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TC 107-CSP GUIDELINES FOR THE FORMULATION OF CREEP AND SHRINKAGE PREDICTION MODELS

Justification and refinements of model B3 for concrete creep and shrinkage 2. Updating and theoretical basis

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Following statistical evaluation in part 1, this part deals with the improvement of prediction by updating one or two parameters of the model on the basis of short term tests and theoretical derivation of some formulae. The updating of model parameters is particularly important for high strength concretes and other special concretes containing various admixtures, superplasticizers, water-reducing agents and pozzolanic materials. For the updating of shrinkage prediction, a new method is presented in which the shrinkage half-time is calibrated by simultaneous measurements of water loss. This approach circumvents the ill-posedness of the shrinkage extrapolation problem. Theoretical justifications of various aspects of the model are given and a new formula for the additional creep due to drying (or stress-induced shrinkage) is derived. The new model should allow a more realistic assessment of the creep and shrinkage effects in concrete structures, which significantly affect durability and long term serviceability of civil engineering infrastructure.

1. UPDATING CREEP AND SHRINKAGE PREDICTIONS BASED ON SHORT TERM MEASUREMENTS

An important advantage of the present model is that all the free parameters for creep with elastic deformation, i.e., q_1, q_2, q_3, q_4, q_5, are contained in the formulas linearly. Therefore, linear regression based on the least-squares method can be used to identify these parameters from test data, which minimizes the value of $\bar{\omega}^2_{\text{all}}$. The same is true of parameters $\varepsilon_{\text{sh}\infty}$ for shrinkage. Thus the only nonlinear parameter of the entire model B3 is the shrinkage half-time τ_{sh}.

The largest source of uncertainty in the creep and shrinkage prediction model is the dependence of model parameters on the composition and strength of concrete. This uncertainty can be reduced greatly by carrying out on the given concrete short term creep and shrinkage measurements (of duration less than 1–3 months) and adjusting the values of some model parameters accordingly. This is particularly important for special concretes such as high strength concretes or fibre-reinforced concretes. Various types of admixtures, superplasticizers and pozzolanic ingredients used in these concretes have been found to have a significant effect [29, 30]. Empirical formulas for the effects of all these ingredients on the model parameters would be very difficult to formulate because of the great variety of additives and their combinations.

2. UPDATING CREEP PREDICTIONS

Compared with other models, including the original BP model, the solidification theory, which is the basis of model B3, has the advantage that the adjusted values of model parameters except τ_{sh} can be obtained easily by linear regression of the short term test data. Consider, for example, the data for basic creep by L'Hermite *et al.* [13], for which the present formulae for the effect of composition and strength do not give a good prediction, as is apparent from Fig. 1 (for information on these data see [3]). We now pretend we know only the first 5 data points for the

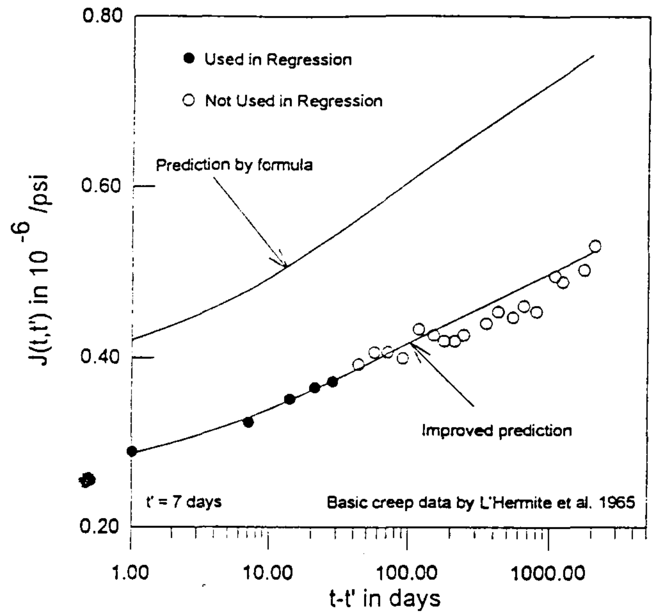

Fig. 1 Example of improving the prediction of creep by the use of short term test data. Basic creep data by L'Hermite *et al.* [14].

