0)$  representing the stress at age  $t$  caused by a unit constant strain applied at age  $t_0$ . One of these functions can be obtained from the other by numerical solution of a simple Volterra integral equation. Approximately, but with very good accuracy, the relaxation function can also be obtained from the compliance function using the following one-line formula proposed by Bažant and Kim (1979) (in which  $t$  and  $t_0$  must be given in days):

$$R(t, t_0) = \frac{0.992}{J(t, t_0)} - \frac{0.15}{J(t, t - 1)} \left[ \frac{J(t - \Delta, t_0)}{J(t, t_0 + \Delta)} - 1 \right],$$

with  $\Delta = \frac{t - t_0}{2}$  (2)

In the age-adjusted effective modulus method, one assumes that the strain history is a linear function of the compliance function, that is,  $\epsilon(t) = \lambda + \mu J(t, t_0)$ , where  $\lambda$  and  $\mu$  are two arbitrary parameters. As proven in Bažant (1972), the corresponding stress history is then  $\sigma(t) = \lambda R(t, t_0) + \mu$ . This property is easily understood if one notes that the strain history corresponding to constant stress  $\mu$  introduced at age  $t_0$  is  $\mu J(t, t_0)$ , and the stress history corresponding to constant strain  $\lambda$  introduced at age  $t_0$  is  $\lambda R(t, t_0)$ . Then the superposition of these two histories yields the aforementioned result.

This result is translated to a more convenient relation as follows: the initial strain at the time of first loading,  $t_0$  is  $\epsilon(t_0) = \lambda + \mu E_0^{-1} = \sigma(t_0)/E_0$ , where  $\sigma(t_0) =$  initial stress at the first loading and  $E_0 =$  initial elastic modulus. Note that, by definition,  $E(t_0) = 1/J(t_0, t_0)$ . Using  $\Delta$  to denote the changes from  $t_0$  to the current time  $t$ , e.g.,  $\Delta\epsilon(t) = \epsilon(t) - \epsilon(t_0)$ , we have for these strain and stress histories  $\Delta\epsilon = \mu(J - E_0^{-1})$ , and  $\Delta\sigma = \lambda(R - E_0)$ . Then we solve  $\lambda$  and  $\mu$  from the last two equations and substitute them into the foregoing expres-

sions for  $\epsilon(t)$  and  $\sigma(t)$ . Thus, defining the creep coefficient as  $\phi(t, t_0) = E(t_0)J(t, t_0) - 1$ , we obtain the following familiar form of the basic relation of the age-adjusted effective modulus method:

$$\Delta\epsilon(t) = \frac{\Delta\sigma(t)}{E''} + \frac{\sigma(t_0)}{E(t_0)} \phi(t, t_0), \quad E'' = \frac{E(t_0) - R(t, t_0)}{\phi(t, t_0)} \quad (3)$$

In other words, the strain increment is the sum of the elastic strain increment based on age-adjusted effective modulus  $E''$  and  $\phi$ -times the initial elastic strain. We now apply this rule to the stress and strain changes in the paste from  $t_0$  to  $t$  in the model in Fig. 1(a).

Let us now denote the stresses in the aggregate and in the portion of paste coupled in parallel as  $\sigma_a$  and  $\sigma_p$ , and their common strain as  $\epsilon_a$ . To calculate the compliance function, we apply stress  $\sigma(t_0)$  on the model at age  $t_0$  and then hold it constant, i.e.,  $\Delta\sigma = 0$ . The stress-strain relations for the aggregate and the parallel portion of the paste (the latter based on the age-adjusted effective modulus method), and the equilibrium condition for the parallel coupling, are

$$\Delta\sigma_a = E_a\Delta\epsilon_a; \quad \Delta\sigma_p = E_p'' \left[ \Delta\epsilon_p - \frac{\sigma_p(t_0)}{E_p(t_0)} \phi \right] \quad (4a,b)$$

$$\alpha\Delta\sigma_a + (1 - \alpha)\Delta\sigma_p = 0 \quad (5)$$

in which

$$\sigma_p(t_0) = \sigma(t_0) \frac{E_p(t_0)}{\alpha E_a + (1 - \alpha)E_p(t_0)} \quad (6)$$

where  $\sigma(t_0)$  can be chosen as 1. From these relations

$$\Delta\epsilon = (1 - \alpha)E_p'' \frac{\sigma(t_0)}{\alpha E_a + (1 - \alpha)E_p(t_0)} \frac{\phi_p(t, t_0)}{E_{ap}''} \quad (7a)$$

$$E_{ap}'' = \alpha E_a + (1 - \alpha)E_p'' \quad (7b)$$

where  $E_p''$  is according to (3) expressed for the portion of paste coupled in parallel. Adding the compliance function for the portion of paste coupled in series with the parallel coupling, we then finally obtain the compliance function of the composite that we have been seeking

$$J(t, t_0) = \frac{\beta}{\alpha E_a + (1 - \alpha)E_p(t_0)} \left[ 1 + (1 - \alpha) \frac{E_p''}{E_{ap}''} \phi_p(t, t_0) \right] + (1 - \beta)J_p(t, t_0) \quad (8)$$

in which

$$E_p'' = \frac{E_p(t_0) - R_p(t, t_0)}{\phi_p(t, t_0)}, \quad \phi_p(t, t_0) = E_p(t_0)J_p(t, t_0) - 1 \quad (9)$$

This formula was presented in a recent conference paper by Bažant (1993).

### INTERPRETATION OF MAXIMUM AGGREGATE COMPACTNESS

An intricate question is the determination of parameters  $\alpha$  and  $\beta$ , which characterize the subdivision of the paste between the parallel and series coupling for concrete [Fig. 1(a)]. First several hypotheses regarding  $\alpha$  and  $\beta$  were examined: (1) have  $\beta$  fixed for all the cases and then  $\alpha = A_g/\beta$ ; (2) have  $\alpha$  fixed for all the cases and then  $\beta = A_g/\alpha$ ; (3) choose a new parameter  $\gamma$  and then  $\alpha = \sqrt{\gamma A_g}$  and  $\beta = \sqrt{A_g/\gamma}$ . However, none of these three hypotheses gave satisfactory results in the fitting of test data described later. Therefore, a different idea was tried—the maximum compactness of aggregates (Fig. 2). This idea was inspired by de Larrard and Le Roy (1992), who exploited it for determining the elastic modulus of concrete.

