Materials and Structures, 1995, **28**, 415–430

RILEM TECHNICAL COMMITTEES
COMMISSIONS TECHNIQUES DE LA RILEM

107-GCS GUIDELINES FOR THE FORMULATION OF CREEP AND SHRINKAGE PREDICTION MODELS

Justification and refinements of Model B3 for concrete creep and shrinkage 1. Statistics and sensitivity

ZDENĚK P. BAŽANT, SANDEEP BAWEJA
Department of Civil Engineering and Materials Science, Northwestern University, Evanston, IL 60208, USA

Model B3 for creep and shrinkage prediction in the design of concrete structures, presented as a RILEM Recommendation in *Mater. Struct.* **28** (1995) 357–365, is calibrated by a computerized data bank comprising practically all the relevant test data obtained in various laboratories throughout the world. The coefficients of variation of deviations of the model from the data are distinctly smaller than for the latest CEB model, and much smaller than for the previous ACI model (which was developed in the mid-1960's). The effect of concrete composition and design strength on the model parameters is identified as the main source of error. The model is simpler than the previous models (BP and BP-KX) developed at Northwestern University, yet it has comparable accuracy and is more rational.

1. INTRODUCTION

Realistic prediction of concrete creep and shrinkage is of crucial importance for the durability and long term serviceability of concrete structures, and in some cases also for the long term stability and load capacity. Mispredictions of this phenomenon, which contribute to excessive deflections and cracking, have been one of the important reasons for problems with longevity of the civil engineering infrastructure in all countries. Errors in the prediction of concrete creep and shrinkage have generally been larger than those caused by simplifications in the methods of structural analysis. It is now clear that, for creep sensitive structures, it makes little sense to use finite element analysis or other sophisticated computational approaches if a realistic model for creep and shrinkage is not introduced in the input. If a simplistic and grossly inaccurate prediction model for creep and shrinkage is used for a creep sensitive structure, one can hardly justify anything more than simple hand calculations of stresses and deformations in structures. In such a case it makes no sense for the analyst to spend weeks on the structural analysis while spending half an hour to determine creep and shrinkage properties to use as the input. The design will be better if more time is devoted to the latter than the former.

Realistic prediction of creep and shrinkage of concrete is a formidably difficult problem because the phenomenon is a result of several interacting physical mechanisms and is influenced by many variable factors. In view of this fact it is not surprising that improvements have been coming only slowly, gradually. No major breakthrough has occurred in the history of the research of this phenomenon; however, the accumulated advancement of knowledge since the early systematic researches in the 1930's, and especially during the last two decades, has been enormous. It is now possible to formulate a much better prediction model than two decades ago.

Four major advances have made a significant prediction improvement possible:

1. Improved theoretical understanding, for example mathematical modelling of the solidification process of cement, diffusion processes, thermally activated processes, cracking damage, residual stresses, and non-uniformity of stress and pore-humidity profiles.
2. Gradual accumulation of test data and formulation of an extensive computerized data bank, which started with the Northwestern University data bank in the 1970's having over 10,000 data points and, in collaboration with ACI and CEB, resulted in the RILEM Data Bank, compiled by Subcommittee 5 of RILEM Committee TC-107 (chaired by H. Müller). (This data bank now comprises about 600 measured time curves from about 100 test series in various laboratories around the world, with about 15,000 data points.)

3. Progress in statistical evaluation of test data and optimization of the creep and shrinkage prediction model, and optimization which minimizes the sum of the squares of errors. This task has been facilitated by the computerized form of the aforementioned RILEM Data Bank.
4. Numerical studies of the response of test specimens and structures (especially by finite elements) and their comparisons with observed behaviour, which has shed light on various assumptions on the material model used in the input.

Much of the complexity and error of the prediction model is caused by the fact that the design offices still analyse most structures according to beam theory, which requires the average material characteristics for the cross-section of the beam as a whole. Because the material creep and shrinkage properties inevitably become non-uniform throughout the cross-section (due to diffusion phenomena, residual stresses, cracking, damage localization and fracture), the model for the average material properties in the cross-section is not a material constitutive model. It depends on many more influencing factors than the constitutive model, e.g., on the shape of the cross-section, environmental history, ratios of the bending moment, normal force and shear force, etc.

For this reason, a really good model for the prediction of the average shrinkage properties and average creep at drying in the cross-section under general loading and environmental conditions will never be possible. One must accept significant errors, increased complexity and a greater number of empirical parameters if one insists on characterizing the behaviour of the cross-section as a whole by its average properties. In the future, the design approach should move away from characterizing the cross-section creep and shrinkage properties of the cross-section as a whole, and towards an approach in which the cross-sections are subdivided into individual small elements. However, for the time being, an integral prediction model for the cross-section as a whole is needed.

In the special case of constant temperature and constant moisture content, the average characterization of creep in the cross-section is identical to a constitutive model for a material point. In this case, the model becomes far simpler and is more accurate.

The present model, which is for brevity labelled model B3 and is based on a recent report [1], is the third major refinement in a series of models developed at Northwestern University, beginning with the BP model [2] and BP-KX model [3, 4]. Model B3 is simpler, better supported theoretically and as accurate as these previous models. Research progress will not stop, and no doubt further improved versions will become possible in the future. Compared with the latest BP-KX model, the improvement in model B3 consists of simplification of the formulae achieved by sensitivity analysis, incorporation of a theoretically derived rather than empirical expression for the drying creep, and calibration of the model by an enlarged data set including the data published in the last few years.

Model B3 conforms to the guidelines that have been formulated recently by RILEM Committee TC-107 [5a], as a refinement and extension of the conclusions of preceding RILEM Committee TC-69 [5b]. These guidelines summarize the basic properties of creep and shrinkage that have been well established by theoretical and experimental research and represent the consensus of the Committee. The existing prediction models of major engineering societies violate many of these guidelines. The present model must nevertheless be regarded as only an example of a model satisfying these guidelines, because formulation of a model based on the guidelines is not unique and conceivably partly different models satisfying these guidelines could be formulated, too.

The purpose of this paper is to provide a justification for, as well as some refinements of, Model B3. It may be noted that the same model as described here has been proposed to ACI Committee 209, and has been approved by the Committee's vote. All the notations from the preceding RILEM Recommendation [6] are retained.

2. UNBIASED STATISTICAL EVALUATION BASED ON THE COMPUTERIZED DATA BANK

The development of a data bank comprising practically all the relevant test results on creep and shrinkage of concrete obtained in various countries and laboratories up to the present time facilitates the evaluation and calibration of creep and shrinkage prediction models. No longer does tediousness limit such evaluation to a few subjectively selected test data. It is important to use the complete set of available test data, because subjective selections of some data for verification of a creep model have been shown capable of greatly distorting the conclusions.

The statistical evaluation and optimization of model B3 have been carried out in a similar manner as for the preceding models BP and BP-KX (see [2] part 6, [3] and [7]). Optimum values of the model parameters which minimize the sum of squared deviations from the data in the data bank have been determined. The deviations of the model from the test data (errors) have been characterized by their coefficient of variation $\bar{\omega}$ which is defined for the data set number j as

$$\bar{\omega}_j = \frac{s_j}{\bar{J}_j} = \frac{1}{\bar{J}_j}\left[\frac{1}{n-1}\sum_{i=1}^{n}(w_{ij}\Delta_{ij})^2\right]^{1/2} \qquad (1)$$

in which

$$\bar{J}_j = \frac{1}{n}\sum w_{ij}J_{ij}, \quad w_{ij} = \frac{n}{\;} \qquad (2)$$

Table 1 Coefficients of variation of errors (expressed as percentage) of basic creep predictions for various models

Test data	Model B3 $\bar{\omega}$	ACI $\bar{\omega}$	CEB $\bar{\omega}$
1. Keeton	19.0	37.5	42.8
2. Kommendant *et al.*	15.3	31.8	8.1
3. L'Hermite *et al.*	49.4	133.4	66.2
4. Rostasy *et al.*	15.2	47.6	5.0
5. Troxell *et al.*	4.6	13.9	6.2
6. York *et al.*	5.6	37.7	12.8
7. McDonald	6.9	48.4	22.2
8. Maity and Meyers	33.8	30.0	15.7
9. Mossiossian and Gamble	18.6	51.5	47.3
10. Hansen and Harboe *et al.* (Ross Dam)	14.1	51.2	31.1
11. Browne *et al.* (Wylfa vessel)	44.7	47.3	53.3
12. Hansen and Harboe *et al.* (Shasta Dam)	22.7	107.8	43.1
13. Brooks and Wainwright	12.6	14.9	15.4
14. Pirtz (Dworshak Dam)	12.5	58.2	32.5
15. Hansen and Harboe *et al.* (Canyon ferry Dam)	33.3	70.2	56.9
16. Russel and Burg (Water Tower Place)	15.7	19.3	31.5
17. Hanson	14.1	63.3	12.1
$\bar{\omega}_{all}$	23.6	58.1	35.0

