Materials and Structures, 1992, **25**, 219–223

Improved prediction model for time-dependent deformations of concrete: Part 6—Simplified code-type formulation

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The complexity of the full BP–KX model for creep and shrinkage prediction presented in the previous five parts is due to the large number of influencing factors taken into consideration rather than an inherent complexity of the theory. However, a sophisticated model with all these factors is needed only for the design of special structures, such as bridges of a record span, buildings of a record height, or nuclear reactor structures. For most practical applications, many of the factors can be eliminated and can be fixed at their typical values. This yields a simplified BP–KX model which is presented in this last part. The simplified model should suffice for many practical applications and is suitable for standard code-type recommendations of engineering societies. The design of every structure can begin with this simplified model, and only if the effects of creep and shrinkage are found rather significant is analysis according to the full BP–KX model necessary.

1. INTRODUCTION

The BP–KX model for creep and shrinkage prediction presented in the foregoing five parts [1–5] is relatively complex. This is not due to inherent theoretical complexity but to the fact that there are many factors that influence creep and shrinkage and most of them are taken into account, with a broad range of variation. In most practical situations, however, many of these factors have similar values and can be considered to vary only over a rather limited range. For such situations, the present prediction model can be considerably simplified, which is the objective of this last part of the present series.

The simplified BP–KX model that we are going to present now is suitable for code-type standard recommendations of engineering societies. However, the full BP–KX model, which was calibrated by virtually all the existing, sufficiently documented test data from the literature (consisting of 347 data series), will be needed for special situations and for structures of high sensitivity to the effects of creep and shrinkage, such as bridges of a record span, buildings of a record height or nuclear power-plant structures.

2. SIMPLIFIED FORMULAE FOR SHRINKAGE

For the mean shrinkage strain in the cross-section

$$\varepsilon_{sh}(t, t_0) = \varepsilon_{sh\infty} k_h S(\hat{t}) \qquad \hat{t} = t - t_0 \tag{1}$$

For the time curve

$$S(\hat{t}) = \tanh\left(\frac{\hat{t}}{\tau_{sh}}\right)^{1/2} \tag{2}$$

For humidity dependence

$$k_h = \begin{cases} 1 - h^3 & \text{for } h \le 0.98 \\ -0.2 & \text{for } h = 1 \\ \text{linear interpolation} & \text{for } 0.98 \le h \le 1 \end{cases} \tag{3}$$

For size dependence

$$\tau_{sh} = 0.033D^2 \qquad D = 2v/s \tag{4}$$

Here t = time, representing the age of concrete, t_0 = age when drying begins, $\hat{t}$ = duration of drying (all the times must be given in days), $\varepsilon_{sh\infty}$ = ultimate shrinkage strain, h = relative humidity of the environment ($0 \le h \le 1$), τ_{sh} = shrinkage half-time, D = effective cross-section thickness in millimetres, v/s = volume-to-surface ratio in millimetres.

The final shrinkage strain may be approximately predicted from the mix composition and strength of concrete as follows:

$$\varepsilon_{sh} = \alpha_1\alpha_2\{1.12(w/c)^{1.5}c^{1.1}(f'_c)^{-0.2} \times [1 - (a/c)/(\rho_c/c)] + 0.16\} \quad (\text{in } 10^{-3}) \tag{5}$$

where

$$\alpha_1 = \begin{cases} 1.0 & \text{for type I cement} \\ 0.85 & \text{for type II cement} \\ 1.1 & \text{for type III cement} \end{cases} \tag{6}$$

and

$$\alpha_2 = \begin{cases} 0.75 & \text{for steam-cured specimens} \\ 1.0 & \text{for specimens cured in water or 100\% RH} \\ 1.4 & \text{for specimens sealed during curing} \end{cases} \tag{7}$$

‡ Deceased 1989.

Here c = cement content in lb ft^{-3} (1 lb ft^{-3} = 16.02 kg m^{-3}), w/c = water/cement ratio by weight, f'_c = 28-day standard cylinder strength in psi (1 psi = 6895 Pa), a/c = aggregate/cement ratio by weight, $\rho_c/c = 1 + (w/c) + (a/c)$ = relative concrete density, and $a/c = (g/c) + (s/c)$ where g/c is the gravel/cement ratio and s/c is the sand/cement ratio (all by weight).