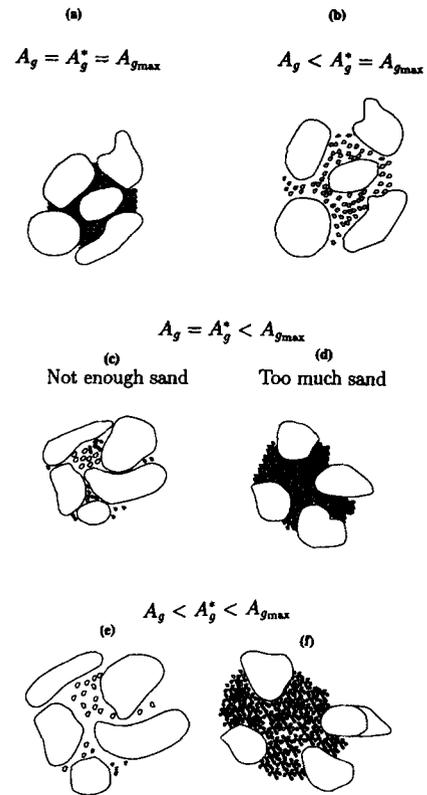


FIG. 2. Interpretation of Maximum Compactness  $A_g = A_{gmax}$  of Aggregates for Parallel and Series Model

The maximum compactness of the aggregate,  $A_{gmax}$  [Fig. 2(a)], is defined as the maximum volume content (total volume of all grains per unit volume of granular material) that can be reached for aggregates with given maximum and minimum grain diameters  $d_{min}$  and  $d_{max}$ . The  $A_{gmax}$ -value is attained when the aggregate size distribution (grading) follows a certain optimum grading curve (granulometric curve). Caquot (1935) experimentally established the following empirical formula:

$$A_{gmax} = 1 - 0.47(d_{min}/d_{max})^{1/5} \quad (10)$$

The coefficient 0.47 pertains to essentially round aggregate shapes (for aggregates of other shapes, e.g., with sharp corners or elongated, larger coefficients need to be used). For the aggregate content  $A_g = A_{gmax}$ , the concrete must be expected to exhibit the least creep possible because the number of contacts between the aggregate pieces is maximum and the paste is merely filling the remaining voids.

This fact is suggested by the studies of shrinkage of the prepacked (or preplaced) aggregate concrete (also called "prepakt") (Neville and Dilger 1970; Neville et al. 1981) and is also known for the so-called sifcon (slurry infiltrated fiber-reinforced concrete). To produce such concretes, the forms for placing concrete are first filled with the aggregate alone. After compacting it, the aggregate is infiltrated by a cement (or cement mortar) slurry.

The prepacked concrete should be describable quite well by the parallel coupling only. The reason is that this coupling yields the greatest possible stiffness of model, and that the maximum aggregate compactness yields the stiffest possible concrete. The layers of paste separating the adjacent aggregate pieces (Fig. 3) are coupled basically in series with the aggregate pieces. So we assume that the amount of paste that is needed to fill the voids at maximum aggregate compactness corresponds to the parallel coupling, and that the excess amount of paste used in the concrete corresponds to the series cou-

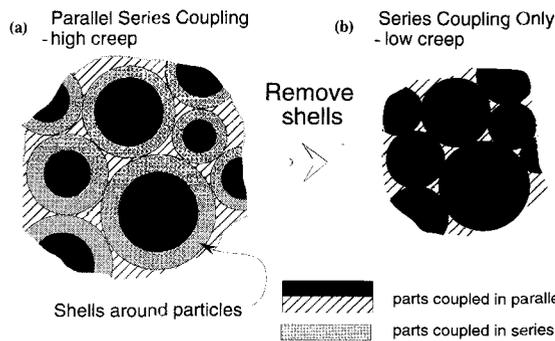


FIG. 3. Parts of Soft Matrix that Act Approximately in Parallel and Series Couplings with Hard Aggregates

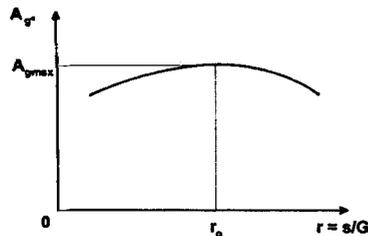


FIG. 4. Interpretation of  $A_g^*$  as Function of  $r = s/g$

pling (Fig. 3). This is the key idea for estimating the portion coupled in parallel.

It is thus clear that, for our parallel-series model, the minimum creep is reached if all the paste is coupled in parallel with the aggregates [Fig. 1(b)], in which case  $\alpha = A_{g_{\max}}$  and  $\beta = A_g/A_{g_{\max}} = 1$  [Fig. 1(b)]. For real concretes, for which the grading between  $d_{\min}$  and  $d_{\max}$  is not optimum [Figs. 2(e) and 2(f)], we can define  $A_g^*$  as the maximum relative compactness obtainable when the maximum possible amount of paste is withdrawn from the concrete without creating further voids [Fig. 2(b)]. During this withdrawal, the aggregates may be imagined to be brought into contact with each other to the maximum possible extent by means of homogeneous contraction, but the configurations of adjacent grains are not rearranged optimally as in vibration. So this is not the same as compacting the aggregates by vibration, as done by Caquot. This means that to fill the remaining voids more paste is needed than for Caquot's optimum case [Fig. 2(b)]; therefore  $A_g^* < A_{g_{\max}}$ .