first 28 days of creep duration, which are shown by the solid circles. We consider that the compliance function is updated by only two update parameters p_1 and p_2, introduced as follows:

$$J(t, t') = p_1 + p_2 F(t, t') \qquad (1)$$

in which

$$F(t, t') = C_0(t, t') + C_d(t, t', t_0) \qquad (2)$$

Function $F(t, t')$ is evaluated according to the model, using the formulae for the effect of composition parameters and strength (in our subsequent example, $F(t, t') = C_0(t, t')$ because only basic creep is considered). If the data agreed with the form of model B3 exactly, the plot of $J(t, t')$ versus $F(t, t')$ would have to be a single straight line for all t, t' and t_0. The vertical deviations of the data points from this straight line represent errors which may be regarded as random and are to be minimized by least-squares regression. So we consider the plot of the known (measured) short term values $Y = J(t, t')$(up to 28 days of creep duration) versus the corresponding values of $X = F(t, t')$, calculated from model B3, and pass through these points the regression line $Y = AX + B$. Then the slope A and the Y intercept B of this line give the values of p_1 and p_2 that are optimum in the sense of the least-squares method; $A = p_2$ and $B = p_1$. According to the well known normal equations of least-squares linear regression,

$$p_2 = \frac{n \sum (F_i J_i) - (\sum F_i)(\sum J_i)}{n \sum (F_i^2) - (\sum F_i)^2}, \quad p_1 = \bar{J} - p_2 \bar{F} \qquad (3)$$

where subscripts $i = 1, 2, \ldots n$ label the known data points, n is their total number, $F = J(t, t')$, $J = J(t, t')$, $\bar{J}$ is the mean value of all the measured J_i values, and $\bar{F}$ is the mean value of all the corresponding F_i values. Obviously, the improvement of long term prediction achieved by short term measurements is in this example very significant. The well known formulae of linear regression also yield the coefficients of variation of p_1 and p_2, which in turn provide the coefficients of variation of $J(t, t')$ for any given t and t'. (However, the uncertainty of long term predictions obtained by updating from the short term data should properly be handled by the Bayesian statistical approach [31, 32].)

For the planning of short term creep measurements, note that prediction improvement based on short term data is more successful if the creep measurements begin at very short times after loading (and likewise for shrinkage, if the measurements begin immediately after the stripping of the mold). Also the measurements should be taken at approximately constant intervals in log-time, e.g., at 30, 100, 300, 1000, 3000 . . . s. The reason is that the creep curves rise smoothly through the entire range from 0.0001 s to 30 years. In our example, the first reading was taken as late as 1 day after loading, as is often done, and therefore as many as 28 days of creep data were needed for prediction improvement. In a similar example using a different data set it was shown [7] that if the first reading is taken as soon as possible after loading (within 1 min) and about six readings are taken during the first 2 days of load duration (uniformly spaced in log-time), a similar improvement can be achieved using those readings only. Thus the required duration of short term test could be reduced if the readings begin immediately after loading. Anyhow, for reliable prediction of creep values for over five years of creep duration, it is recommended to carry out short term tests of at least 28 day duration (with the first reading immediately after loading and further readings equally spaced on the logarithmic scale of creep duration).

3. PROBLEMS IN UPDATING SHRINKAGE PREDICTIONS

The problem of updating is much harder for shrinkage than for creep because of the recently discovered ill-posedness of the shrinkage extrapolation problem [7]. The updating based on only short term measurements of shrinkage values is not possible unless the measurements extend into the final stage in which the shrinkage curve begins to level off on approach to the final value. Before reaching this final stage, it is impossible to tell, without additional information, how much longer the curve of ε_{sh} versus log $(t - t_0)$ will rise at a non-decreasing slope and when it will start to level off.

If the time range of shrinkage measurements is not sufficiently long, the problem of fitting the shrinkage formula of model B3 in Equations (9) and (10) of the RILEM recommendation to the measured strain values