Table 2 Coefficients of variation of errors (expressed as percentage) of shrinkage predictions for various models

Test data	Model B3 $\bar{\omega}$	ACI $\bar{\omega}$	CEB $\bar{\omega}$
1. Hummel *et al.*	27.0	30.0	58.7
2. Rüsch *et al.* (1)	31.1	35.2	44.8
3. Wesche *et al.*	38.4	24.0	36.1
4. Rüsch *et al.* (2)	34.7	13.7	27.8
5. Wischers and Dahms	20.5	27.3	35.9
6. Hansen and Mattock	16.5	52.9	81.5
7. Keeton	28.9	120.6	48.3
8. Troxell *et al.*	34.1	36.8	47.4
9. Aschl and Stökl	57.2	61.3	44.2
10. Stökl	33.0	19.5	29.6
11. L'Hermite *et al.*	66.7	123.1	69.4
12. York *et al.*	30.6	42.8	8.9
13. Hilsdorf	11.7	24.7	29.6
14. L'Hermite and Mamillan	46.1	58.7	45.5
15. Wallo *et al.*	22.0	33.0	55.6
16. Lambotte and Mommens	39.1	30.7	31.3
17. Weigler and Karl	31.3	29.6	21.3
18. Wittmann *et al.*	23.7	65.4	40.0
19. Ngab *et al.*	20.4	45.3	64.6
20. McDonald	5.1	68.8	21.4
21. Russel and Burg (Water Tower Place)	38.5	51.0	58.1
$\bar{\omega}_{all}$	34.3	55.3	46.3

Here J_{ij} are the measured values (labelled by subscript i) of the compliance function in the data set number j; n is the number of all data points in the data set number j; Δ_{ij} is the deviation of the value given by the model from the measured value; w_{ij} are the weights assigned to the data points, n_d is the number of decades on the logarithmic time scale spanned by measured data in data set number j; and n_1 is the number of data points in the decade to which point i belongs.

The weight assigned to a data point in a decade on the logarithmic scale is taken as inversely proportional to the number n_1 of data points in that decade, and the weights are normalized such that

$$\sum_i w_{ij} = n.$$

The use of $n-1$ in Equation 1 is required for an unbiased estimate of s_j. The overall coefficient of variation of the deviations of the model from the measured values for all the data sets in the data bank has been defined as

$$\bar{\omega}_{all} = \left[\frac{1}{N}\sum_{j=1}^{N} \bar{\omega}_j^2\right]^{1/2} \tag{3}$$

in which N is the number of data sets in the bank. Similar expressions, with J replaced by shrinkage strain ε_{sh}, have been used for shrinkage. Note that the alternative formula

$$\bar{\omega}_{all} = \frac{1}{\bar{J}}\left[\frac{1}{N}\sum_j s_j^2\right]^{1/2} \quad \bar{J} = \frac{1}{N}\sum_j \bar{J}_j$$

which is based on the sum of squared deviations for all the data sets combined, would give more weight to weaker concretes. Equation 3 gives equal weight to each data set.

The statistics of the errors of model B3 in comparison with the test data sets in the RILEM data bank are given in Tables 1–3 for basic creep, shrinkage and creep at drying. For comparison, the statistics of errors of the current ACI model [8], developed in the mid 1960's by Branson *et al.*, and the latest CEB model [9] are also shown (for a comparison with another recently proposed model [10] see the discussion of [10]). Table 4 presents the statistics of the extension of the model to basic creep at constant elevated temperatures. Figs 1–4 show some typical comparisons with selected important test data from the data bank.

In evaluating a creep and shrinkage model, it is important to avoid subjective bias. This has not been true of the evaluations of some other models recently presented in the literature. For example, consider that there are over 1,000 data points for creep durations

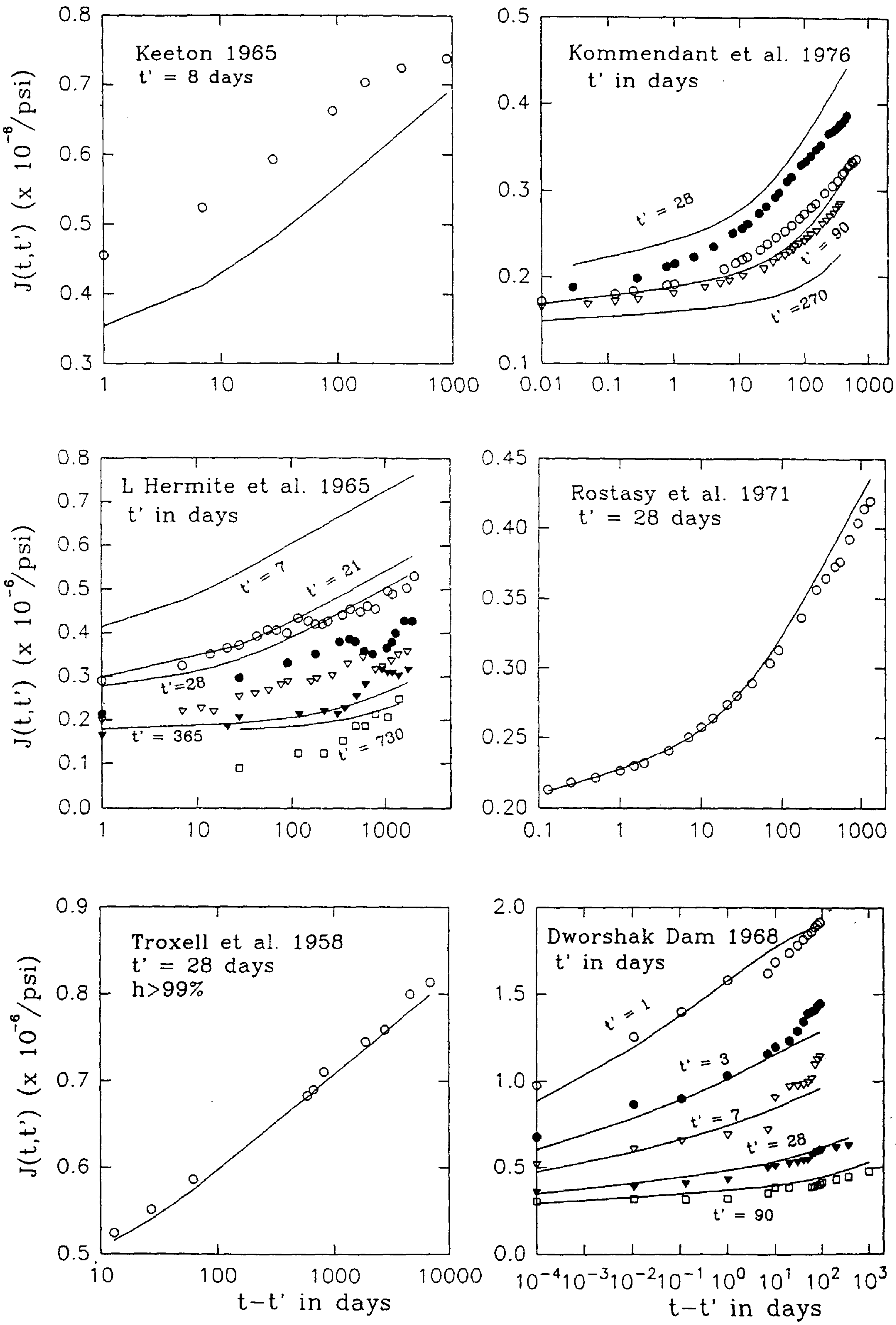

Fig. 1 Comparison of the predictions of the B3 model (solid curves) with some important test data for basic creep from the literature.