Because Caquot's formula applies to the optimum size distribution (grading) between  $d_{\min}$  and  $d_{\max}$ , one must expect the difference  $A_g^* - A_{g_{\max}}$  to depend on the grading, or at least on the sand-gravel ratio  $r = s/g$ , as the simplest characteristic of grading. Because  $A_g^*$  attains a maximum at  $A_{g_{\max}}$ , the dependence of  $A_g^*$  on  $s/g$  near this minimum must obviously be a concave function. We take it as a quadratic function of  $r$  (Fig. 4)

$$A_g^* = 1 - \lambda(d_{\min}/d_{\max})^{1/5}[1 + \rho(r - r_0)^2], \quad r = s/g \quad (11)$$

in which  $\lambda$ ,  $\rho$ , and  $r_0$  are empirical constants to be found by data fitting.

There are various interesting points about the aforementioned configuration with  $\alpha = A_g^*$  and  $\beta = A_g/A_g^*$  [Fig. 1(a)]. If we consider constant  $A_g$  (same volume of the paste) and increase  $A_g^*$  by optimizing the grading of the aggregates or by taking another set of aggregates, we will increase the creep, which is at first surprising. But this can be understood upon realizing that if  $\beta = A_g/A_{g_{\max}}$  decreases, the portion of the paste coupled in series increases. Furthermore, if we consider constant  $A_g^*$ , any increase of  $A_g$  will reduce creep.

## VARIATION OF CEMENT PASTE COMPOSITION CAUSED BY HYDRATION

Before treating the hardened cement paste as a composite, we need to formulate a physical model for the variation of its composition caused by the chemical reaction of cement hydration as well as some related long-time processes, such as polymerization. We will consider a unit mass of cement paste and we will denote by lowercase letters the masses of various constituents per unit mass of cement paste (i.e., mass concentrations, dimensionless). Furthermore, we will denote by capital letters the volumes of various constituents per unit mass of cement paste (not of the constituent) (dimension  $\text{cm}^3/\text{g}$ ), i.e., the inverses of mass densities of the constituents within the paste. Thus,  $c$ ,  $w$ , and  $v_p$  will be the masses of anhydrous cement before hydration, water, and cement paste per unit mass of paste (of course  $v_p = 1$ ), and  $C$ ,  $W$ , and  $V_p$  will be the corresponding volumes of cement, water, and the paste. Obviously, the mass density of the paste is  $\rho_p = V_p^{-1}$ . The total initial volume of a unit mass of the paste (e.g., 1 g) can be expressed as

$$V_{p0} = \frac{1}{1-p} \left( \frac{c_0}{\rho_c} + \frac{w_0}{\rho_w} \right) \quad (12)$$

in which  $p$  = volume of the pores filled by entrained air and vapor per unit volume of paste (air porosity);  $c_0$  = initial mass of anhydrous cement;  $\rho_c = 3.15 \text{ g/cm}^3$ ; and  $\rho_w = 1 \text{ g/cm}^3$  (mass densities of anhydrous cement and water).

The progress of hydration may be characterized by function  $h(t)$  of concrete age  $t$ , representing the ratio of the mass of hydrated cement to the initial mass of cement. Although accurate analysis calls for different functions for different cements and mixtures, we choose for the sake of simplicity only one function

$$h(t) = \frac{h_\infty \sqrt{t}}{h_\infty + \sqrt{t}} \quad (13)$$

where, by definition,  $h_\infty = 1$  if the cement has become fully hydrated. The volume of anhydrous cement decreases with age  $t$  as

$$C(t) = \frac{c_0}{\rho_c} [1 - h(t)] \quad (14)$$

The full hydration of cement is known to require a mass of water equal to about  $r_w = 20\%$  of the mass of anhydrous cement (Taylor 1964; Powers 1968; Lea 1971). The cement gel produced by hydration is very porous. Its porosity may be approximately considered as  $r_p = 20\%$ , which characterizes the volume of pores filled by evaporable (not chemically bound) water (which consists of water adsorbed on the surfaces of the pores and interlayer water in the hydrates, but not the capillary water that is present in the hardened cement paste but is not counted as part of the cement gel; the capillary pores have typical dimensions of  $10^{-6} \text{ m}$  and the gel pores of  $3 \times 10^{-10} \text{ m}$  to  $100 \times 10^{-10} \text{ m}$ ).

Let  $W$ ,  $A$ , and  $G$  be the volumes of water, air, and cement gel per unit mass of the paste. To calculate  $G$  from the mix proportions, we need the relative volume of gel,  $r_g$ . Although the volume of hydration products with their pores is almost precisely equal to the initial volume of cement and water before they react, the volume of gel (without the capillary pores of the paste) is about 10% less than the combined volume of cement and water used in the chemical reaction of hydration (this phenomenon is related to Le Chatelier contraction). So we may set  $1 - r_g = 0.1$  or  $r_g = 0.9$ . The remaining volume becomes the pores filled with air and water vapor. This shrinkage, of course, is much smaller (typically

less than  $10^{-4}$ ) because of the restraining effect of the hardened framework of cement paste. The deficiency of gel volume caused by hydration is the source of autogenous shrinkage.

The aforementioned changes can be described by two equations: the equation of mass conservation stating that the masses of cement, chemically bound water and pore water must equal the mass of cement gel

$$c_0 - c(t) + r_w[c_0 - c(t)] + r_p G(t) = g(t) \quad (15)$$

and the equation of constancy of volume, stating that the volume of hydrated cement, bound water, and pore water, reduced according to Le Chatelier contraction, must equal the volume of gel

$$r_g[\rho_c^{-1}[c_0 - c(t)] + r_w[c - c(t)] + r_p G(t)] = G(t) = \rho_g^{-1}g(t) \quad (16)$$

where  $\rho_g$  = mass density of the gel. From (15) and (16), we can solve

$$G(t) = k_g c_0 h(t); \quad \rho_g = 2.31 \text{ g/cm}^3;$$

$$k_g = \frac{r_g(\rho_c^{-1} + r_w)}{1 - r_p r_g} = 0.568 \quad (17a-c)$$

Then the volumes of evaporable (free) water and air (with water vapor) are

$$W(t) = W_0 - r_w c_0 h(t) - r_p G(t) \quad (18)$$

$$A(t) = \frac{p}{1-p} \left( \frac{c_0}{\rho_c} + w_0 \right) + \left( \frac{1}{r_g} - 1 \right) G(t) \quad (19)$$

in which the subscripts 0 denote the initial values in the mix. The last equation states that the volume of air equals the initial volume of entrained air plus Le Chatelier contraction.