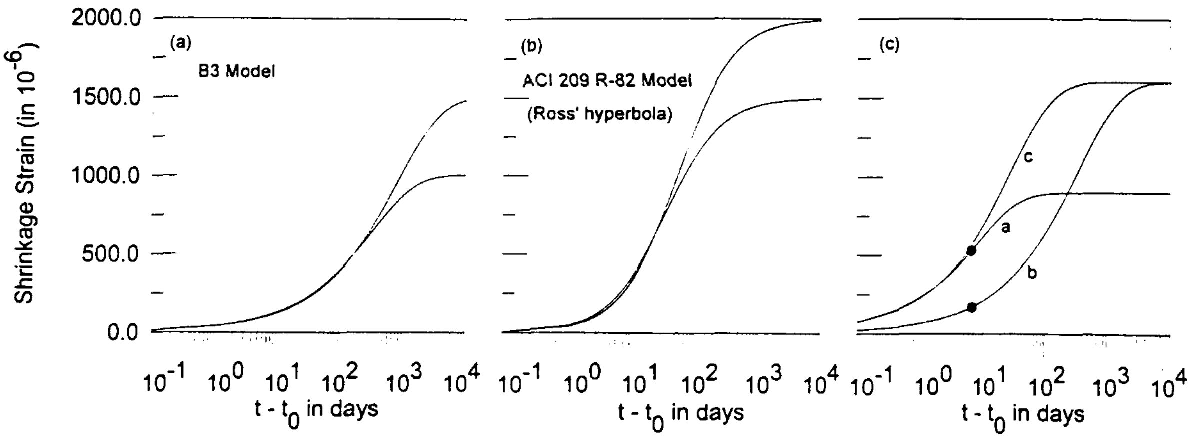

Fig. 2 (a, b) Example of shrinkage–time curves giving nearly the same development of shrinkage for short times but very different final values, and (c) possible shrinkage curves of identical specimens of different concretes.

is what is known in mathematics as an ill-posed problem. In other words, very different values of parameters $\varepsilon_{sh\infty}$ and τ_{sh} can give almost equally good fits of short term data, as documented in Fig. 2a, b (taken from [7]). This is true not only for the present model B3 formulae but also for all other shrinkage formulae, including Ross' hyperbola used in the ACI model (but this formula does not give a good shape of the shrinkage curves and disagrees with the asymptotic forms for short and long times required by the RILEM Committee Guidelines [5]). The problem is clear from Fig. 2a,b, in which two shrinkage curves according to the present model or the ACI model, corresponding to very different parameter values, are shown to nearly coincide for a long period of time. If the data do not reach beyond the time at which the two curves shown in Fig. 2 begin to diverge significantly, there is no way to determine the model parameters unambiguously.

It must be warned that comparisons of shrinkage measurements of two concretes can be misleading if the test durations are not long enough. Two possible shrinkage curves of two concretes a and b are shown in Fig. 2c. Curve a is the shrinkage curve of a relatively porous concrete that dries quickly and reaches moisture equilibrium soon but has a low final shrinkage. Curve b is the shrinkage curve of a dense concrete which dries slowly but has a large final shrinkage. If these concretes are compared based on the short term tests terminating at the points marked, one would think that concrete b is much better. However, the opposite is true. (Unfortunately many comparisons of shrinkage in the literature might be afflicted by this problem.)

From such plots it is concluded that even a crude approximate updating of the final value of shrinkage would require, for 6 in (15 cm) diameter cylinders, measurements of at least 5 years duration, which is unacceptable for a designer. Even with a 3 in (7.5 cm) diameter cyclinder, this would exceed 15 months. Increasing the temperature of the shrinkage tests to about 50°C would not shorten these times drastically and would raise further uncertainties due to the effect of temperature. A greater increase in temperature would change the shrinkage properties so much that inferences for those at room temperature would become questionable. Significant acceleration of shrinkage would require reducing the thickness of the shrinkage specimen to less than about 1 in (2.54 cm). But in that case the specimens would have to be saw-cut from larger specimens and this would cause the three-dimensional composite interaction between the mortar matrix and the aggregate pieces to be very different from bulk concrete.

3.1 Updating shrinkage prediction from short term data when τ_{sh} is known

The nonlinear parameter in the shrinkage model is τ_{sh}. So let us first describe the updating, assuming that the value $\bar{\tau}_{sh}$ of shrinkage half-time τ_{sh}, has somehow been determined using short term measurements. The updated values of shrinkage prediction, labelled by primes, are considered as follows:

$$\varepsilon'_{sh}(t, t_0) = p_6[\bar{\varepsilon}_{sh}(t, t_0)] \qquad (4)$$

in which $\bar{\varepsilon}_{sh}$ are the values predicted from model B3 based on the given value of $\bar{\tau}_{sh}$, ignoring Equation 12 of [6], and p_6 is an update parameter. Consider that values $\varepsilon^*_{sh_i}$ at times t_i have been measured. The optimum update should minimize the sum of squared deviations Δ_i of the updated model from the data, that is

$$S = \sum_i \Delta_i^2 = \sum_i (p_6\bar{\varepsilon}_{sh_i} - \varepsilon^*_{sh_i})^2 = \min \qquad (5)$$

where $\bar{\varepsilon}_{sh_i} = \bar{\varepsilon}_{sh}(t_i, t_0)$. A necessary condition of minimum is that $dS/dp_6 = 0$. This yields the condition

$$\sum_i (p_6\bar{\varepsilon}_{sh_i} - \varepsilon^*_{sh_i})\bar{\varepsilon}_{sh_i} = 0.$$

From this, the value of the update parameter is calculated as

$$p_6 = \sum_i \bar{\varepsilon}_{sh,i} \varepsilon^*_{sh,i} / \sum_i \bar{\varepsilon}^2_{sh,i} \tag{6}$$

3.2 Updating shrinkage prediction from short term data and water-loss measurements

Second, consider how τ_{sh} can be estimated. To circumvent the aforementioned ill-posedness of the shrinkage updating problem, the following approach is proposed in [1].