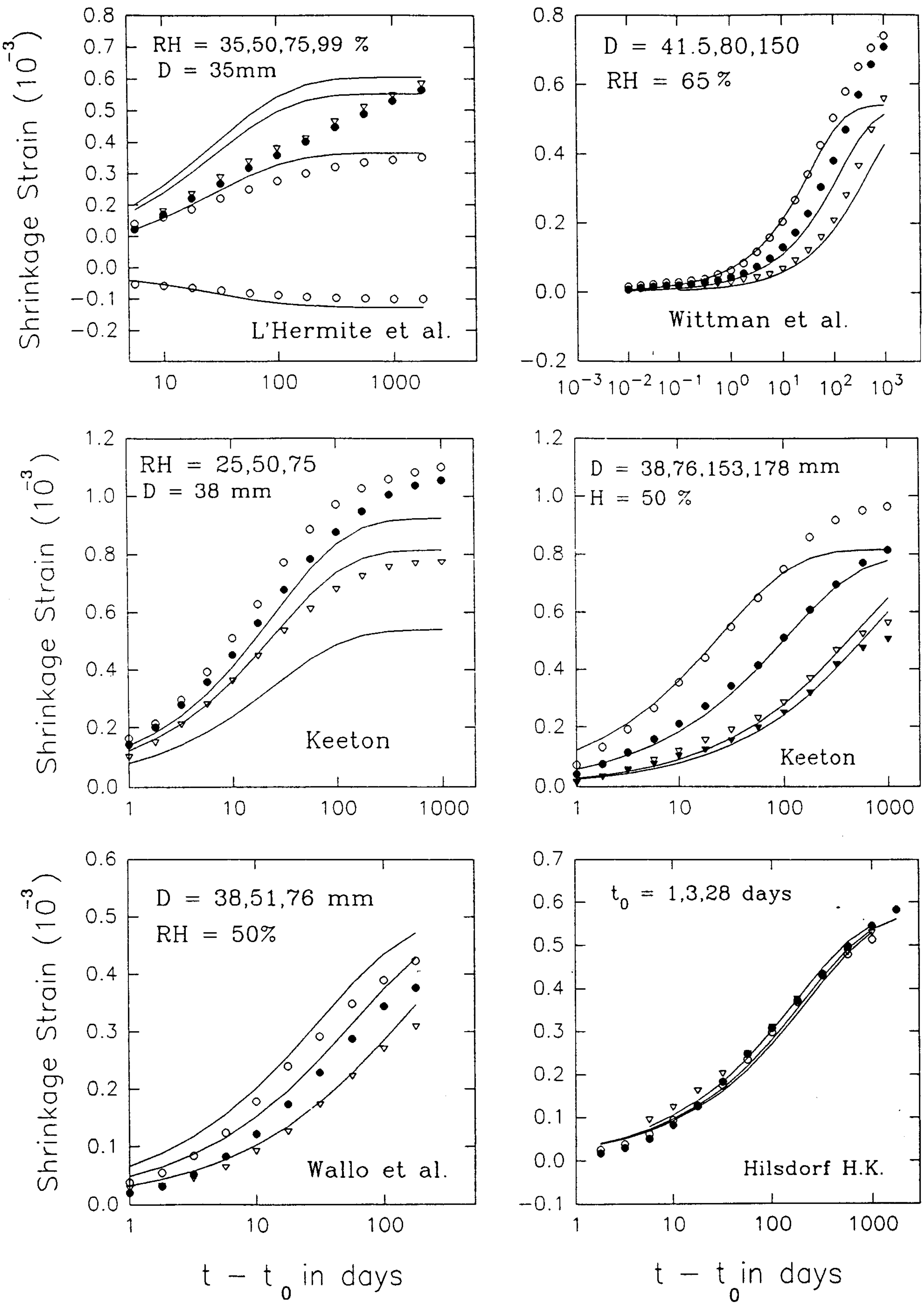

Fig. 2 Comparison of the predictions of the B3 model (solid curves) with some important test data for shrinkage from the literature.

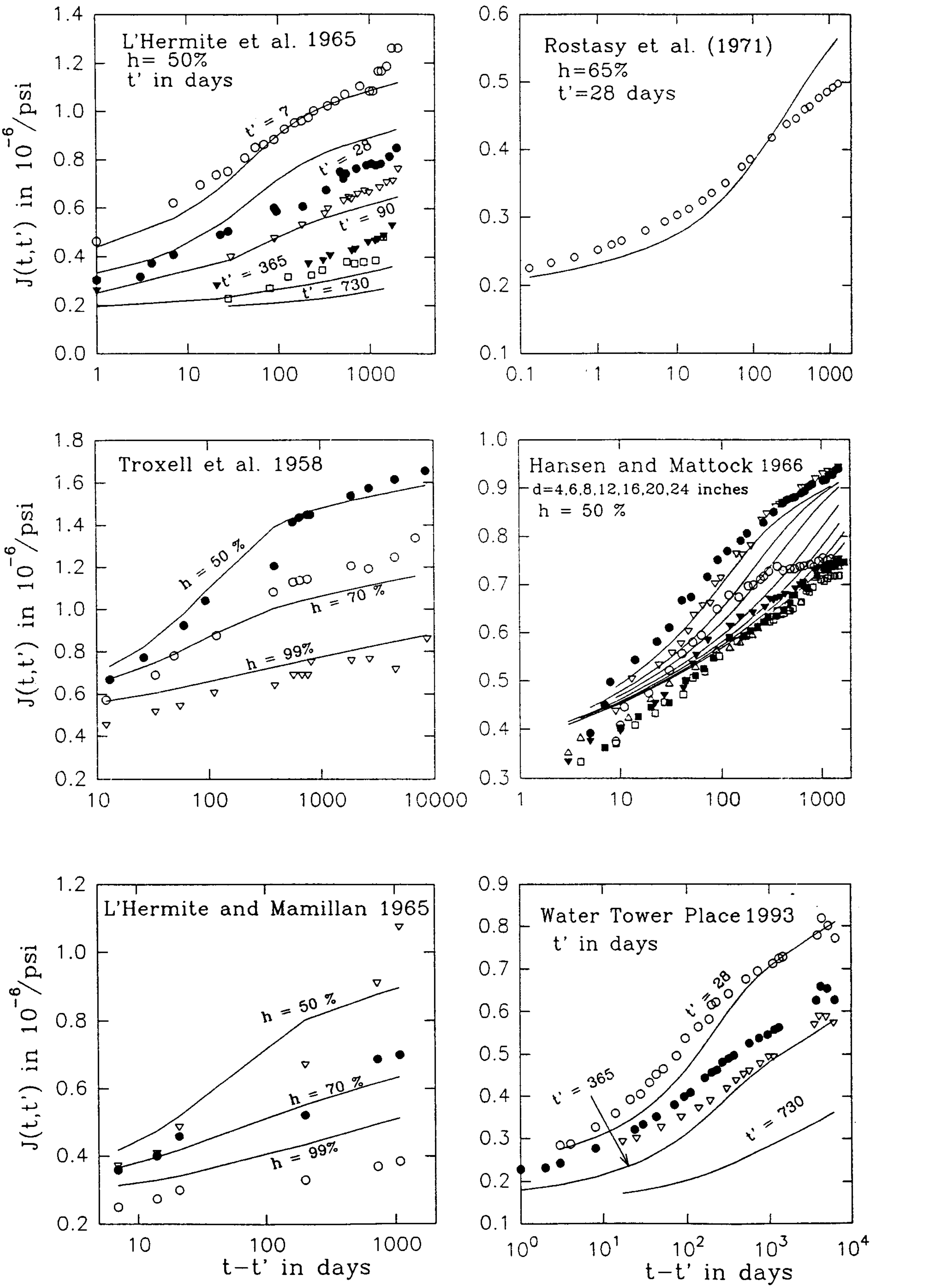

Fig. 3 Comparison of the predictions of the B3 model (solid curves) with some important test data for creep at drying from the literature.