3. SIMPLIFIED FORMULAE FOR CREEP

The average compliance function for a cross-section of a long member, representing the sum of the instantaneous deformation, the basic creep, and the additional creep due to drying, can be expressed as

$$J(t, t') = q_1 + C_0(t, t') + C_d(t, t', t_0) \tag{8}$$

$$C_0(t, t') = q_2 Q(t, t') + q_3 \ln[1 + (t - t')] + q_4 \ln\left(\frac{t}{t'}\right) \tag{9}$$

$$C_d(t, t', t_0) = q_5 k_h \varepsilon_{sh\infty}\left[S\left(\frac{t - t_0}{2\tau_{sh}}\right) - S\left(\frac{t' - t_0}{2\tau_{sh}}\right)\right]^{1/2} \tag{10}$$

This expression contains five empirical parameters, $q_1, q_3 \cdots q_5$ (constants), of the dimensions 1/psi (1 psi = 6895 Pa). The terms containing q_2, q_3 and q_4 represent the ageing viscoelastic compliance, the non-ageing viscoelastic compliance and the flow compliance, respectively. The term with q_5 represents the drying creep, i.e. the apparent additional creep due to drying. Furthermore, $q_1 = 1/E_0$, where E_0 is the asymptotic elastic modulus, which characterizes the strain for extremely short load durations and is obtained by extrapolating the short-time creep measurements to zero time (infinitely fast loading). Function $C_0(t, t')$ represents the basic creep compliance and $C_d(t, t', t_0)$ represents the additional creep compliance due to drying simultaneous with creep; t' = age at loading, t_0 = age at the start of drying, and t = time = current age of concrete (all in days). S is a function already defined for shrinkage and $\varepsilon_{sh\infty}$ = final shrinkage strain for zero humidity and reference conditions. Furthermore,

$$Q(t, t') = Q_f(t')\left[1 + \left(\frac{Q_f(t')}{Z(t, t')}\right)^r\right]^{-1/r} \tag{11}$$

with

$$Z(t, t') = (t')^{-1/2} \ln[1 + (t - t')^{0.1}] \tag{12}$$

$$Q_f(t') = [0.086(t')^{2/9} + 1.21(t')^{4/9}]^{-1} \tag{13}$$

$$r = 1.7(t')^{0.12} + 8 \tag{14}$$

in which t and t' must be given in days. Function $Q(t, t')$ represents an approximate (but rather accurate) solution of a simple integral equation based on the solidification theory.

The material parameters $q_1, \ldots, q_5$ all appear in Equations 8–10 linearly, and so they can be easily determined from test data by linear regression. Wherever possible, parameters $q_1, \ldots, q_5$, or at least some of them should be calibrated by fitting of test data for the particular concrete to be used, or at least a similar concrete used in a given geographic zone. Even the use of short-time data, with calibration of only one or two parameters among $q_1, \ldots, q_5$ (as described in Parts 1 to 3 [1–3]) is preferable to using no data at all. In the absence of test data, one may predict the values of $q_1, \ldots, q_5$ from the concrete strength and composition using the following formulae, which represent a simplification of those from Parts 1 to 3.

For instantaneous (asymptotic) strain $q_1 = 1/E_0$

$$q_1 = \frac{10^6}{1.5E_{28}} \qquad E_{28} = 57\,000(f'_c)^{1/2} \tag{15}$$

For ageing viscoelastic strain

$$q_2 = 0.011(w/c)^{0.8}c^{1.5} \times [1 - (a/c)/(\rho_c/c)]^{-0.9}(0.001f'_c)^{-0.5} - 0.39 \quad (\text{in } 10^{-6}) \tag{16}$$

For non-ageing viscoelastic strain

$$q_3 = 0.025q_2 \quad (\text{in } 10^{-6}) \tag{17}$$

For ageing viscous strain (flow)

$$q_4 = 0.072(w/c)^{2.3}c^{0.2} \times [1 - (a/c)/(\rho_c/c)]^{0.39}(0.001f'_c)^{0.46} \quad (\text{in } 10^{-6}) \tag{18}$$

For drying creep

$$q_5 = 40(f'_c)^{-1/2} \tag{19}$$

in which f'_c is the 28-day standard cylinder strength in psi (1 psi = 6895 Pa), and E_{28} (also in psi) is the conventional elastic modulus at 28 days (which is taken here according to the well-known ACI formula).