In the fresh mix, the anhydrous cement grains are far from each other, separated by layers of water. Therefore, during the first hours up to the time of set, the gel that is produced by the chemical reaction of hydration lends no stiffness to the fresh paste. Initially, the solid particles can move in response to hydration but, after a few hours, the moment of set,  $t_s$ , is reached, at which the cement grains get close enough to form a solid framework. From that moment on, the gel that is produced further serves to increase the stiffness of the existing microstructure. It is thus clear that we must distinguish the time periods before and after the set. Let  $h_s$  be the degree of hydration at the moment of set, that is,  $h_s = h(t_s)$ . As is well known,  $t_s$  depends on the water-cement ratio  $w/c$ , and for this purpose we choose the empirical relation

$$t_s = \frac{17}{24} \left( \frac{w}{c} - 0.1 \right)^{1.2} \text{ (days)} \quad (20)$$

The numerical coefficients would certainly change somewhat for various types of cement (depending, for example, on the amount of  $C_3A$ ). Prior to the set, one has mainly anhydrous cement grains surrounded by envelopes of gel in the state of suspension in water. Therefore, the volume deficiency created by Le Chatelier contraction prior to the set is manifested as a decrease of volume of the hardened paste. Only the volume deficiency created after the set creates voids (or pores) since the microstructure has become stiff enough to prevent significant volume contraction (other than small strains due to stress). To take this behavior into account, we modify (19) as

$$A(t) = \frac{p}{1-p} \left( \frac{c_0}{\rho_c} + w_0 \right) + \left( \frac{1}{r_g} - 1 \right) [G(t) - G(t_s)] \quad (21)$$

Now we can also calculate the volume of the hardened cement paste

$$V_p(t) = A(t) + C(t) + W(t) + G(t), \text{ for } t < t_s, \quad (22)$$

while  $V_p(t) = V_p(t_s) = \text{constant}$  for  $t \geq t_s$ .

As with concrete, we will now adapt, for reasons already explained, the parallel-series model to the cement paste. As illustrated in Fig. 5, the anhydrous cement grains are represented by an element of cross section  $a(t)$  and length  $b(w/c)$ . The cement gel, which represents a contiguous matrix, is imagined to be separated into a part of cross section  $v(t)$  and length  $b(w/c)$  that is coupled in parallel with anhydrous cement grains, and another part of length  $1 - b(w/c)$  and the same cross section  $v(t)$  that is coupled in series, as shown in Fig. 6 (for the sake of simplicity, we chose the parts of cement gel coupled in series and in parallel to have equal cross section,  $v(t)$ , at all times). The increase of  $v(t)$  models the solidification process involving the dissolution of cement grains and the precipitation of the hydrates. Parameters  $a$  and  $b$  for the anhydrous cement are analogous to  $\alpha$  and  $\beta$  for the aggregates in Fig. 1. Parameter  $b$  for the length of the parallel and series parts of cement gel is empirical and constant in time but can vary depending on the composition of the paste, particularly the water-cement ratio  $w/c$ .

Then, according to the model in Fig. 6

$$a(t)b = C(t)/V_p; \quad v(t) = G(t)/V_p \quad (23, 24)$$

According to the parallel and series couplings of the model in Fig. 6, the overall elastic modulus of the hardened paste,  $E_{p0}$ , at the time of first loading  $t_0 (> t_s)$  is given by the equation

$$\frac{1}{E_{p0}} = \frac{b}{E_g^*} + \frac{1-b}{v(t_0)E_g}, \text{ with } E_g^* = a(t_0)E_{ac} + v(t_0)E_g \quad (25)$$

where the Young's elastic moduli of anhydrous cement grains and cement gel are denoted as  $E_{ac}$  and  $E_g$ .

A similar equation could be written for the modulus of the paste  $E_p$  at any time  $t$ . This modulus must always increase

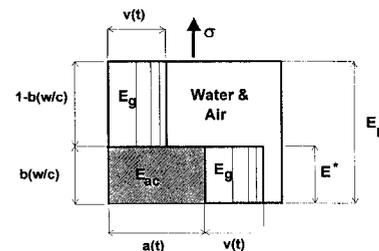


FIG. 5. Parallel and Series Model for Solidifying Cement Paste

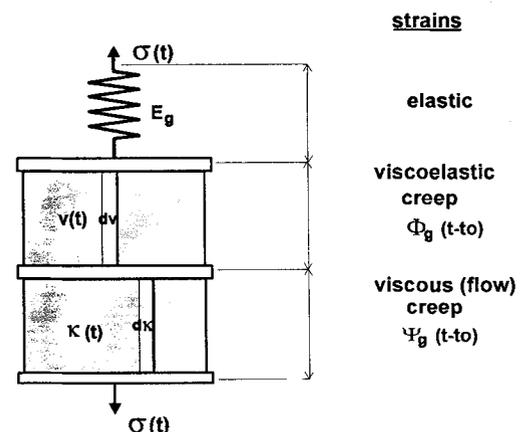


FIG. 6. Model for Role of Solidification in Creep for Cement Gel

due to hydration, i.e.,  $dE_p/dt$  or  $dE_p/d[h(t)] > 0$ . A sufficient but not necessary condition to satisfy this inequality is

$$1.79E_g b(w/c) > E_{ac} \quad (26)$$

which restricts the choice of empirical parameter  $b$ .

### COMPLIANCE FUNCTION OF PASTE BASED ON SOLIDIFICATION THEORY

The solidification process of cement paste may be regarded as a gradual precipitation of layers of cement gel, characterized by volume growth  $v(t)$ . These layers are in a stress-free state at the moment of their precipitation. Our mathematical formulation will be similar to that used by Bažant and Prasannan (1989) for concrete as a single composite system; however, the solidification theory is now applied only to the gel, which is on the submicroscopic level.