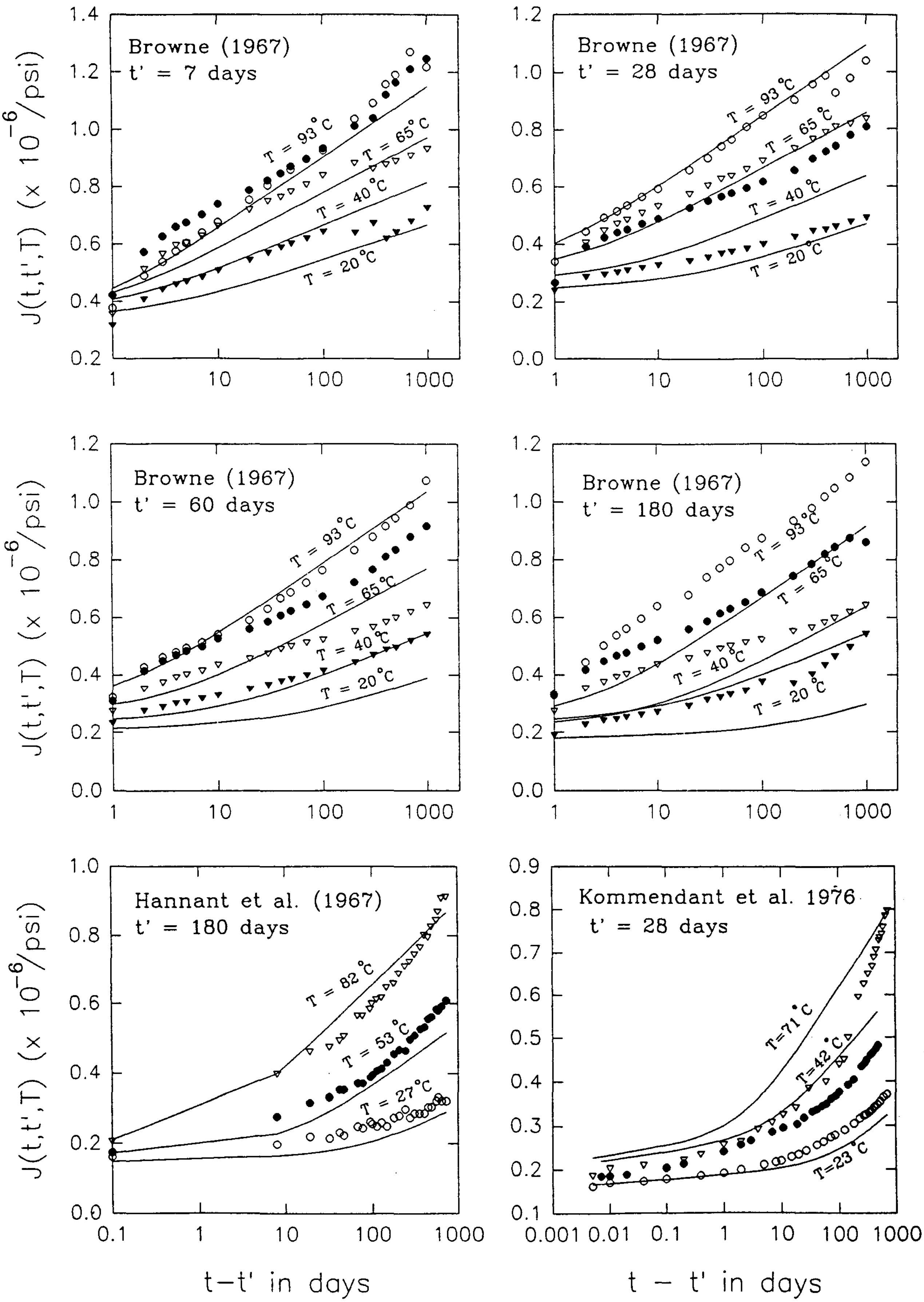

Fig. 4 Comparison of the predictions of the B3 model (solid curves) with some important test data for the effect of constant elevated temperature on basic creep.

Table 3 Coefficients of variation of errors (expressed as percentage) of the predictions of creep at drying for various models

Test data	Model		
	B3 $\bar{\omega}$	ACI $\bar{\omega}$	CEB $\bar{\omega}$
1. Hansen and Mattock	5.8	32.1	11.9
2. Keeton	31.4	46.3	37.9
3. Troxell *et al.*	5.9	33.0	7.9
4. L'Hermite *et al.*	14.0	55.8	25.5
5. Rostasy *et al.*	6.5	20.9	14.8
6. York *et al.*	5.8	42.1	45.1
7. McDonald	10.9	40.4	38.9
8. Hummel	15.3	46.2	24.6
9. L'Hermite and Mamillan	20.6	62.5	15.2
10. Mossiossian and Gamble	11.3	71.7	30.8
11. Maity and Meyers	62.8	45.9	83.7
12. Russel and Burg (Water Tower Place)	10.7	41.2	19.1
$\bar{\omega}_{all}$	23.1	46.8	35.5

Table 4 Coefficients of variation of errors (expressed as percentage) of Model B3 predictions of the effect of constant elevated temperature on basic creep.

Test data	$\bar{\omega}$
1. Johansen and Best	20.3
2. Arthanari and Yu	35.5
3. Browne *et al.*	27.2
4. Hannant D. J.	14.0
5. York, Kennedy and Perry	41.1
6. Kommendant *et al.*	18.8
7. Okajima *et al.*	6.9
8. Ohnuma and Abe	23.7
9. Takahashi and Kawaguchi (a)	38.8
10. Takahashi and Kawaguchi (b)	43.9
$\bar{\omega}_{all}$	28.1

under 100 days, and only 10 data points in excess of 1,000 days. If all these data were used in the statistical evaluation with the same weight, the error in predictions for times over 1,000 days would obviously have a negligible influence on the resulting statistics. Thus, a very low coefficient of variation of errors would be obtained if the model fitted the data well only in the range of up to 100 days. Yet predictions of the long term behaviour are most important. A similar problem arises when the data bank has many values for loading ages under 100 days and very few over 1,000 days. Yet, in long term relaxation, significant stress changes can occur in structures even after 1,000 days of age, and if the model does not predict the creep well for such older ages at loading, the stress relaxation cannot be predicted correctly from the principle of superposition. This point is important to realize, since some recent comparisons with test data suffered from this kind of bias.

Therefore one must ensure that the data measured within each decade of load or shrinkage duration (in logarithmic time scales) have equal weights. If possible this should also be ensured for the age at loading. Ideally, equal weights would be achieved and subjective bias eliminated if the measured data had equal spacing in the logarithmic scales of load or drying duration and age at loading, and if they covered the entire range. But, unfortunately, creep measurements have not been made in this way. In a previous study [2, 3], the subjective bias was eliminated by hand-smoothing in log time scale the experimentally measured curves and then placing points equally spaced in the log time scale on such hand-smoothed curves. However, there is a slight objection to this approach, since it inevitably involves some manipulation of test data and suppresses part (albeit only a very small part) of the scatter of data. A different approach has been adopted for the evaluation of model B3.

The data points in each decade in the logarithmic scale of load duration or drying duration are considered as one group, and each group of data points is assigned equal weight as a whole. Thus an individual data point in a particular group is weighted in inverse proportion to the number of points in that group. Since most data sets have only few data points for load durations less than one day, all the data points for load durations under 10 days have been treated as part of one group. As for the ages at loading, it turned out to be impractical to treat these in the same manner because very high and very low ages at loading are missing from most data. For this reason, statistical comparisons of model B3 predictions with the values from the data bank have also been calculated separately for various decades of load duration and of ages at loading (grouped together in decades on the log scale); see Table 5 for creep both without and with drying and Table 6 for shrinkage. It is worth noting that the coefficients of variation of model B3 remain low even for the last decade of age at loading (over 1,000 days), while for some other models they become very large for that range. Correct representation of creep for loading ages over 1,000 days is important for calculating long term stress relaxation from the principle of superposition, as well as for general solutions of the stress variations in structures over long periods of time. In Table 6 for shrinkage, the coefficients of variation are not given separately for individual decades of the logarithm of the age t_0 at the start of drying because the effect of t_0 is relatively small and the differences in the coefficients of variation are not large.

It must be stressed that Figs 1–4 (data from [11–24]) are not intended as a verification of the model. For that purpose, the fits of all the data sets in

Table 5 Statistics of errors of various models for basic creep and creep at drying, compared for different ranges of age at loading and creep duration (in days)

$\bar{\omega}$	$t' \leq 10$	$10 < t' \leq 100$	$100 < t' \leq 1000$	$t' > 1000$
Model B3				
$t - t' \leq 10$	17.8	24.0	19.8	
$10 < t - t' \leq 100$	13.7	23.1	25.3	29.3
$100 < t - t' \leq 1000$	13.9	20.5	22.6	33.6
$t - t' > 1000$	12.7	14.6	17.8	
ACI Model				
$t - t' \leq 10$	60.3	30.7	33.3	
$10 < t - t' \leq 100$	45.7	36.7	49.9	97.1
$100 < t - t' \leq 1000$	34.6	39.9	51.7	93.9
$t - t' > 1000$	36.8	39.9	40.9	
CEB Model				
$t - t' \leq 10$	40.5	23.1	11.2	
$10 < t - t' \leq 100$	25.8	23.5	21.2	40.8
$100 < t - t' \leq 1000$	17.5	22.8	25.0	41.3
$t - t' > 1000$	11.6	20.5	24.7	

Table 6 Statistics of errors of various models for shrinkage, compared for different ranges of drying duration (in days)

$\bar{\omega}$	$t - t_0 \leq 10$	$10 < t - t_0 \leq 100$	$100 < t - t_0 \leq 1000$	$t - t_0 > 1000$
B3	38.5	29.3	22.4	19.6
ACI	67.8	50.4	43.3	44.8
CEB	53.5	40.2	44.7	37.4

the data bank must be considered, which is presented in Tables 1–4. Visual comparisons with all the data sets have been generated by the computer but are too extensive for publication.

It should also be pointed out that the deviations from the data points seen in the figures are caused mainly by errors in the prediction of model parameters from the composition and strength of concrete. If the model parameters are adjusted, all these data can be fitted very closely, but then one is not evaluating the prediction capability of the model. The figures showing most of the data from the data bank were presented in a previous paper [3], along with basic information on the tests.

Other models in the literature have been verified by only a limited selection of the available test data. Before the age of computers this was understandable, due to the tediousness of such comparisons, but with the availability of a computerized data bank such selective comparisons with test data are unjustifiable. There is always the danger of grossly distorting the results by a convenient selection of some test data sets to use for justifying some proposed model. For example, note that if among the 21 data sets used for shrinkage only the 12 most favourable data sets were selected (which might seem plenty for justifying a model), the $\bar{\omega}_{all}$ value would be reduced from 34.3% to 23.7%. Likewise, if among the 17 data sets used for basic creep, only 7 of the most favourable of the data sets were selected, the $\bar{\omega}_{all}$ value would be reduced from 23.6% to 10.7%. These observations, and similar ones made in [2] and [7], document the dangerous deception that could be hidden in selective use of test data. Justifying some model by 12 or 7 data sets may look like plenty, yet it can be greatly misleading unless the data to be used for justification were chosen truly randomly (e.g., by casting dice).