It has been known for a long time that shrinkage strains are approximately proportional to the water loss, denoted as Δw. Now, the idea is that (i) the water loss can be measured easily and simultaneously with shrinkage tests, and (ii) the final value $\Delta w_\infty(0)$ of water loss at nearly zero environmental humidity can be estimated easily by heating the test specimen in an oven to 110°C after the short term test is terminated.

Although heating to 110°C causes a slightly higher water loss [33] than drying up to hygral equilibrium at constant temperature and at $h \approx 0$, the difference can be neglected. It is nevertheless important to measure the weight loss immediately upon heating.

As another spoiling influence, one might point out carbonation of $Ca(OH)_2$ which occurs quickly upon heating and increases the weight of the specimen. But, because of the time the diffusion of CO_2 requires, this effect is probably significant only on thin cement paste specimens. Thus the sum of the weight losses during shrinkage and the subsequent heating can be assumed to be approximately equal to the final weight loss $\Delta w_\infty(0)$ that a shrinkage specimen would experience if it attained hygral equilibrium at $h \approx 0$.

To estimate the final water loss $\Delta w_\infty(h)$ for shrinkage at environmental humidity h, we need an approximation for the desorption isotherm. Assuming that the shrinkage is proportional to water loss, this isotherm should be approximately a linear expression in function h^3 used in k_h in Equation 11 of [6]. Although the shapes of the desorption isotherms of concrete vary considerably [34–36], the following expression seems reasonable (Fig. 3).

$$\Delta w_\infty(h) \approx 0.75 \left[1 - \left(\frac{h}{0.98}\right)^3\right] \Delta w_\infty(0) \quad \text{for } 0.25 \le h \le 0.98 \tag{7}$$

It satisfies the condition that there is no water loss for $h \approx 0.98$ (in water immersion, i.e., for $h = 1.0$ there is water gain). For $h < 0.25$ this expression is invalid (Fig. 3), but environmental humidities below 25% normally are not of interest.

An alternative way to estimate $\Delta w_\infty(h)$ might be to estimate first the final degree of hydration in the shrinkage specimen, and on that basis calculate the final water content from the total water content and cement content in the mix. Sealed curing would be required before the shrinkage test to prevent imbibition of water. However, the uncertainty involved in that approach would no doubt be higher.

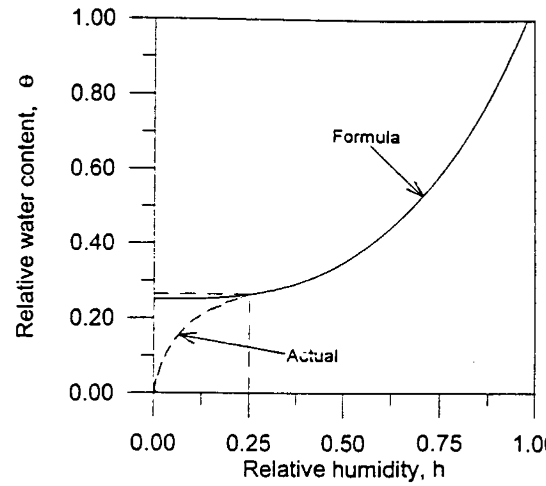

Fig. 3 Relative water content, $\theta = 1 - \Delta w_\infty(h)/\Delta w_\infty(0)$, versus relative humidity h.

Because Equations 9 and 10 of [6] were derived from diffusion theory, under the assumption of proportionality to water loss, the evolution of water loss with time should follow approximately the same equation as Equation 9 in [6], that is,

$$\frac{\Delta w}{\Delta w_\infty(h)} = \tanh\left(\frac{t - t_0}{\tau_{sh}}\right)^{1/2} \tag{8}$$

where Δw is the weight loss of the shrinkage specimen up to time t. This equation can be rearranged easily to a linear form:

$$t - t_0 = \tau_{sh}\psi, \quad \text{with } \psi = \left[\tanh^{-1}\left(\frac{\Delta w}{\Delta w_\infty(h)}\right)\right]^2 \tag{9}$$