4. ADMISSIBLE PARAMETER RANGES

The simplifications of the formulae from Parts 1 to 3 [1–3] were achieved by restricting the ranges of certain parameters. For the following parameter ranges, the present formulae usually give very good results, their deviations from the formulas in Parts 1 to 3 being relatively small.

For shrinkage

$$\begin{array}{ll} 1.0 \le k_s \le 1.3 & t_0 \le 7 \text{ days} \\ 13°\text{C} \le T \le 37°\text{C} & 1 \le a/s \le 2.6 \end{array} \tag{20}$$

where $a/s = (a/c)/(s/c)$. For creep (basic and drying)

$$\begin{array}{ll} |\sigma| \le 0.55f'_c & 7 \text{ days} \le t_0 \le 40 \text{ days} \\ 7 \text{ days} \le t' \le 40 \text{ days} & t' \ge t_0 \end{array} \tag{21}$$

The ranges of cement content and sand/gravel ratio s/g that appear in Equation 20 can be all the values characteristic of normal concretes. This simplified model is basically applicable for type I (ordinary) Portland cement, and also for normal (wet) curing conditions.

For the following parameter values the predictions of shrinkage and basic creep coincide with the predictions based on Parts 1 and 2: $k_s = 1.15$ (shape factor for a cylinder), $t_0 = 10$ days, $T = 23°C$, $a/s \leq 2.6$, $t' = t_0$, $\sigma = 0.3f'_c$, $c = 20\ \mathrm{lb\ ft^{-3}}$ ($320\ \mathrm{kg\ m^{-3}}$) (for parameter q_3), and $s/g = 0.5$ (for parameter q_4). Replacing Equation 4 by $\tau_{sh} = 0.025D^2k_s^2$ with $k_s = 1$ for a slab or $k_s = 1.25$ for a prism, improved predictions can be obtained for these shapes.

Figs 1–6 demonstrate some predictions with the present formulae and compare these with typical test data. These figures, however, do not represent a verification of the model. It has been amply demonstrated before how a selective use of only some of the existing data can be grossly misleading [12]. The verification of the present model has been provided by the systematic analysis of all the existing sufficiently documented test data as presented in Parts 1 to 5.

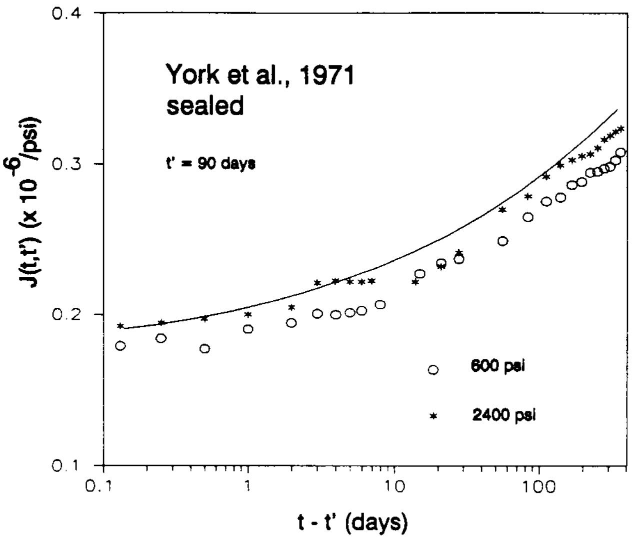

Fig. 3 Predictions of basic creep data by York *et al.* [8] (1 psi = 6895 Pa).

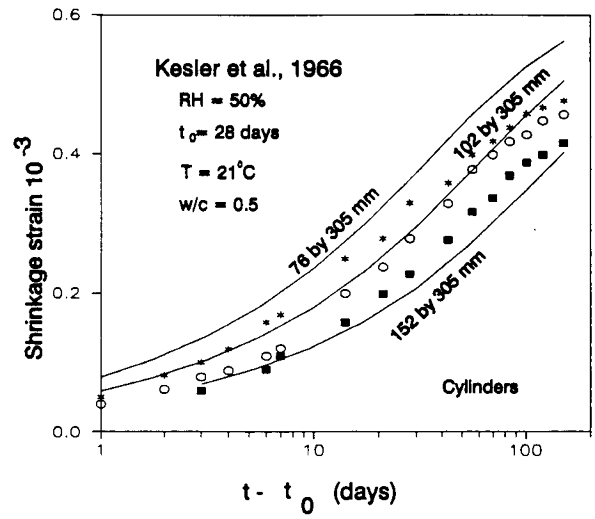

Fig. 1 Predictions of shrinkage test data by Kesler *et al.* [6].