If the aforementioned weights were not used, a smaller coefficient of variation of errors could be achieved; however, the fit of the long term data would be much poorer. This fact shows clearly that the weights must be used in statistical evaluations. Better weighting, aside from applying sensitivity analysis to simplify the model, is one reason why the present coefficients of variation come out slightly higher than for the BP-KX model.

A further assessment of the degree of scatter can be obtained from the plots of measured value X_k versus corresponding predicted values Y_k of creep and shrinkage ($k = 1, 2, 3 \ldots N_p$ are all the points in the plot). They are shown in Fig. 5, not only for the present B3 model but also for the previous ACI and the new CEB-FIP models. The basic creep and creep at drying are combined in the same plots. If the models were

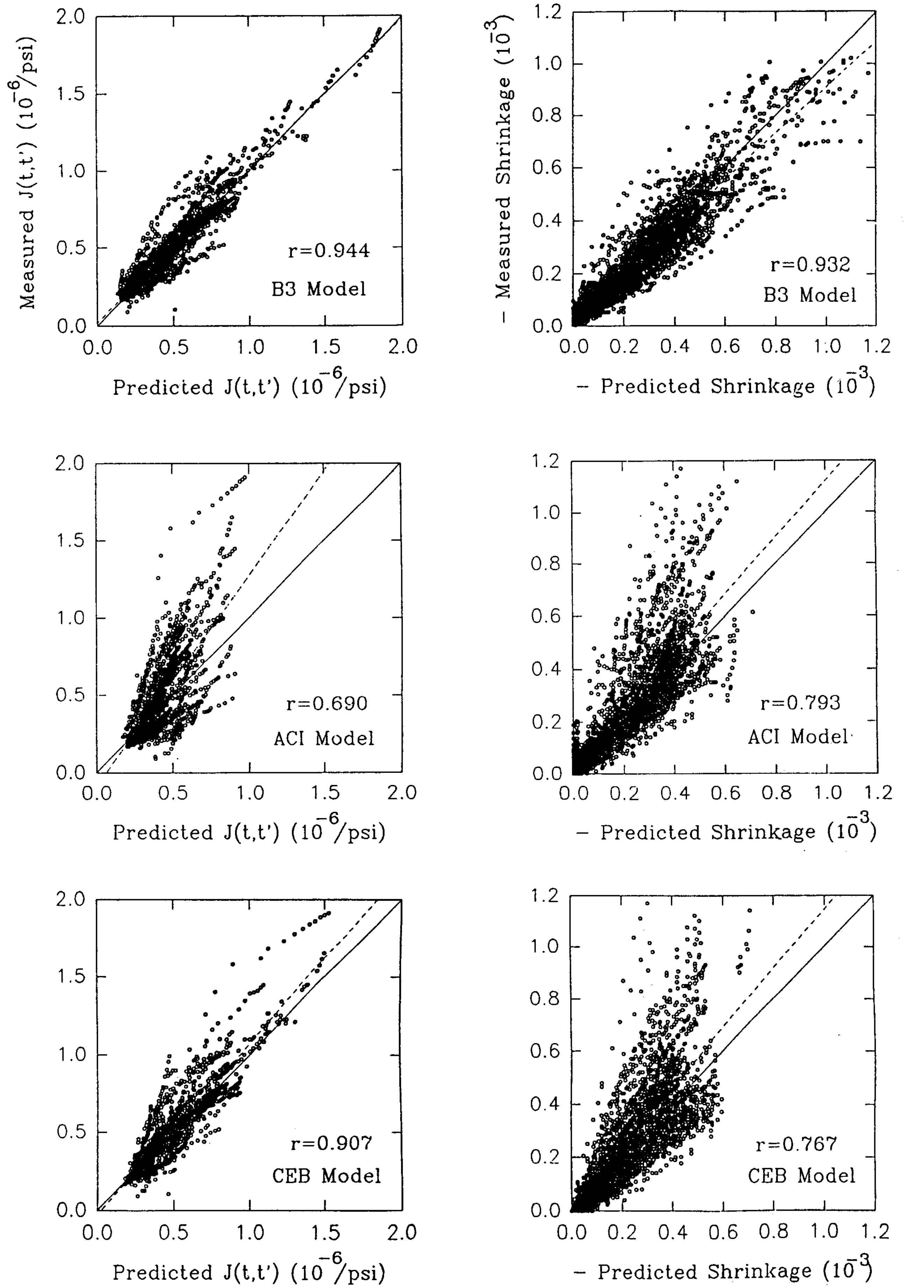

Fig. 5a. Scatter plots of the measured versus predicted values of creep and shrinkage (dashed lines are regression lines).

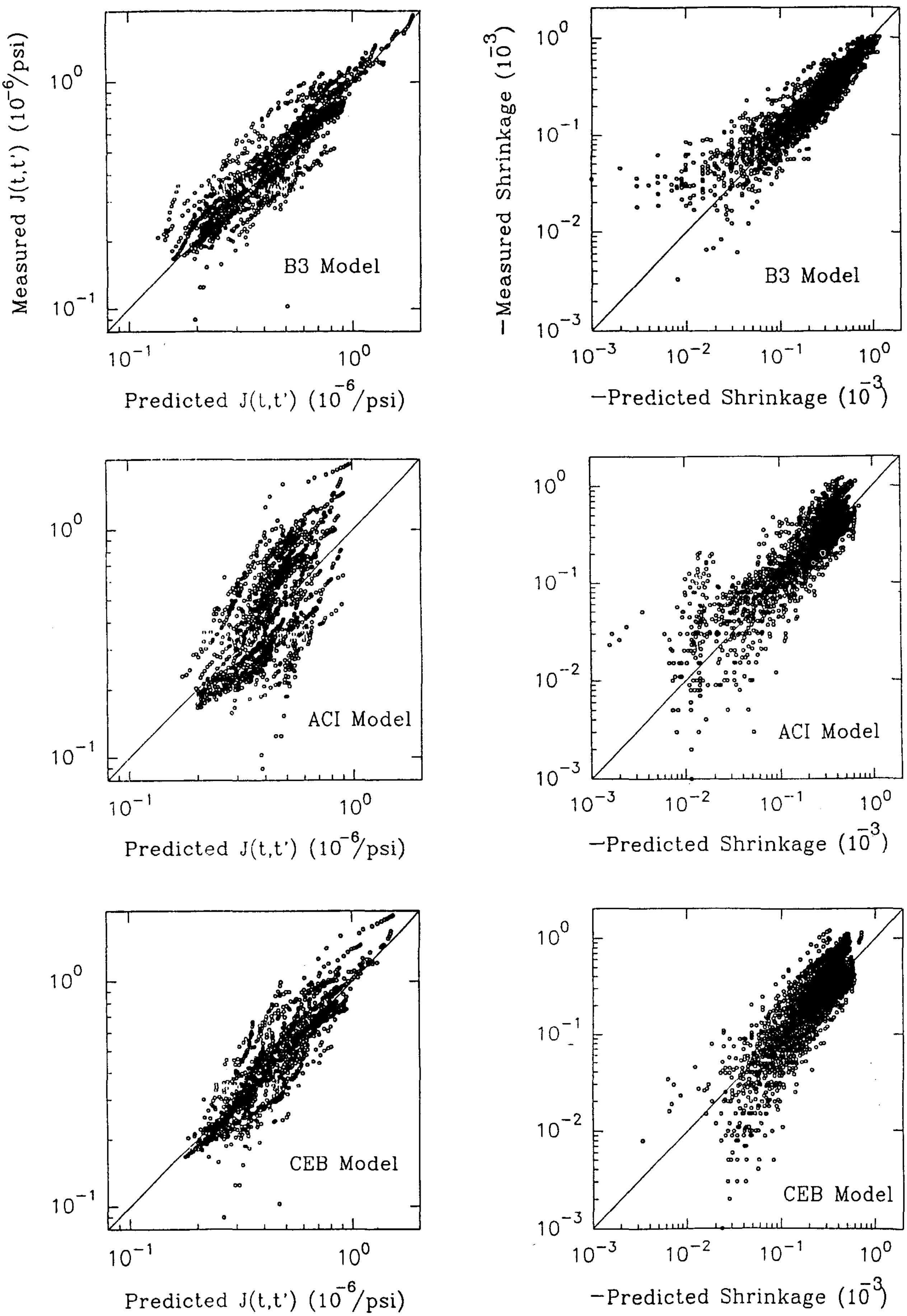

Fig. 5b. Scatter plots on logarithmic scales (showing relative error) of the measured versus predicted values of creep and shrinkage.

