269

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RILEM DRAFT RECOMMENDATIONS
PROJETS DE RECOMMANDATION DE LA RILEM

TC 89-FMT FRACTURE MECHANICS OF CONCRETE – TEST METHODS
MÉCANIQUE DE LA RUPTURE DU BÉTON – MÉTHODES D'ESSAI

Size-effect method for determining fracture energy and process zone size of concrete

The text presented hereunder are drafts which are being submitted to enquiry. Comments should be sent to Prof. S. P. Shah, The Technological Institute, Northwestern University, Evanston, Illinois 60208-3109, USA, before 1 July 1991.

CONTENTS

1. SCOPE AND DEFINITION

The recommendation describes the size-effect method for determining the fracture energy of concrete by measuring the maximum loads of geometrically similar notched concrete specimens of different sizes. The fracture energy G_f which is obtained by this method is defined as the specific energy (i.e. energy per unit crack plane area) required for fracture growth in an infinitely large test specimen. This definition is, in theory, independent of both the specimen size and shape.

The method is described here for Mode I fracture only, although the same approach is possible for Mode II and Mode III shear fractures. The method is also extended to yield other non-linear fracture characteristics such as the effective length of the process zone.

2. SPECIMENS

2.1 Material

All specimens must be cast from the same batch of concrete. The quality of concrete must be as uniform as possible. The curing procedure and the environments to which the specimens are exposed, including their histories, must be the same for all the specimens.

2.2 Geometry of specimens

Although the present method works equally well for several geometries, three-point bend beams are recommended for the purpose of standardization. The recommended specimen is shown in Fig. 1. It is a beam of width b, depth d and length L. The beam is loaded at mid-span by a concentrated load, and is simply supported over span l. A notch of depth a_0 is cut into the cured beam at mid-length (Fig. 1). The loads are applied through one hinge and two rollers with the minimum possible rolling friction. The concentrated load as well as the support reactions, which represent uniformly distributed line loads across the beam width, are applied over bearing plates whose thickness is such that they could be considered as rigid. The bearing plates are either glued with epoxy or are set in wet cement. The distance from the end of the beam to the nearest support must be sufficient to prevent spalling and cracking at beam ends.

The span-to-depth ratio of the specimen, l/d, should

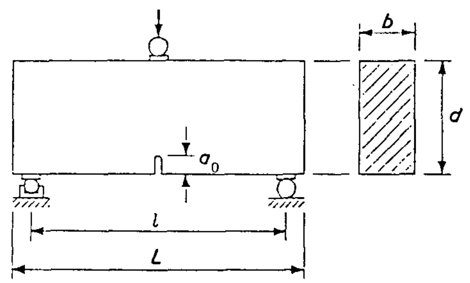

Fig. 1 Suggested specimen geometry.

be at least 2.5. The ratio of the notch depth to the beam depth, a_0/d, should be between 0.15 and 0.5. The notch width should be as small as possible and should not exceed 0.5 times the maximum aggregate size d_a. The width b and the depth d must not be less than $3d_a$.

Specimens of at least three different sizes, characterized by beam depths $d = d_1, \ldots, d_n$, and spans $l = l_1, \ldots, l_n$ must be tested. The smallest depth d_1 must not be larger than $5d_a$, and the largest depth d_n must not be smaller than $10d_a$. The ratio of d_n to d_1 must be at least 4. The ratios of the adjacent sizes should be approximately constant. Optimally, the size range should be as broad as feasible. Thus, for instance, the choice $d/d_a = 4, 8, 16$ is usually acceptable, but the choice $d/d_a = 3, 6, 12, 24$ is preferable.

At least three identical specimens should be tested for each specimen size. All the specimens should be geometrically similar in two dimensions, with the same third dimension (thickness b). This means that the ratios l/d, a_0/d, and L/d should be the same for all specimens.

3. APPARATUS

It is sufficient to use an ordinary uniaxial testing machine without high stiffness. However, closed-loop control and a high stiffness of the loading frame lead to more consistent results. The machine must be capable of accurately registering the maximum load. The post-peak response need not be measured. The same machine must be used to test all the specimens.

4. TEST PROCEDURE

Specimens should be loaded at constant (or almost constant) displacement rates. The loading rates should be such that the maximum load is reached in about 5 min.

5. TEST RESULTS

The test results that are needed to determine the fracture energy are only the maximum load values $P_1, \ldots, P_n$ for specimens of various sizes $d_1, \ldots, d_n$, and the Young's modulus of the concrete, E_c. The Young's modulus can be obtained using any conventional test method, e.g. compression tests on cylinders. The following data should also be reported: all dimensions of the beams and of the bearing plates; maximum aggregate size; the ratios (by weight) of water, cement, sand and gravel in the mix; type of cement, its fineness, admixtures; mineralogical type of aggregate; curing and storing conditions; temperature and humidity during the test; standard compressive strength; and the mean mass density of the concrete.