Now consider that, at times t_i of shrinkage measurements, the values of water loss Δw_i up to times t_i have been measured also and the corresponding values of ψ_i have been calculated. The optimum value of τ_{sh} must minimize the sum of squared deviations, i.e.,

$$S = \sum_i [\tau_{sh}\psi - (t_i - t_0)]^2 = \min \tag{10}$$

A necessary condition of minimum is that $dS/d\tau_{sh} = 0$. This yields the linear equation

$$\sum_i [\tau_{sh}\psi_i - (t_i - t_0)]\psi_i = 0.$$

It follows that

$$\tau_{sh} = \sum_i (t_i - t_0)\psi_i / \sum_i \psi_i^2 \tag{11}$$

Based on this value one may then use Equation 1 to obtain the updating parameter p_6 for the final shrinkage values.

In the foregoing procedure, it has been assumed that the shrinkage half-time τ_{sh}, is equal to the water-loss half-time τ_w. Fitting of the shrinkage and water loss data for very thin cement paste specimens [37] has shown τ_{sh} to be somewhat higher than τ_w. This could be explained by the existence of a certain time-lag Δ caused by the local microdiffusion of water from gel micropores to capillary pores. Thus $\tau_{sh} = \tau_w + \Delta$. Now realize that Δ ought to be a constant determined by the pore structure of the paste, mortar or concrete and independent of the specimen size. Therefore the difference between τ_{sh} and τ_w is likely to be significant only for thin specimens (probably less than about 3 mm). For the standard 4 in shrinkage specimen, or even for 3 in specimens, this difference is likely to be negligible. Besides, high accuracy in the updating of τ_{sh} is not needed.

It must be emphasized that, although the approximately linear relationship of water loss and shrinkage underlying the foregoing equations is quite well established and widely accepted, systematic checks of the proposed updating procedure, using the proposed method for estimating the final water loss by heating the specimen, and of the assumption that $\tau_{sh} \approx \tau_w$ have not yet been made. This new method deserves deeper evaluation of its accuracy.

To illustrate the procedure, consider the recent shrinkage and water-loss data obtained by Granger *et al.* [38] (Fig. 4). They measured shrinkage on cylinders of diameter 16 cm and length 100 cm over a gauge length of 50 cm. Weight loss was measured on cylinders with diameter 16 cm and length 15 cm. The tests were carried out in an environment of 50% RH. The ends of both the shrinkage and the weight-loss specimens were sealed to ensure radial drying. The concrete had a cylindrical compressive strength of 34.3 MPa. The composition of concrete was water:cement:sand:coarse aggregate = 0.57:1.0: 0.94:2.0. The cement content was 350 kg m^{-3}. Simultaneous measurements on companion sealed specimens revealed significant autogenous shrinkage. For our updating, however, we may assume that the autogenous shrinkage in the drying specimens was negligible for reasons explained after Equation 14 of the RILEM Recommendation [6].

We now pretend that we know only the first nine points of data (up to a drying duration of 64 days) which are shown as solid black points in Fig. 4. To estimate the final water loss, $\Delta w_\infty(0)$, at $h \approx 0$, we assume that the water used up in the hydration reaction is about 20% by weight of cement. $\Delta w_\infty(50)$ is calculated using Equation 7. Using this value of $\Delta w_\infty(50)$ and the first nine points of weight-loss data we determine $\tau_{sh} = \tau_w$ from Equations 8–11. This value of τ_{sh} and the drying shrinkage data up to 64 days duration are used in Equation 4–6 to determine the updated values of shrinkage prediction. Curves, a and b in Fig. 4, showing the prediction by the formula and the updated prediction, clearly confirm the significant improvement achieved. Fig. 4 also shows curve c obtained by trying to match the short term data solely