The principal idea of the solidification theory is that the aging property of creep is not a true property of the material but an apparent property. The material must be decomposed into constituents of nonaging properties, and the aging comes about as a change in volume concentration of one component, whose mechanical properties are age-independent. This type of formulation is not only physically justified but is also necessary from the thermodynamic viewpoint. Chemical thermodynamics does not deal with substances of time-dependent properties; rather, it deals with compounds of age-independent properties but varying concentrations or volumes. Moreover, by virtue of modeling the solidification process in this manner, one can take into account one obvious restriction, namely, that a layer of the solidified constituent at the moment of its precipitation must be free of stress. This restriction is by no means automatically implied, and in fact some creep models proposed in the past violated this restriction if interpreted in terms of solidification.

According to Bažant and Prasannan (1989), the compliance function of the cement gel is chosen as the sum of the viscoelastic strain characterized by function  $\Phi_g(t, t_0)$  and viscous flow characterized by function  $\Psi_g(t, t_0)$ , that is

$$\Phi_g(t - t_0) = \frac{1}{E_g} + q_2 \ln \left[ 1 + \left( \frac{t - t_0}{\lambda_0} \right)^n \right],$$

$$\Psi_g(t, t_0) = \frac{t - t_0}{\eta_0} \quad (27)$$

in which  $E_g$  = elastic modulus of the gel;  $q_2$  and  $\eta_0$  = constants;  $\eta_0$  represents the viscosity of the gel; and  $\lambda_0$  = an empirical parameter that is approximately  $\lambda_0 = 1$  day (Bažant and Prasannan 1989). The expression for the viscoelastic strain is the log-power law, which is justified by the observation that for short times the creep curves are approximately power curves and for long times they are approximately logarithmic curves. Indeed, for short  $t - t_0$ , the logarithmic term in (27) approximately equals  $q_2(t - t_0)^n$ , but for long  $t - t_0$  it approximately equals  $q_2 n \ln(t - t_0)$ , and (27) gives the simplest transition between these two functions.

Because of the solidification process, the compliance function  $J_h(t, t_0)$  for the hydrates of growing volume concentration must be obtained in terms of rates, which is written as follows:

$$J_h(t, t_0) = \frac{1}{v(t)} \dot{\Phi}_g(t - t_0) + \frac{1}{\eta_0 \kappa(t)} \quad (28)$$

where  $v(t)$  = volume associated with  $\Phi_g$  and  $\kappa(t)$  = volume associated with  $\Psi_g$  as in Fig. 7. The overdots denote differentiation with respect to  $t$ , e.g.,  $\dot{J}(t, t_0) = \partial J(t, t_0) / \partial t$ . [Note that if the compliance rates in (28) were replaced by the compliance values, we would imply that the material solidifies

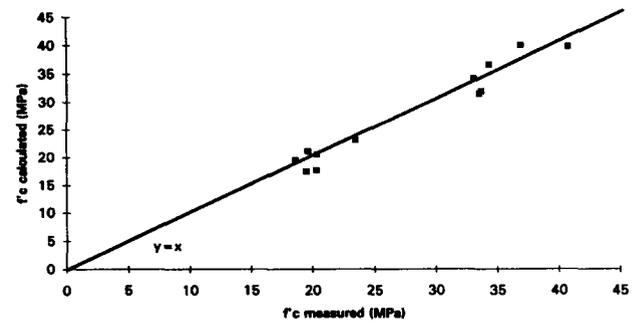


FIG. 7. Comparison between Measured Strength and Feret Law Taking Into Account Volume of Entrained Air

in a stressed state, which would be physically unacceptable.] According to (24)

$$\frac{1}{v(t)} = \frac{V_p}{k_g c_0} \left( \frac{1}{\sqrt{t}} + \frac{1}{h_x} \right) \quad (29)$$

Function  $\kappa(t)$ , which characterizes the growth of viscosity and is important for long-time creep of a paste loaded at early age, must also be quantified in relation to the growth of gel volume. A simple way to describe this function is to assume that, at the moment of loading, it is proportional to  $v(t_0)$  and later increases proportionally to  $t$  because this preserves the logarithmic form of the long-time creep curves; thus

$$\kappa(t) = v(t_0) \frac{t}{q_4 \eta_0} \quad (30)$$

In a strict sense, however, parameter  $\kappa(t)$  must be interpreted more generally than just the volume of cement gel. The reason is that after several months the volume increase of the cement gel is very small, yet the creep properties still vary significantly up to many years of age. As one possible explanation, it was proposed that the cement gel undergoes polymerization in which new bonds are being formed for a long time even after the volume growth has almost ceased. Because creep is probably caused by debonding in the cement gel, polymerization must be expected to reduce creep. Also, addition of new bonds increases the load-bearing part of cement paste volume that is coupled to the solid framework. Some parts of hardened cement gel may be loose lumps when formed and require further bonds to start carrying the load. To take this phenomenon into account, volume  $\kappa(t)$  is better interpreted as the volume of the load-bearing portion of the cement gel that represents the part that is sufficiently bonded to function as part of the solid framework [see also Bažant and Prasannan (1989)].

The compliance function of the hydrates can be obtained by integration of equation (28), which may be written as

$$J_h(t, t_0) = q_1 + \frac{V_p}{k_g} c_0 q_2 \{ Q(t, t_0) + q_3 \ln[1 + (t - t_0)^n] \} + \frac{q_4}{v(t_0)} \ln \left( \frac{t}{t_0} \right) \quad (31)$$

This is analogous to the expression obtained for the solidification theory in Bažant and Prasannan (1989)

$$q_1 = \frac{1}{E_g v(t_0)}; \quad q_3 = \frac{1}{h_x} \quad (32a,b)$$

and  $q_2$  and  $q_4$  are two parameters that can be adjusted according to experimental data.