6. CALCULATION PROCEDURE

(a) The corrected maximum loads, $P_1^0, \ldots, P_n^0$, which take the weight of the specimen into account, have to be calculated. If L_j is almost the same as l_j,

$$P_j^0 = P_j + \frac{1}{2} m_j g \qquad (j = 1, \ldots, n) \tag{1}$$

in which m_j is the mass of specimen j, g = acceleration due to gravity, and n = number of tests conducted. If L_j is much larger than l_j,

$$P_j^0 = P_j + \frac{2l_j - L_j}{2l_j} m_j g \qquad (j = 1, \ldots, n) \tag{2}$$

(b) Now carry out linear regression, considering the plot of the ordinates Y_j against the abscissae X_j where

$$Y_j = \left(\frac{bd_j}{P_j^0}\right)^2, \quad X_j = d_j \tag{3}$$

Determine the slope and intercept of the regression line $Y = AX + C$:

$$A = \frac{\sum_j (X_j - \bar{X})(Y_j - \bar{Y})}{\sum_j (X_j - \bar{X})^2}, \quad C = \bar{Y} - A\bar{X} \tag{4}$$

where

$$\bar{X} = \frac{1}{n}\sum_j X_j, \qquad \bar{Y} = \frac{1}{n}\sum_j Y_j \tag{5}$$

$(\bar{X}, \bar{Y})$ is the centroid of all data points. Also check whether the plot of data points is approximately linear. If not, then some errors or disturbing effects have probably occurred in the test procedure.

(c) Calculate auxiliary values for the extrapolation to very large specimen sizes for which linear elastic fracture mechanics applies. Defining relative crack length $\alpha = a/d$, where a = crack length, For $l/d = 2.5$:

$$F_{2.5}(\alpha) = \frac{1.0 - 2.5\alpha + 4.49\alpha^2 - 3.98\alpha^3 + 1.33\alpha^4}{(1-\alpha)^{3/2}} \tag{6}$$

For $l/d = 4$:

$$F_4(\alpha) = \frac{1.99 - \alpha(1-\alpha)(2.15 - 3.93\alpha + 2.7\alpha^2)}{\pi^{1/2}(1+2\alpha)(1-\alpha)^{3/2}} \tag{7}$$

For $l/d = 8$:

$$F_8(\alpha) = 1.11 - 2.12\alpha + 7.71\alpha^2 - 13.55\alpha^3 + 14.25\alpha^4 \tag{8}$$

Linear interpolation can be used for other values of l/d. For example, for $4 < l/d < 10$,

$$F(\alpha) = F_4(\alpha) + \frac{(l/d) - 4}{4}[F_8(\alpha) - F_4(\alpha)] \tag{9}$$

The non-dimensional energy release rate is

$$g(\alpha) = \left(\frac{l}{d}\right)^2 \pi\alpha[1.5F(\alpha)]^2 \tag{10}$$

(note that for values of l/d which are far from those corresponding to Equations 6 to 8, interpolation is not sufficiently accurate).

For $\alpha_0 = a_0/d$, compute $g(\alpha = \alpha_0)$.

(d) Now calculate the fracture energy G_f (mean prediction):

$$G_f = \frac{g(\alpha_0)}{E_c A} \tag{11}$$

(e) Finally, calculate the statistics:

$$s_X^2 = \frac{1}{n-1}\sum_j (X_j - \bar{X})^2$$

$$s_Y^2 = \frac{1}{n-1}\sum_j (Y_j - \bar{Y})^2 \tag{12}$$

$$s_{Y|X}^2 = \frac{1}{n-2}\sum_j (Y_j - Y_j')^2$$

$$= \frac{n-1}{n-2}(s_Y^2 - A^2 s_X^2) \tag{13}$$

$$\omega_{Y|X} = s_{Y|X}/\bar{Y} \qquad \omega_X = s_X/\bar{X} \tag{14}$$

$$\omega_A = \frac{s_{Y|X}}{A s_X (n-1)^{1/2}}$$

$$\omega_C = \frac{s_{Y|X}}{C(n-1)^{1/2}}\left(1 + \frac{1}{\omega_X^2}\right)^{1/2}$$

$$m = \frac{\omega_{Y|X}}{\omega_X} \tag{15}$$

and the approximation:

$$\omega_G^2 \approx \omega_A^2 + \omega_E^2 \tag{16}$$

where $j = 1, 2, \ldots, n$; X_j are the specimen sizes, Y_j are the measured data points (not averages of some groups of data points); $(Y_j - Y_j')$ are the vertical deviations of these points from the regression line, ω_X = coefficient of variation of the sizes chosen, $\omega_{Y|X}$ = coefficient of variation of the errors (vertical deviations from the regression line), ω_A = coefficient of variation of the slope of the regression line, ω_C = coefficient of variation of the intercept, m = relative width of scatter-band, ω_E = coefficient of variation of the values of the elastic modulus E_c used in Equation 11. If the values of E_c and G_f are assumed to be uncorrelated, the coefficient of variation of G_f is $\omega_{G_f} \approx \omega_G$, and if they are assumed to be perfectly correlated, $\omega_{G_f} \approx \omega_A$. In reality, $\omega_A < \omega_{G_f} < \omega_G$.