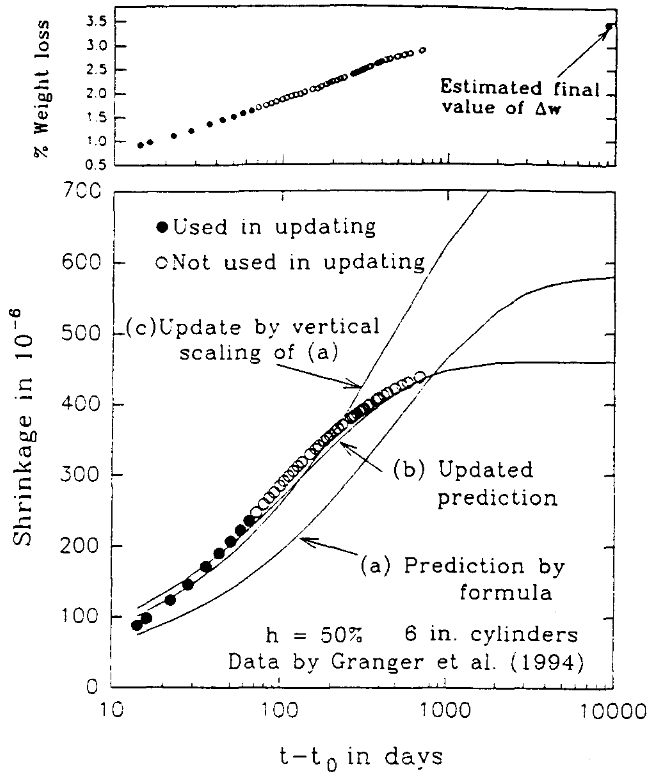

Fig. 4 Updating of shrinkage prediction using short term test data. Data by Granger [38].

by vertical scaling of curve a, which is what one would have to do if water-loss data were unavailable. Although this curve also matches the short term data quite well the long term prediction would in this case become even worse than with curve a before updating. So, updating of τ_{sh} by other than shrinkage data is essential for achieving improved predictions.

The ill-posedness of extrapolation in time of course occurs also for the portion of creep due to drying because, being governed by diffusion phenomena, it is based on the same function τ_{sh}. Even though this is a lesser problem (because only part of creep is affected), it is advisable to eliminate this ill-posedness in extrapolation from short term data. To do that one should use the τ_{sh} value obtained from the water-loss data also for the function $F(t, t')$, in Equation 2, used in extrapolation of creep, instead of using in function $F(t, t')$ the τ_{sh} value from the prediction formula.

4. IMPORTANCE OF MEASURING THE INITIAL SHRINKAGE

Theoretically, the initial shrinkage curve after the surface dries should evolve in proportion to $(t - t_0)^{1/2}$ [39] if the exposure to the environment is sudden. This has been, for example, verified quite well by the data of Wittmann, Bažant and coworkers [15, 32]. However, some shrinkage data, especially older ones, do not quite agree with this rule. The most likely reason is that the first reading was not taken right at the time of stripping of the mold but sometime later, which means that some initial shrinkage strain $\Delta\varepsilon_{sh}$ has been missed. Thus, the erroneous readings $\tilde{\varepsilon}_{sh}$ should be corrected by constant $\Delta\varepsilon_{sh}$. Neglecting other possible influences this may be done by optimally fitting to the initial data the relation, $\tilde{\varepsilon}_{sh} + \Delta\varepsilon_{sh} = k(t - t_0)^{1/2}$, in which k is some constant. This relation can be represented by a linear regression in the plot of $\tilde{\varepsilon}_{sh}$ versus $(t - t_0)^{1/2}$.

Another error may occur when, during the curing before shrinkage test, the seals leak moisture. This may cause $\Delta\varepsilon_{sh}$ to come out negative. But no meaningful correction to data is possible in this case. Such data should be discarded if $|\Delta\varepsilon_{sh}|$ is large.

In the present data sets, these corrections have been found to be relatively small and at the same time quite uncertain in most cases, due to the high initial scatter of the data. Therefore, these corrections have not been used in the optimum fitting of model B3. These corrections, however, could be important for the evaluation of short term data with the purpose of extrapolating to longer times.

5. EXTENSION TO SPECIAL CONCRETES

Special concretes such as high strength or fibre-reinforced concretes contain various admixtures and pozzolanic materials. Experimental research has indicated [29, 30] significant influence of these additives on creep and shrinkage. Parameter prediction formulae based on composition are, for such concretes, difficult to formulate because of the wide variety of additives used. However, model B3 can be applied to such special concretes if the material parameters are calibrated by short term tests, provided that certain special behaviour is taken into account.

The observed autogenous shrinkage, which is very small for normal concretes, represents a significant portion of the total shrinkage in high strength concretes [40]. The reason is that, because of small ratios of water to cementitious compounds, significant decrease of pore humidity due to self-desiccation occurs in such concretes. Despite limited test data [40], the following formula can be recommended for the total shrinkage of high strength concretes:

$$\varepsilon_{sh}^{total}(t, t_0) = \varepsilon_a(t) + \varepsilon_{sh}(t, t_0) \tag{12}$$

where ε_a is the autogenous shrinkage, and ε_{sh} is the drying shrinkage according to model B3 but updated on the basis of short term measurements. The autogeneous shrinkage can be described approximately by the formula

$$\varepsilon_a(t) = \varepsilon_{a\infty}(0.99 - h_{a\infty})S_a(t);$$
$$S_a(t) = \tanh\left(\frac{t - t_s}{\tau_a}\right)^{1/2} \tag{13}$$