Function  $Q(t, t_0)$  is defined by its rate with respect to  $t$

$$\dot{Q}(t, t_0) = \left(\frac{\lambda_0}{t}\right)^{1/2} \frac{n(t-t_0)^{n-1}}{\lambda_0^n + (t-t_0)^n} \quad (33)$$

Integration of this expression leads to the binomial integral, which, unfortunately, cannot be evaluated in a closed form for a realistic value of  $n$ . A simple approximate formula for the integral  $Q(t, t')$ , however, exists, and a numerical table has been given (Bažant and Prasanna 1989).

Now we have to introduce the condition of parallel coupling of the hydrates (gel) with the anhydrous cement grains. The model in Fig. 6 is loaded at  $t_0$  by stress  $\sigma_0$ , which is kept constant afterwards. For the parallel coupling we have the equilibrium relations

$$a(t_0)\sigma_{ac}(t_0) + \sigma_h(t_0) = \sigma_0, \text{ at } t = t_0 \quad (34a)$$

$$a(t)\sigma_{ac}(t) + \sigma_h(t) = \sigma(t) = \sigma_0, \text{ at } t > t_0 \quad (34b)$$

where  $\sigma_{ac}$  = stress transmitted by the element representing the anhydrous cement, considered as completely elastic (Young's modulus  $E_{ac}$ ); and  $\sigma_h$  (subscript  $h$  is for hydrate) = stress transmitted by the growing gel

$$\sigma_h(t) = \int_{\tau=t_0}^{\tau=t} \sigma_h(\tau, t_0) \frac{d\nu(\tau)}{d\tau} d\tau \quad (35)$$

Subtracting (34) we have

$$a(t)\Delta\sigma_{ac} + \Delta a\sigma_{ac}(0) + \Delta\sigma_h = 0 \quad (36)$$

Furthermore

$$\Delta\varepsilon_{ac} = \Delta\varepsilon_h; \quad \sigma_{ac}(t_0) = E_{ac} \frac{\sigma_0}{E_0^*} \quad (37a,b)$$

$$\sigma_h(t_0) = \sigma_0 - a(0)\sigma_{ac}(t_0); \quad \Delta\varepsilon_{ac} = \frac{\Delta\sigma_{ac}}{E_{ac}} \quad (37c,d)$$

Because  $\varepsilon_h = \sigma_h J_h(t, t_0)$ , we have  $\Delta\varepsilon_h = \Delta\sigma_h/E_h^* + \sigma_h^0 \phi_h/E_h$ , and so

$$\Delta\varepsilon_h = \frac{\sigma_0}{E_0^*} \frac{E_h - R_h - \Delta a E_a}{a(t)E_{ac} + E_h^*} \quad (38)$$

Consequently, the compliance function of the hardened cement paste, corresponding to the entire model in Fig. 6 is

$$J_p(t, t_0) = \frac{1}{E_p^0} + \frac{1-b}{E_h} \phi(t, t_0) + \frac{b}{E_0^*} \frac{E_h - R_h - \Delta a E_a}{a(t)E_{ac} + E_h^*} \quad (39)$$

This compliance function can then be combined with the effect of the aggregate according to the previously derived equation (8), to predict the overall compliance function of concrete.

## ANALYSIS OF TEST DATA

Although the effects of concrete composition on creep were experimentally studied by many investigators (Bažant et al. 1976), the tests of Ward et al. (1969) seem to provide the only sufficiently comprehensive data set permitting numerical calibration and verification of the present model. These investigators performed 14 tests of basic creep using the same materials: two pastes, four mortars, and eight concretes (labeled as P, M, and C), with or without entrained air (labeled with or without A); see Table 1, in which  $A_g$  = aggregate volume within a unit volume of concrete;  $s/g$  = ratio of fine to coarse aggregates (sand to gravel) by mass; and  $f'_c$  = standard compression strength at the age of loading  $t_0 = 7$  days for all the tests. Cement of type III was used. The coarse aggregates, of sizes from 2.4 to 10 mm, consisted of 55% quartz and 25% limestone (ASTM absorption was 0.92). The fine aggregate (or sand), of grain sizes from 0.1 to 2.4 mm,

TABLE 1. Composition and Main Characteristics of Ward et al. (1969) Data

Mix number (1)	w/c (2)	g (3)	s/G (4)	Air content (%) (5)	$f'_c$ (MPa) (6)	$E$ (GPa) (7)
P7	0.80	—	—	4.00	12.5	4.4
P7A	0.63	—	—	10.70	14.9	7.1
M5	0.88	0.555	—	4.00	17.6	15.3
M5A	0.69	0.514	—	12.50	21.0	12.6
M6	0.60	0.502	—	3.75	40.0	18.8
M6A	0.45	0.396	—	11.00	39.8	15.9
C1	0.80	0.705	1/1.23	2.75	23.1	24.2
C1A	0.68	0.683	1/1.38	8.00	20.5	16.8
C2	0.88	0.716	1/1.16	3.25	17.4	17.7
C2A	0.70	0.697	1/1.33	8.00	19.5	18.7
C3	0.63	0.687	1/1.31	2.50	31.5	21.2
C3A	0.48	0.659	1/1.87	7.75	31.9	26.4
C4	0.62	0.690	1/1.28	3.00	34.1	24.2
C4A	0.47	0.651	1/2.12	8.00	36.6	23.2

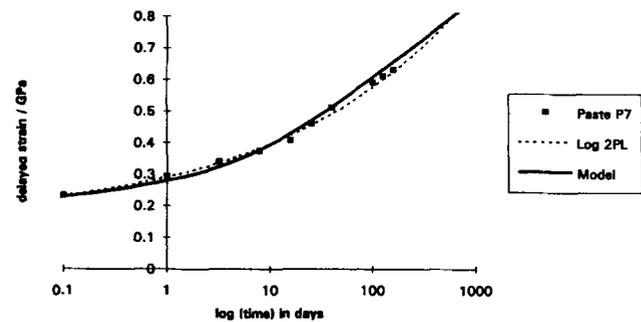


FIG. 8. Paste P7: Best Fit with Model and with Double Power Law

was a natural sand (with ASTM absorption 2.0). The test specimens were cylinders of 76 mm in diameter, loaded by compressive stress  $0.3f'_c$ . The mass densities of the fine and coarse aggregates were 2.6 and 2.65 g/cm<sup>3</sup>, respectively.