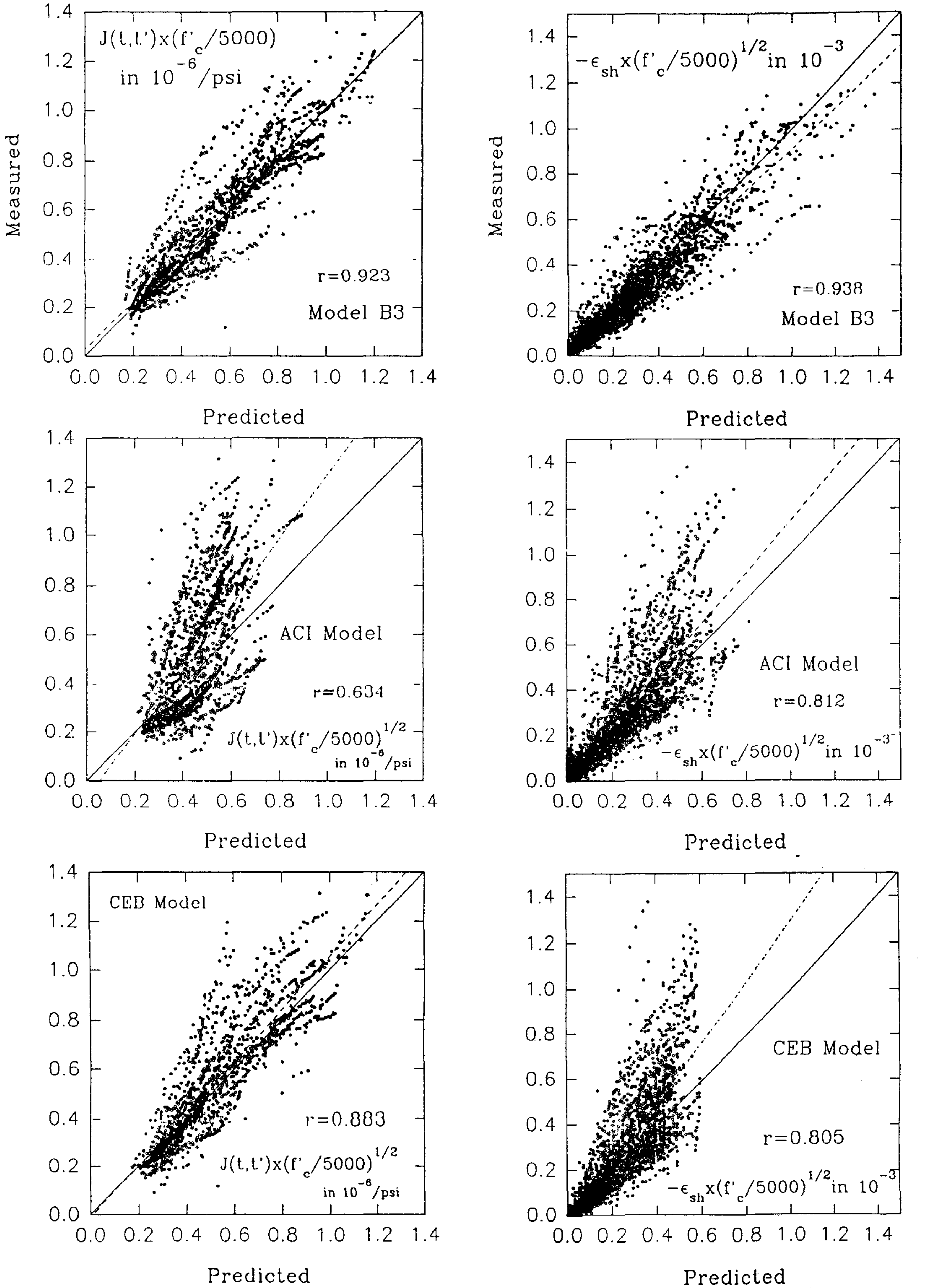

Fig. 5c. Scatter plots of the measured versus predicted values of creep and shrinkage with coordinates multiplied by $(f'_c/5000)^{1/2}$ (dashed lines are regression lines).

perfect and scatter did not exist, these plots would be straight lines of slope 1. Thus, deviations from this line represent errors of the model predictions plus inevitable scatter of the measurements. As seen, the errors of the current B3 model are significantly less than for the previous ACI 209 model and distinctly less than for the new CEB model. It should be noted that the B3 model achieves the most significant improvements for large strains (or long times) which are most important. This is revealed by observing that the high strain points for the B3 model lie relatively close to the line of slope 1, while those for the ACI and CEB-FIP models lie high above this line (which implies underprediction).

The correlation coefficient r of the population of the measured values and the corresponding model predictions is given in each figure. It has been calculated as

$$r = \frac{\sum_k (X_k - \bar{X})(Y_k - \bar{Y})}{(N_p - 1)s_X s_Y},$$

where

$$s_X^2 = \frac{\sum_k (X_k - \bar{X})^2}{(N_p - 1)}, \qquad s_Y^2 = \frac{\sum_k (Y_k - \bar{Y})^2}{(N_p - 1)},$$

$$\bar{X} = \frac{\sum_k X_k}{N_p}, \qquad \bar{Y} = \frac{\sum_k Y_k}{N_p},$$

Note that r characterizes only the grouping of the data about the regression lines of the plots. The regression lines are also drawn in Fig. 5. However, these regression lines do not have slope 1 and do not pass through the origin. This represents another kind of error which is not reflected in the value of r (this is the basic deficiency of this kind of plot compared with statistical regression). We see from these plots that in the case of the B3 model the regression line (dashed) is close to the line of slope 1 through the origin, and also the value of r is close to 1.

The same plots of measured versus calculated values of creep and shrinkage are plotted on logarithmic scales in Fig. 5(b). These plots show the relative errors. As seen from these plots, especially for shrinkage, the relative error for all the models decreases with increasing shrinkage strain, as opposed to absolute error seen in Fig. 5(a), which increases with increasing strain. These plots also show that the overall predictions are best for model B3.

In the coordinates of Fig 5(a,b) the strains for higher strength concretes are generally smaller than those for lower strength concretes. This is statistically undesirable as low strength concretes receive in such plots larger weights. To correct it we note that, roughly, the shrinkage and creep strains are proportional to $1/\sqrt{f'_c}$. Thus a plot of measured versus predicted values in which the coordinates are multiplied by $\sqrt{(f'_c/5000)}$ (strength values normalized by a mean strength of 5000 psi) gives roughly the same weight to concretes of high and low strengths. Such plots are shown in Fig. 5(c). From these plots it may be seen that the data which were crowded in Fig. 5(a) have become more dispersed. The regression lines, too, have been shown on these plots and the correlation coefficient r has been calculated. In this case also the regression line for model B3 is seen to be close to the line of slope 1 and the correlation coefficient r is also close to 1.

As mentioned in the preceding RILEM recommendation [6] the ACI formula for elastic modulus, $E = 57000\sqrt{f'_c}$ gives values approximately equal to $1/J(28 + \Delta, 28)$ where $\Delta \approx 0.01$ day (or 5–20 min). This is verified by Fig. 6(a), which shows the values of $E_{28} = 1/J(28 + \Delta, 28)$ obtained in all the creep tests in the data bank for various Δ values. Fig. 6(b) demonstrates that the value of $q_1 = 1/E_0$ can be considered as age independent. Fig. 7 shows the values of $E_{28} = 1/J(28 + \Delta, 28)$ versus $\sqrt{f'_c}$ plotted for $\Delta = 0.01$ day, and indicates that the discrepancy between the values of the ACI formula (line marked $E = 57000\sqrt{f'_c}$ in Fig. 7) and the values of $1/J(28.01, 28)$ is not large, even though the method of measuring $E_{28} = 1/J(28.01, 28)$ in a creep test differs from the ASTM standards for measuring the elastic modulus. The deviation of the regression line of the points in Fig. 7 indicates that a closer fit would be possible (with $E \propto \sqrt{f_c'^{2.5}}$) but the improvement in creep predictions would be minor.

3. SENSITIVITY ANALYSIS

The formulae in Equations 18 and 19 of [6] for predicting model B3 parameters from concrete composition and strength are simpler than those in the previous models. The simplification has been achieved mainly through sensitivity analysis [25]. First, all the parameters of the model are assumed to depend on all the composition and strength parameters, in the form of products of power functions (which means that the logarithms of these parameters are assumed to depend linearly on the logarithms of composition parameters and of strength). Then the parameter values are optimized for the entire data bank. In the case of basic creep, the following expressions are then obtained:

$$\left.\begin{aligned} q_2 &= 0.77(c)^{0.73}(a/c)^{-0.09}(0.001 f'_c)^{-1.25}, \\ q_3 &= 0.36(w/c)^5 q_2 \end{aligned}\right\} \quad (4)$$

$$q_4 = 0.001\,(c/30)^{3.57}(0.001 f'_c)^{1.29}(a/c)^{3.16} \quad (5)$$

Then the plots of the model parameters versus the composition and strength parameters over their typical ranges are considered; see Fig. 8. As seen from this figure, various model parameters are almost insensitive to some of the composition and strength variables (see the nearly horizontal rows of points in Fig. 8).