where t_s is the time of final set of cement; ε_{sh} is the same as given by Equation 9 of [6]; τ_a is the half-time of autogenous shrinkage which depends on the rate of hardening of the type of high strength concrete; and $h_{a\infty}$ is the final self-desiccation humidity (for very low w/c, $h_{a\infty}$ may be assumed to be about 80% because hydration almost ceases below this humidity). Note that there is no size or shape effect in the above formula because self-desiccation is not caused by diffusion of water (except that the non-uniform heating due to the rapid hydration reaction during autogenous shrinkage can induce moisture diffusion and some associated size effect, but this is neglected here). The material parameters in the foregoing formula may be calibrated by carrying out shrinkage measurements on sealed specimens (autogenous shrinkage) and drying specimens (total shrinkage). The autogenous shrinkage measured in high strength concrete tends to terminate early. After self-desiccation, there is not much water left in the specimen for drying by diffusion because the water contents of high strength concretes tend to be small.

6. THEORETICAL JUSTIFICATIONS OF MODEL B3

6.1 New theoretical formula for drying creep

The formula used for drying creep in the preceding BP and BP-KX models was semi-empirical. However, a more rational formula can be derived theoretically as follows [1]. We assume that the additional creep due to drying is essentially the stress induced shrinkage, that is, we neglect the complex and hard to quantify influence of cracking. Note that because creep is tested under compression, the effect of microcracking is reduced. But even if it were considered, the basic aspect of the following derivation (especially the role of τ_{sh}) would still apply because the microcracking is also associated with water diffusion. (For a theoretical background to the drying creep problem see [41–43].)

According to [41], the average rate of the stress-induced shrinkage within the cross-section may be expressed approximately as $\dot{C}_d = \kappa\dot{H}$ in which H is the spatial average of pore relative humidity over the cross-section and κ is a coefficient. This coefficient may be considered a function of H as well as the total stress-induced strain C_d. We assume the relation:

$$\dot{C}_d(t, t', t_0) = \frac{k\rho(H)}{C_d}\dot{H} \tag{14}$$

where ρ is a coefficient depending on H, and k is a constant. This can be rewritten as $d(C_d^2)/dt = 2k\rho(H)\dot{H}$. Integrating from age at loading t' to the current time t,

$$C_d^2 = 2k\int_{t'}^{t}\rho(H)\dot{H}\,d\tau = 2k\int_{H(t')}^{H(t)}\rho(H)\,dH \tag{15}$$

Since drying creep, like shrinkage, is caused by water content changes governed by the diffusion theory

[39], the following equation gives a good approximation (having the correct asymptotic forms for very short and very long times $t - t_0$):

$$H(t) = 1 - (1 - h)S(t) = 1 - (1 - h)\tanh\left(\frac{t - t_0}{\tau_{sh}}\right)^{1/2} \tag{16}$$

where h is the environmental humidity and $S(t)$ is given by Equation 10 of [6]. According to diffusion theory, the water loss from the specimen is initially (for small $t - t_0$) proportional to the square root of drying time. Equation 16 satisfies this property because, for $t - t_0 \ll \tau_{sh}$, $1 - H = (1 - h) \times [(t - t_0)/\tau_{sh}]^{1/2}$. For longer times $t - t_0 \gg \tau_{sh}$, Equation 16 approaches the final asymptotic value exponentially, as required by the diffusion theory [39].

Studies of the data show that the function $\rho(H)$ may be assumed approximately in the form $\rho(H) = e^{aH}$. Evaluation of the integral in Equation 15 then yields:

$$C_d^2 = 2k[e^{aH(t)} - e^{aH(t')}] \tag{17}$$

which is the expression used in Equation 15 of [6].

6.2 Basic creep and shrinkage

The theoretical justification of the formulae used in model B3 is the same as stated in [3] for the previous BP-KX model and for some formulae already in [2]. Briefly, Equation 7 of [6] is derived from the solidification theory [44, 45] in which it is assumed that the chemical constituents of cement paste are not ageing and the ageing is due exclusively to volume growth and interlinking of layers of a non-ageing constituent of viscoelastic properties.

In the previous simplified version of this model [7], the log-double power law, which is simpler, was used instead of the present formula for basic creep. However, this law is not much simpler and gives poor long term predictions for concrete loaded at a very young age. Also, this law is not entirely free of the problem of divergence identified in previous works [5]. For this reason the formulation based on solidification theory is now preferred. It also gives better data fits. The log double power law used in [7] was again better for long term creep than the double power law used in [2].