One advantage of these data is that the water cement ratio  $w/c$  of the paste in these concretes and mortars ranged from 0.45 to 0.88 in the mix, and from 0.41 to 0.80 after a correction for the water absorbed by the aggregates. This is a broad range. Therefore, the expression of the compliance function for pastes P7 and P7A cannot be used for other pastes. The compliance function  $J_p(t, t')$  for the other pastes must be deduced from that for paste P7 by using the present theory for the paste composition. Thus, comparisons with the data for the other pastes and concretes provide a verification of this theory.

Furthermore, the measurement of the comprehensive strength  $f'_c$  provides an additional check on the data (especially on the amount of air, which is difficult to measure). The relation to  $W/c$  and  $A/c$  is known to be quite well described by Feret (1896) law, and can be written as

$$f'_c = KR_c \left(1 + 3.1 \frac{W + V}{c}\right)^{-2} \quad (40)$$

in which  $W$  and  $V$  = volumes of water and of voids (pores) per unit mass of concrete;  $c$  = mass of cement per unit mass of concrete;  $R_c$  = compressive strength of the cement, measured on a 1:0.5:2 mortar specimen and taken here as 60 MPa; and  $K$  = empirical factor depending on the type of aggregate. Fig. 7 demonstrates that Ward et al.'s (1969) data are quite consistent from a mechanical point of view with Feret (1896) law in which  $K = 6.26$ .

The measured values of the compliance function  $J(t, t_0)$  are shown by the data points in Figs. 8–11, and the predictions of the present model are shown by the curves. The analysis

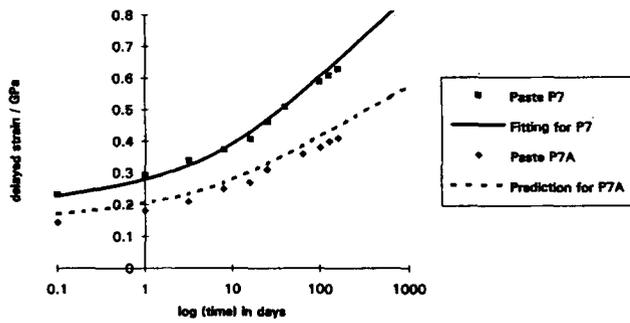


FIG. 9. Results for Pastes P7 and P7A; Optimum Adjustment of Parameter  $b(w/c)$

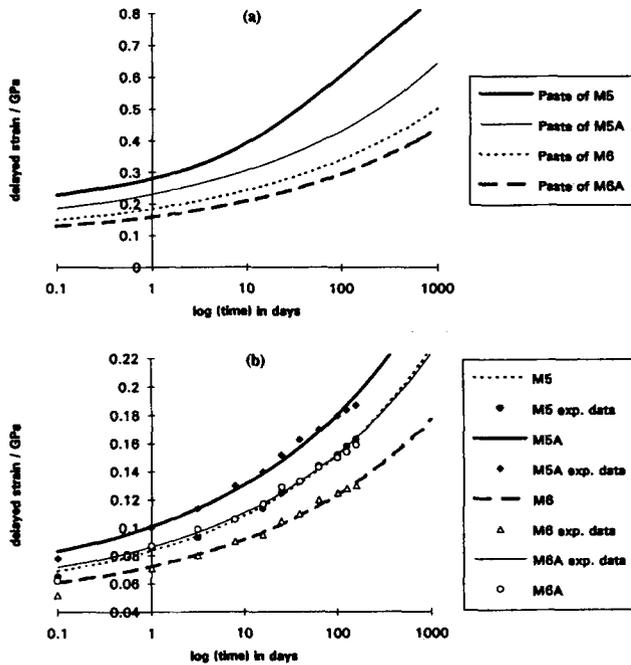


FIG. 10. Prediction of Compliance Function for Pastes Extracted from Mortars and Best-Fit Value of  $\alpha$  for Mortars ( $\beta = A_g/\alpha$ )

of concrete as a composite begins with the known values of the compliance function of the paste. The optimum fitting of paste P7 with the log-double power law (Bažant 1986; Bažant and Chern 1985) provided the expression

$$J_p(t, t_0) = \frac{1}{6.6} \{1 + 134.1 \ln[1 + 0.0278(t_0^{-3/4} + 0.016)(t - t_0)^{0.242}]\} [(GPa)]^{-1} \quad (41)$$

in which  $t$  and  $t_0$  must be given in days, and the exponent  $3/4$  was chosen a priori because the age effect had not been measured in this experimental study. In fitting the log-double power law to test data, it was assumed that the asymptotic modulus  $E_0$  for extrapolation of the creep curve to infinitely fast loading is 1.5 times the conventional static modulus of elasticity (which corresponds to load duration of about 0.1 day).

The parameters of the model, that is, the elastic moduli of anhydrous cement and of cement gel, and parameters  $q_2$  and  $q_4$  of the compliance function for the gel, were obtained by optimum fitting of the data P7 in Fig. 9. The following values were found:  $E_{ac} = 35$  MPa;  $E_g = 80$  MPa;  $q_2 = 0.1768$ ; and  $q_4 = 0.04304$ . The value of parameter  $b$  at  $w/c = 0.8$  was chosen as  $b = 0.25$ .