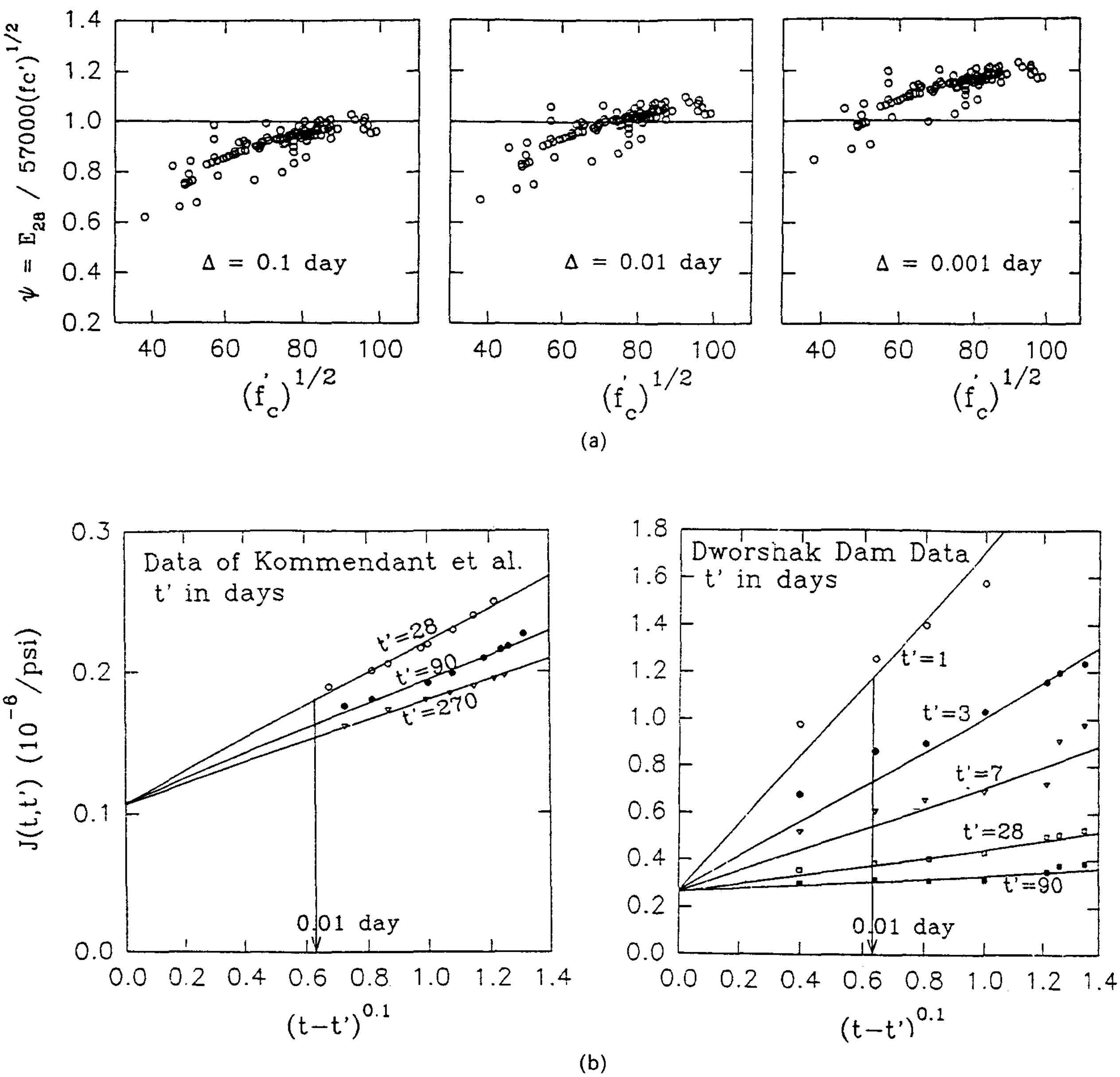

Fig. 6 (a) Ratio of the values of elastic modulus by the formula $E_{28} = 1/J(28 + \Delta, 28)$ for various Δ values, to ACI formula for E, and (b) demonstration that the short term creep data confirm the age-independence of $q_1 = 1/E_0$.

Mathematically, such insensitivity is detected from the following formula for sensitivity factor α_i [26]:

$$\alpha_i = \frac{\bar{X}_i}{\bar{F}} \frac{\partial F}{\partial X_i} \qquad (i = 1, 2, \ldots n) \tag{6}$$

with

$$\frac{\partial F}{\partial X_i} \approx \frac{1}{2\Delta X_i} [F(\bar{X}_1, \ldots, \bar{X}_i + \Delta X_i, \ldots, \bar{X}_n) - F(\bar{X}_1, \ldots, \bar{X}_i - \Delta X_i, \ldots, \bar{X}_n)] \tag{7}$$

where F is the material property which depends upon the random parameters X_i, ΔX_i are chosen small variations of X_i, and the superimposed bars denote the mean values. Then the composition and strength parameters to which a given model parameter is found to be insensitive are deleted from the assumed expression. The data from the data bank are then fitted again and the model optimized. If the new coefficient of variation of errors is not significantly larger, then the simplified expression is accepted.

It must be emphasized that the results are limited to the chosen form of dependence of model parameters on the composition and strength (products of power functions). Somewhat different trends may be found with other assumed types of dependence. Note, however, that the simplified formulae agree with experience, general trends established by experiments and considerations of physical mechanisms [27], e.g., with the fact that creep increases with an increase in cement content and decreases with an increase in strength and aggregate content, and that shrinkage increases with an increase in water content.

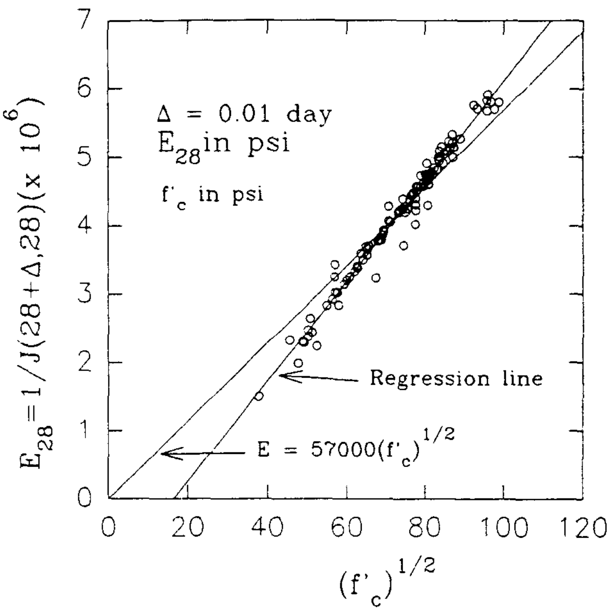

Fig. 7 Elastic modulus predictions by creep formulae.

In the future, the formulae for the dependence of model parameters on the basic characteristics of concrete should be based on the theory of composites. Some useful results in this regard have been achieved already [28]; however, no comprehensive theory for practical use is in sight at present.

4. CLOSING REMARKS

The statistical evaluation of the experimental evidence was taken up in this first part; the second part, which is scheduled for the next issue, will deal with improvements of prediction on the basis of short term tests and will also give a theoretical derivation of a new formula for drying creep introduced in the B3 model.

ACKNOWLEDGEMENT

Partial financial supports from NSF under NSF grant No. MSS-9114476 to Northwestern University and