The value of ω_A should not exceed about 0.10 and the values of ω_C and m about 0.20. These conditions prevent situations in which the size range used is insufficient compared with the scatter of results. Such a situation is illustrated in Fig. 2a, in which a unique regression slope A is obtained but is highly uncertain. Fig. 2b illustrates the case where the large scatter of test results necessitates the use of a very broad size range, while Fig. 2c illustrates the case where a small scatter of test results permits the use of a narrow size range. Obviously, the necessary size range can be reduced by carefully controlled testing which results in a low scatter.

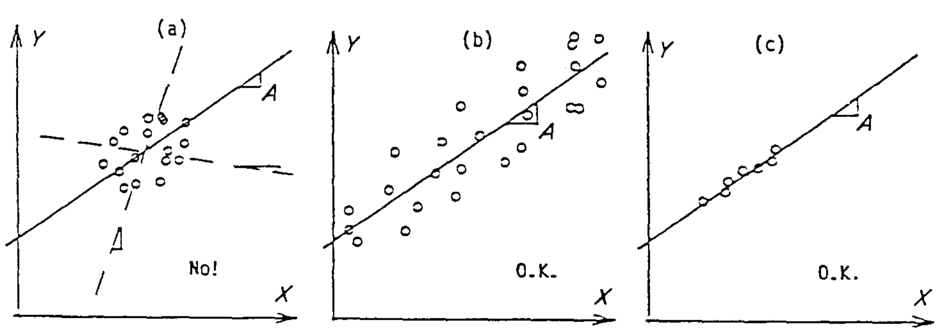

Fig. 2 Scatter of results.

7. CAVEAT: OTHER SIZE EFFECTS

Scattered results can, however, also arise when other size effects are present. This is of particular concern for relatively large and thick specimens, in which hydration heat can cause different heating in small and large specimens, and thus lead to a size effect that is superimposed on that described by non-linear fracture mechanics. In that case the present method can fail, although theoretical corrections to filter out these other types of size effect might be possible. Similarly, it can fail when size effects of different type are produced by drying, which affects small specimens differently from the large ones.

8. EXAMPLE

For a concrete of maximum aggregate size 13 mm, the data in Table 2 have been obtained according to Section 6(a) by correcting the measured maximum loads of three-point bend specimens (Fig. 1) of width b = 38 mm, $l/d = 2.5$, $L/d = 8/3$ and notch depth $a_0 = d/6$; E_c = 27.7 GPa and $\omega_E = 0.02$.

Data for regression are given in Table 3. From Equations 6 and 10, $g(\alpha_0) = 6.07$. For the regression line $Y = AX + C$ (Fig. 4):

$$A = 5.98 \times 10^{-3}\,\text{mm}^{-1}\,\text{MPa}^{-2}$$

$$C = 0.465\,\text{MPa}^{-2}$$

$$s_{Y|X} = 0.142\,\text{MPa}^{-2}$$

$$\omega_{Y|X} = 0.108$$

$$\omega_A = 0.0672 \qquad \omega_C = 0.154 \qquad m = 0.144$$

Result:

$$G_f = \frac{6.07}{(27.7 \times 10^3\,\text{MPa})(5.98 \times 10^{-3}\,\text{mm}^{-1}\,\text{MPa}^{-2})}$$

$$= 36.6\,\text{N m}^{-1}$$

$$\omega_G = 0.0701, \qquad 0.0672 < \omega_{G_f} < 0.0701$$

9. BACKGROUND

For materials in which the fracture front is blunted by a non-linear zone of distributed cracking or damage, the fracture energy G_f may be uniquely defined as the energy required for crack growth in an infinitely large specimen. The foregoing definition is practically useful only if the law for extrapolating to infinite specimen size is known. Although the exact form of the size effect law for blunt fracture is not known, an approximate form which appears to be sufficient for practical purposes is given by the size-effect law (see Fig. 3):

$$\sigma_N = Bf_t'/(1+\beta)^{0.5}\,, \quad \beta = d/d_0 \qquad (17)$$

in which σ_N = nominal stress at failure, f_t' = tensile strength, B and d_0 = empirical constants, and β is called the brittleness number.