The aforementioned parameter values have been kept fixed in all calculations. The compliance functions of the other

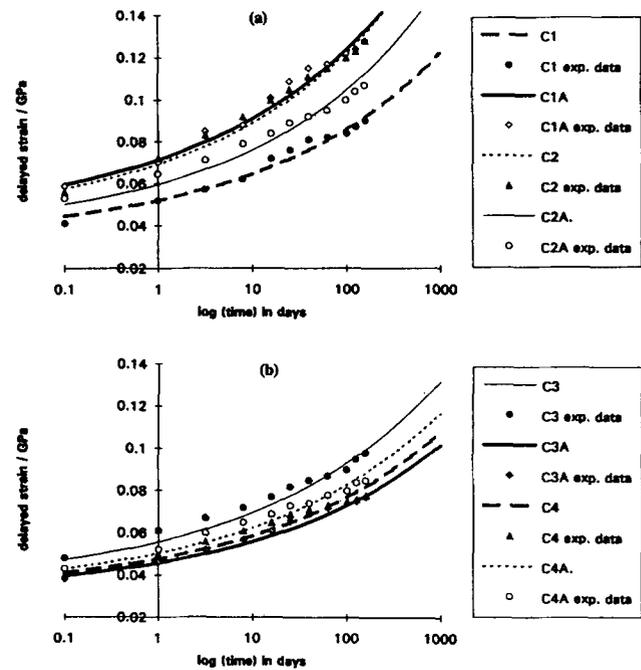


FIG. 11. Best-Fit Value of  $\alpha$  for All Eight Concretes ( $\beta = A_g/\alpha$ )

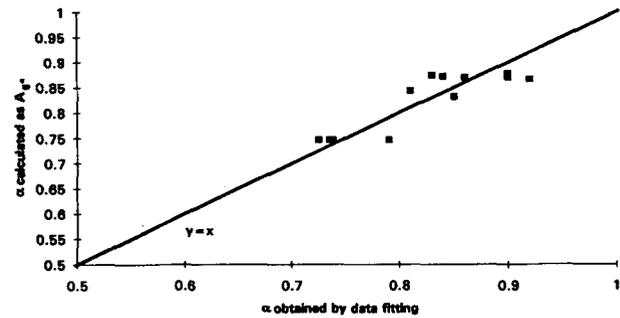


FIG. 12. Comparison between  $\alpha$  Obtained by Fitting and  $\alpha$  Taken as  $\alpha = A_g^*$  According to Modified Caquot's Formula

pastes (Figs. 10 and 11) were then predicted by using the present model. In doing so, parameter  $b$  for the parallel-series coupling with the hydrates, was taken as a decreasing function of  $w/c$

$$b(t_s, w/c) = b(w/c) = 0.574 - 0.787(w/c)^{3.97} \quad (42)$$

The reason for this decrease is that, in a paste of a lower  $w/c$  ratio, the portion of the gel in the thin layers between the cement grains (which corresponds to the series coupling) is smaller. The compliance functions obtained for the pastes by the model were then smoothed by fitting them also with the log-double power law

$$J_p(t, t_0) = \frac{1}{E_p} \{1 + \psi_0 \ln[1 + \psi_1(t_0^{-3/4} + 0.016)(t - t_0)^n]\} \quad (43)$$

which was then easier to introduce in the parallel series model for concrete in (8). The conventional static elastic modulus of the paste is obtained from (43) as  $E_p = 1/J_p(t_0 + 0.1, t_0)$ . The optimum fitting of the test data for various individual tests of mortars and concretes was done individually (Figs. 11 and 4). The optimum fits also indicated for the elastic modulus of the aggregate (which was not reported), the value  $E_a = 70$  MPa.

The values of  $\lambda$ ,  $\rho$ , and  $r_0$  in (11), based on the idea of maximum aggregate compactness, have been left undetermined. They must be identified experimentally from tests.

Optimum fitting of the data by Ward et al. (1969) provided the values  $\lambda = 0.283$ ,  $r_0 = 0.639$ , and  $\rho = 5.15$ . With these values, the modified Caquot's formula led to the results in Fig. 12, which compares the value of  $\alpha$  obtained by fitting the present model to data with the value of  $A_g^*$  calculated from (11). The agreement with test data is much better than that achieved with the alternative assumptions discussed before. It must also be kept in mind that creep and shrinkage tests always exhibit significant scatter (Bažant et al. 1987a, b), and in the light of such scatter the results seen in the figures appear quite acceptable.

The values obtained for  $A_g^*$  are larger than those predicted by Caquot's formula. Finding a greater value for  $A_g^*$  and  $A_{g,max}$  implies a larger creep for the concrete. This might perhaps also be explained by other phenomena, such as possible separation of the paste from the aggregates, or the fact that the layers near the pieces of gravel might have a higher porosity than the rest of the paste and thus exhibit more creep than the paste without the aggregate. Another reason why  $A_g^*$  is larger than expected could be that modeling cement as only a two-phase composite might not be entirely realistic (Nilsen and Monteiro 1993).

## SUMMARY AND CONCLUSIONS

1. The proposed parallel-series model for determining the creep of concrete from the creep of hardened cement paste and the elasticity of aggregate is simple to implement and gives realistic results.
2. The parallel-series model can also be introduced for predicting the effects of age and water-cement ratio on the creep of hardened cement paste. To this end, the previously proposed solidification theory is generalized to describe interaction with elastic grains of anhydrous cement, the influence of changing porosity, and the growth of the hydration products in time. As in the original solidification theory, the basic creeping constituent (i.e., the cement gel) is considered nonaging and the aging property of creep is obtained entirely as a consequence of the growth of volume concentration of the load-bearing gel due to the progress of hydration. In a more precise physical interpretation, this growth may include the effect a gradual increase in the number of loads (or polymerization), which causes that formation of further bonds couples additional hydration products as part of the load-bearing solid framework of the material and thus reduces creep.
3. The subdivision of the matrix (cement paste) between the parts coupled in parallel and in series is based on an adaptation of Caquot's concept of maximum compactness of aggregates. The portion of paste coupled in parallel represents the amount of the paste needed to fill the voids when the aggregates are at their maximum possible compactness. The remaining portion of the paste is then considered to be coupled in series. This new idea appears to give good agreement with comprehensive test data from the literature.
4. To sum up, the basic ideas in the present model are: (1) to use the parallel-series model and the age-adjusted effective modulus method to handle creep with aging; (2) to determine the subdivision of matrix in the model on the basis of maximum aggregate compactness; and (3) to combine the solidification theory with the parallel-series model.

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