Equation 10 of [6] for the time function of shrinkage represents the simplest possible interpolation between two required types of asymptotic behaviour: For short times the shrinkage strain (as well as the weight loss) should evolve as the square root of the drying duration, and for long times the difference from the final value should decay as an exponential. Equation 10 of [6] is justified by the diffusion theory, which requires the shrinkage half-time to scale as D^2. Here a remark on moisture diffusivity in concrete and its effect on evolution of shrinkage is in order. Although k_t in Equation 12 of [6] has the dimensions of the inverse of diffusivity, it represents the diffusivity of moisture only partly. Equation 12 together with Equation 14 of [6] also includes the effect of microcracking which tends to reduce the long-term shrinkage value and at the same time accelerates the shrinkage growth for medium durations (by increasing the diffusivity). The diffusivity, of course, must be expected to increase with w/c ratio (or with $1/f'_c$) and decrease with t_0, but these trends are offset by the effects of Equation 14 on shrinkage evolution.

Like shrinkage, the drying creep, too, must depend on D, which is ensured by the dependence of τ_{sh} on D in Equation 16. This causes the drying creep of a very thick specimen to be negligible within normal lifetimes of structures, the total creep approaching basic creep for $D \to \infty$.

Equation 14 of [6] for $\varepsilon_{sh\infty}$ is justified by the fact that drying shrinkage is caused mainly by forces applied on the solid microstructure as a result of tension in capillary water and adsorbed water layers. According to this mechanism, the shrinkage should decrease with increasing elastic modulus and, because the growth in elastic modulus continues longer in thicker cross-sections (due to slower drying), the decrease in $\varepsilon_{sh\infty}$ for thicker cross-sections should be more pronounced and of longer duration.

7. ENVIRONMENTAL FLUCTUATIONS

Some engineers are skeptical about laboratory tests conducted at constant environmental humidity. They suspect such tests to have little relevance to structures exposed to weather of fluctuating relative humidity. However, this appears to be an exaggeration. Although there is a certain effect (and it is known how to take it into account – see the model for creep at cyclic humidity and temperature in [2] and [3]), analysis of moisture diffusion indicates that, for cross-sections that are not too thin, the effect of such fluctuations cannot be very large and cannot invalidate the present model. Diffusion analysis shows that, for normal concretes, a periodic component of environmental humidity history $h(t)$ having a period T_h does not affect the pore humidity at the centre of the cross-section if $\tau_{sh} \geq 2T_h$, or, for typical concretes, if

$$D \geq \left(\frac{T_h}{10\text{ days}}\right)^{1/2} \text{in} \tag{18}$$

This means that for $D > 6$ in, the annual humidity cycles do not affect the centre of a wall 6 in thick. Furthermore, these humidity cycles do not affect more than 10% of wall thickness when D is about three times larger, i.e., $D > 18$ in. For high strength concretes (much less permeable) these limits for D are much smaller. Normal cracking cannot diminish this limit for D significantly (according to measurements of drying diffusivity of concrete with thin cracks [46]).

On the other hand, for the fluctuating component of temperature, the aforementioned limits for D are much larger, because in concrete the heat diffusivity is about 100 times higher than drying diffusivity. But the effect of temperature on shrinkage and creep is considered secondary and is ignored in present design practice. Nevertheless, the effect of temperature fluctuations deserves deeper study.

8. CONCLUDING COMMENTS

Model B3 (based on the third update of the creep and shrinkage predictions models developed at Northwestern University) is simpler than the previous versions, gives good agreement with available test data, is validated by a larger set of test data, and is better justified theoretically on the basis of an understanding of the mechanisms of creep and shrinkage. Simplification of the effect of concrete composition and strength has been achieved mainly by systematic sensitivity analysis of the parameters of the model. Theoretical improvement has been achieved by replacing an empirical formula with a simplified integration of the previously established formulation for stress-induced shrinkage.

The coefficients of variation of the errors of the prediction model are determined on the basis of the RILEM data bank. They should serve as basis for a statistical analysis of creep and shrinkage effects in structures, making it possible to determine suitable confidence limits (such as 95% confidence limits) rather than average properties.

A method of updating the main parameters of the model on the basis of limited short term tests is presented. Difficulties with the ill-posedness of the updating problem for shrinkage are circumvented by a new method exploiting simultaneous short term measurements of water loss during shrinkage. Such updating can reduce significantly the uncertainty of prediction, and is particularly important for extensions to high strength concrete with various admixtures, superplasticizers, water-reducing agents and pozzolanic materials.

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