Table 2 Example of data obtained

Depth, d (mm)	Corrected maximum loads, P^0 (N)*		
38	1800	1810	1850
76	3010	3140	3160
152	4400	4630	4880
305	7730	7740	7890

*See Fig. 3.

Table 3 Regression data

$X = d$ (mm)	$Y = (bd/P^0)^2$ (MPa^{-2})		
38	0.644	0.636	0.609
76	0.921	0.846	0.835
152	1.723	1.556	1.401
305	2.248	2.242	2.158

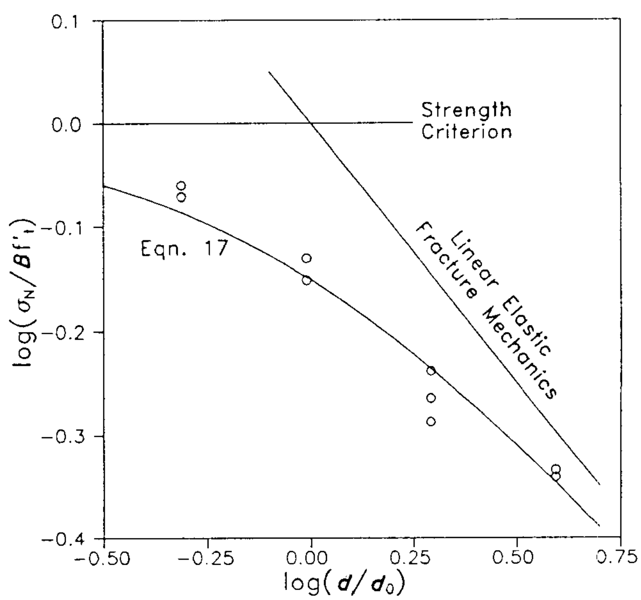

Fig. 3 (○) Test data and size effect (from Bažant and Pfeiffer [1]). $\sigma_N = P^0/bd$, d_0 = 77.8 mm, Bf_t' = 1.47 MPa.

Taking into account the inevitable scatter of test results for a material such as concrete, it appears that the size-effect law in Equation 17 is adequate for a size range up to 1:20, which is sufficient for most practical purposes.

Evaluation of experimental results according to Equation 11 has been shown to yield G_f values that are approximately the same for very different specimen geometries, including edge-notched tension specimens, three-point bend notched specimens, and notched eccentric compression specimens.

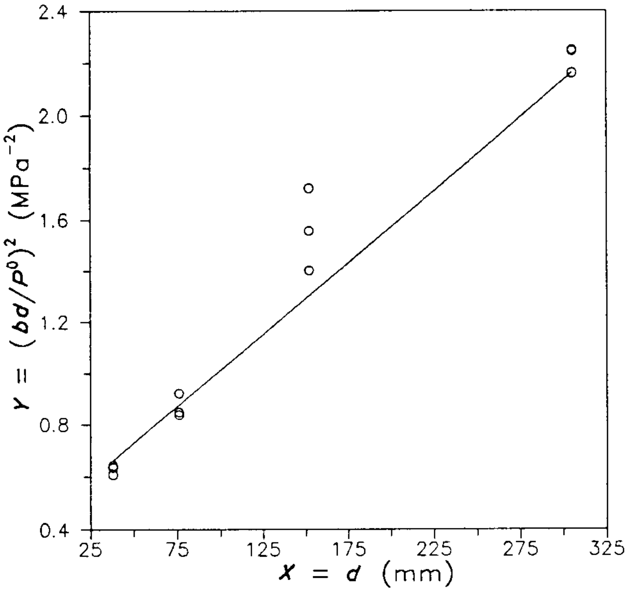

Fig. 4 Linear regression (○) Test data; $Y = AX + C$ where $A = 0.006\ \mathrm{mm}^{-1}\,\mathrm{MPa}^{-2}$, $C = 0.47\ \mathrm{MPa}^{-2}$, $\omega_{Y|X} = 0.109$.

10. EXTENSION: DETERMINATION OF PROCESS ZONE LENGTH AND OTHER FRACTURE CHARACTERISTICS

The size-effect method makes it also possible to determine the effective length c_f of the fracture process zone. This fracture parameter represents the length of the equivalent linear elastic crack that gives the same unloading compliance as the actual crack in an infinitely large specimen at the peak load:

$$c_f = \frac{g(\alpha_0)}{g'(\alpha_0)}\left(\frac{C}{A}\right) \tag{18}$$

where $g'(\alpha) = \mathrm{d}g(\alpha)/\mathrm{d}\alpha$. Equation 18 is valid only if $g'(\alpha_0) > 0$. Also, the critical effective crack-tip opening displacement at the peak load of an infinitely large specimen is

$$\delta_c = (32 G_f c_f / \pi E_c)^{0.5} \tag{19}$$

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