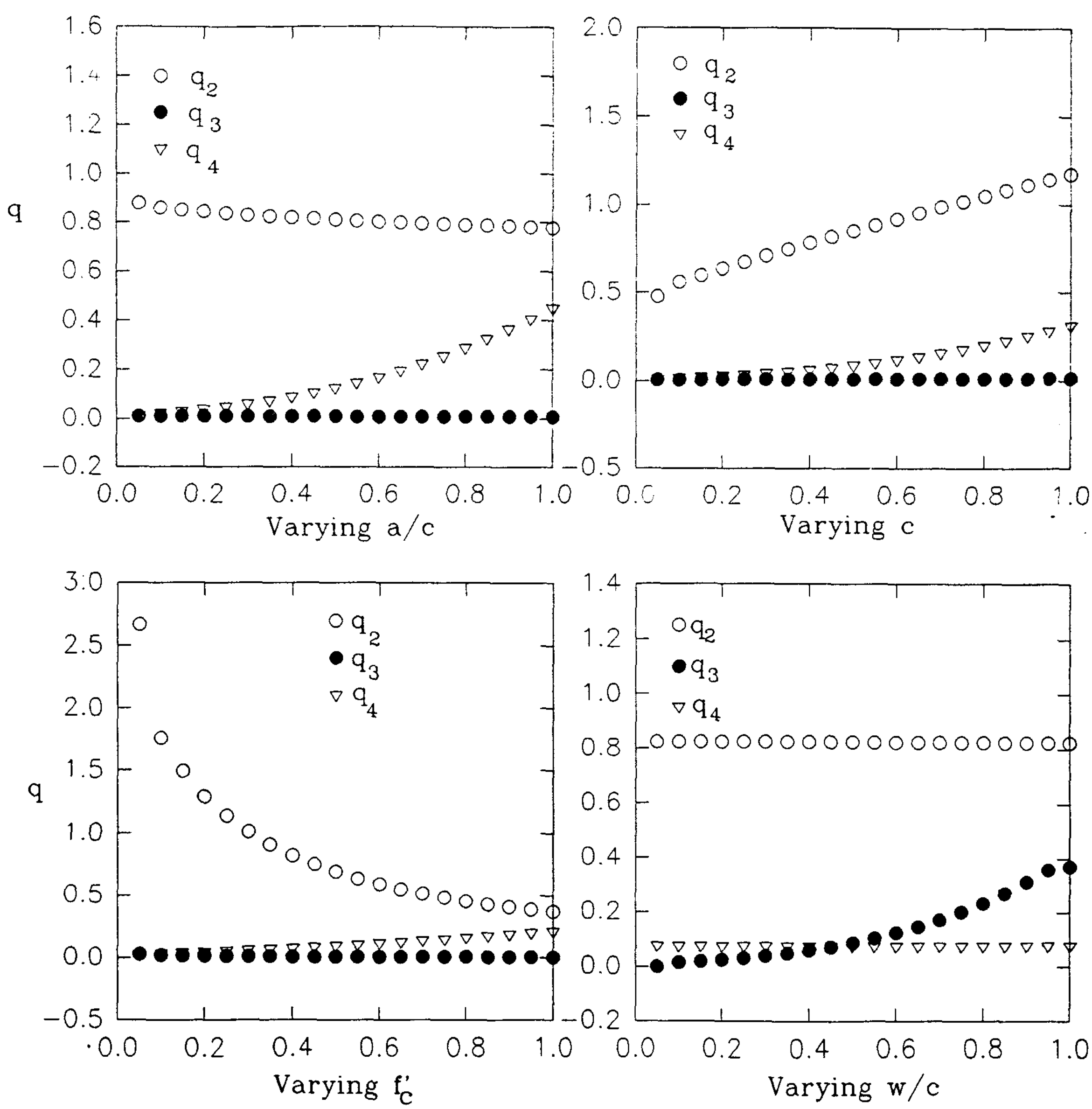

Fig. 8 Results of the sensitivity analysis to determine important influences of composition.

from the NSF Science and Technology Center for Advanced Cement-Based Materials at Northwestern University are gratefully acknowledged.

REFERENCES

1. Bažant, Z. P. and Baweja, S., 'Creep and shrinkage characterization for design of concrete structures - Model B3', Structural Engineering Report 94-10 603c (Northwestern University, 1994).
2. Bažant, Z. P. and Panula, L., 'Practical prediction of time-dependent deformations of concrete', Parts I-VI *Mater. Struct.* **11** (1978) 307-316, 317-328, 425-434; **12** (1979) 169-183.
3. Bažant, Z. P. and Kim, Joong-Koo., 'Improved prediction model for time-dependent deformations of concrete', 'Part 1 - Shrinkage', *Ibid.* **24** (1991) 327-345; 'Part 2 - Basic creep', *ibid.* **24** (1991) 409-442; 'Part 3 - Creep at drying', *ibid.* **25** (1992) 21-28; 'Part 4 - Temperature effects', *ibid.* **25** (1992) 84-94; 'Part 5 - Cyclic load and cyclic humidity', *ibid.* **25** (1992) 163-169.
4. Bažant, Z. P., Panula, L., Kim, Joong-Koo. and Xi, Y., 'Improved prediction model for time-dependent deformations of concrete: Part 6 - Simplified code-type formulation', *Ibid.* **25** (1992) 219-223.
5. (a) RILEM TC-107, 'Guidelines for characterizing concrete creep and shrinkage in structural design codes or recommendations', *Mater. Struct.* **28** (1995) 52-55; (b) RILEM TC 69, 'Conclusions for structural analysis and for formulation of standard design recommendations', in 'Mathematical Modeling of Creep and Shrinkage of Concrete', edited by Z. P. Bažant, Chap. 6 (Wiley, New York, 1988); reprinted in *Mater. Struct.* **20** (1987) 395-398; and in *ACI Mater. J.* **84** (1987) 578-581.
6. Bažant, Z. P. and Baweja, S., 'Creep and shrinkage model for analysis and design of concrete structures', RILEM Recommendation, *Mater. Struct.* **28** (1995) 357-365.
7. Bažant, Z. P., Xi, Y. and Baweja, S., 'Improved prediction model for time-dependent deformations of concrete: Part - 7, Short form of BP-KX model, statistics and extrapolation of short-time data', *Ibid.* **26** (1993) 567-574.
8. 'Prediction of creep, shrinkage and temperature effects in concrete structures', ACI 209 R-92 (American Concrete Institute, Detroit, 1992).
9. 'CEB-FIP Model Code, 1990, Design Code' (Thomas Telford, London, 1990).
10. Gardner, N. J. and Zhao, J. W., 'Creep and shrinkage revisited', *ACI Mater. J.* **90** 1993) 236-246; Discussion by Bažant, Z. P. and Baweja, S., *Ibid.* **91** (1994) 204-216.
11. Keeton, J. R., 'Study of creep in concrete', Technical Reports R333-I, R333-II, R333-III (US Naval Civil Engineering Laboratory, Port Hueneme, California, 1965).
12. Wallo, E. M., Yuan, R. L., Lott, J. L. and Kesler, C. E., 'Sixth progress report on prediction of creep in structural concrete from short time tests', T&AM Report No. 658. (Department of Theoretical and Applied Mechanics, University of Illinois at Urbana, 1965).
13. L'Hermite, R. G. and Mamillan M., 'Influence de la dimension des éprouvettes sur le retrait', *Ann. Inst. Techn. Bâtiment Trav. Publics* **23** (1970) 5-6.
14. L'Hermite, R. G., Mamillan, M. and Lefèvre, C., 'Nouveaux résultats de recherches sur la déformation et la rupture du béton', *Ibid.* **18** (1965) 323-360.
15. Wittmann, F. H., Bažant, Z. P., Alou, F. and Kim, J. K., 'Statistics of shrinkage test data', *Cement Concr. Aggreg. ASTM* **9** (1987) 129-153.
16. Hilsdorf, H. K., 'Unveröffentlichte Versuche an der MPA München', private communication, 1980.
17. Kommendant, G. J., Polivka, M. and Pirtz, D., 'Study of concrete properties for prestressed concrete reactor vessels', Final Report No. UCSESM 76-3 (to General Atomic Company) (Department of Civil Engineering, University of California, Berkeley, 1976).
18. Rostasy, F. S., Teichen, K.-Th. and Engelke, H., 'Beitrag zur Klärung des Zussammenhanges von Kriechen und Relaxation bei Normal-beton', Amtliche Forschungs- und Materialprüfungsanstalt für das Bauwesen, Heft 139 (Otto-Graf-Institut, Universität Stuttgart, 1972).
19. Hansen, T. C. and Mattock, A. H., 'Influence of size and shape of member on the shrinkage and creep of concrete', *ACI J.* **63** (1966) 267-290.
20. Troxell, G. E., Raphael, J. E. and Davis, R. W., 'Long-time creep and shrinkage tests of plain and reinforced concrete', *Proc. ASTM* **58** (1958) 1101-1120.
21. Russell, H. and Burg, R., Private communication, Construction Technology Laboratories, Skokie, Illinois.
22. Browne, R. D., 'Properties of concrete in reactor vessels', in 'Proceedings of the Conference on Prestressed Concrete Pressure Vessels', (Institution of Civil Engineers, London, 1967) pp. 11-31.
23. Hannant, D. J., 'Strain behaviour of concrete up to 95°C under compressive stresses', *Ibid.* pp. 57-71.
24. Takahashi, H. and Kawaguchi, T. 'Study on time-dependent behaviour of high-strength concrete (Part 1) - Application of the Time-Dependent Linear Viscoelasticity Theory of Concrete Creep Behaviour', Report No 21 (Ohbayashi-Gumi Research Institute, Tokyo, 1980) pp. 61-69.
25. Mandel, J., 'The Statistical Analysis of Experimental Data' (Wiley-Interscience, New York, 1964).
26. Tsubaki, T., 'Sensitivity of factors in relation to prediction of creep and shrinkage of concrete', in 'Creep and Shrinkage of Concrete', Proceedings of 5th International RILEM Symposium (ConCreep 5), Barcelona, Spain (Chapman & Hall, London, 1993) pp. 611-622.
27. Neville, A. M., Dilger, W. H. and Brooks, J. J., 'Creep of Plain and Structural Concrete (Construction Press, London, 1983).
28. Granger, L. and Bažant, Z. P. 'Effect of composition on basic creep of concrete', Structural Engineering Report No. 93-8/603e (Northwestern University Evanston, Illinois, 1993); *ASCE J. Engng Mech.*, in press.