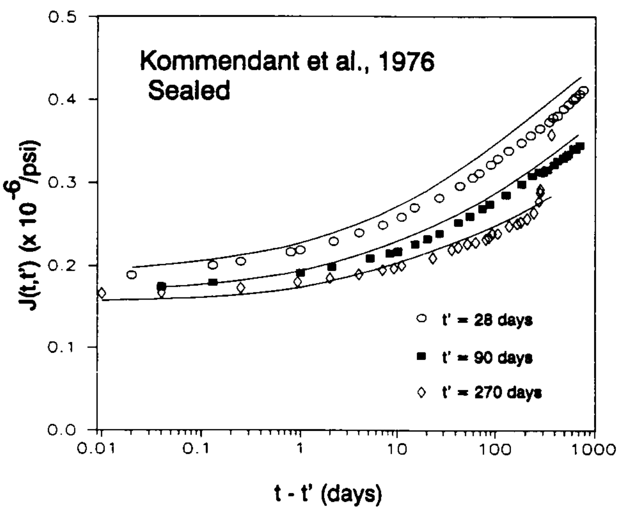

Fig. 4 Predictions of basic creep test by Kommendant *et al.* [9] (1 psi = 6895 Pa).

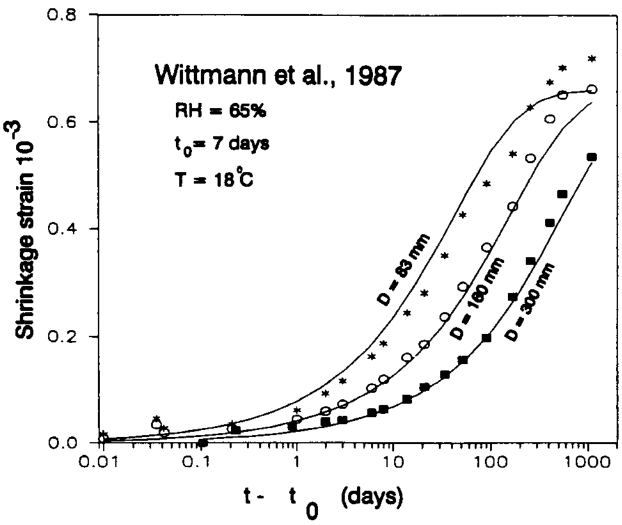

Fig. 2 Predictions of shrinkage data by Wittmann *et al.* [7].

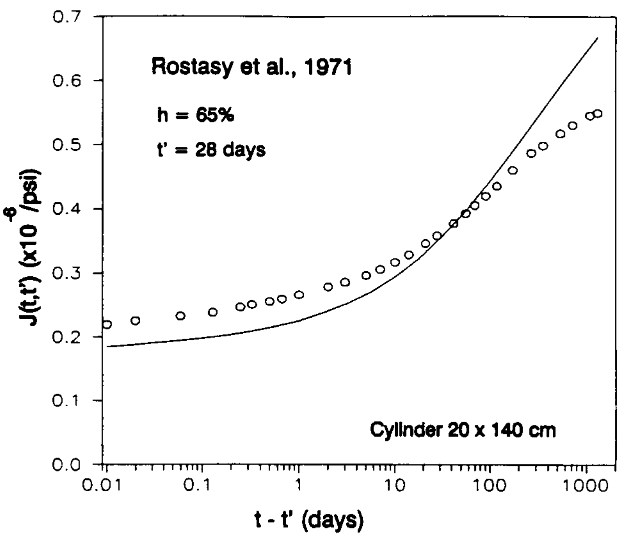

Fig. 5 Prediction of drying creep data by Rostasy *et al.* [10] (1 psi = 6895 Pa).

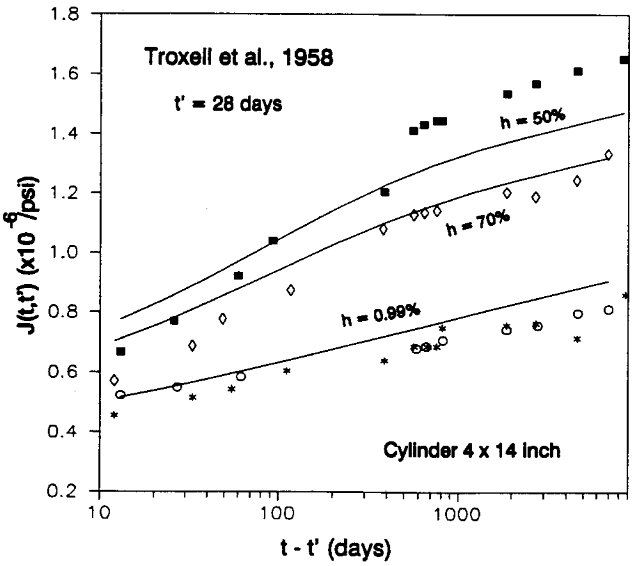

Fig. 6 Predictions of drying creep data by Troxell *et al.* [11] (1 psi = 6895 Pa).

5. SUMMARY OF ADVANTAGES

Although the advantages of the BP–KX model were discussed in detail in Parts 1 to 3 [1–3], it may be useful to give their abbreviated listing:

1. The shrinkage and drying creep formulae agree with all the asymptotic requirements of non-linear diffusion theory for the movement of moisture through concrete.
2. For creep, there are only five free material parameters, and they can all be identified from given test data by linear regression (i.e. no non-linear optimization is needed).
3. The creep formula can be easily converted, by explicit formulae, to a rate-type (Kelvin chain) model with non-ageing properties, the effect of ageing being totally ascribed to transformations of time (this feature is very useful for finite-element analysis).
4. The condition of non-divergence of the creep curves for different t_0 is automatically satisfied.
5. The additional creep due to drying is based on the shrinkage function, which is both the simplest and theoretically best justified (by the theory of stress-induced shrinkage).

6. CONCLUSION

The present formulae are sufficiently simple for practical use in design offices and should be adequate for many structures, except special structures that are very sensitive to creep and shrinkage.

ACKNOWLEDGEMENTS

The present work (Parts 1 to 6) has been partially supported under NSF grant MSM-8700830 to Northwestern University. The work was carried out in close co-operation with Subcommittee 2 of ACI Committee 209, Creep and Shrinkage (chaired by Marvan A. Daye) as well as with RILEM Committee TC 107, Creep and Shrinkage Prediction (chaired by Bažant). Important preliminary studies and tedious data compilation were carried out by Panula at TAMS Inc., New York, until her tragic death in 1989. Y. Xi worked (after the return of J.-K. Kim to Korea in July 1990) on final tuning of the model at the Center for Advanced Cement-Based Materials at Northwestern University, which also provided partial financial support.

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RESUME

Modèle amélioré de prédiction des déformations du béton en fonction du temps: 6ème partie – Formulation simplifiée à l'usage des codes

On doit attribuer la complexité du modèle intégral BP–KX pour la prédiction du fluage et retrait du béton présenté dans les cinq chapîtres précédents au grand nombre de facteur déterminants pris en considération plutôt qu'à une complexité inhérente à la théorie. Cependant, un modèle sophistiqué comportant tous ces facteurs est nécessaire seulement pour le calcul des structures telles que des ponts de très grande portée, des bâtiments de très grande hauteur ou des réacteurs nucléaires. Dans la plupart des applications pratiques, de nombreux facteurs peuvent être éliminés et fixés à leurs valeurs typiques, ce qui donne un modèle BP–KX simplifié que nous présentons dans cette dernière partie. Le modèle devrait suffire pour de nombreuses applications pratiques et convient pour les recommandations à usage de code ou de norme dans l'industrie. On peut commencer le calcul de n'importe quelle structure à l'aide de ce modèle simplifié et, seulement si l'effet du fluage ou du retrait paraît assez marqué, recourir à l'analyse selon le modèle intégral BP–KP.