

# SOLIDIFICATION THEORY FOR CONCRETE CREEP. I: FORMULATION

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**ABSTRACT:** The paper presents a new general constitutive law for creep in which the aging due to continuing hydration of cement is taken into account in a manner that is both simpler and physically better justified than in existing theories. Micromechanical analysis of the solidification process is used to show that the aging may be modeled as a growth of the volume fraction of load-bearing solidified matter (hydrated cement), which itself is treated as nonaging and thus is describable as a nonaging viscoelastic material. The analysis shows that a history integral should be used to express the rate, rather than the total value, of the viscoelastic strain component. Material functions can be chosen in a way that yields previously established simple laws, i.e., the double power law, logarithmic law and log-double power law, as special asymptotic cases. The creep strain is obtained as a sum of aging and nonaging viscoelastic strains and an aging viscous strain (flow). Nonlinearity is introduced by modifying the current creep rate as a function of the current stress. Verification by test results and numerical application is left to Part II, which follows.

## INTRODUCTION

The creep of concrete is profoundly influenced by the process of cement hydration. This influence, commonly called aging, causes the creep at constant stress to decrease significantly as the age at loading increases. The decrease is not confined to young concrete, but occurs through the entire lifetime of structures.

Modeling of the aging aspect of creep has proven to be a major complicating factor. Although integral as well as differential formulations that take the aging into account are available and used in practice ("Mathematical" 1986; "Conclusions" 1987; "Finite" 1982; Bažant 1982), they have several serious shortcomings.

1. The presently used forms of the creep law with aging are not guaranteed to satisfy thermodynamic restrictions. From the theoretical viewpoint this is, in the least, discomforting. The aging is generally described by considering certain material properties to be functions of the age,  $t'$ , of concrete. Although thermodynamic potentials can still be formulated (Bažant 1979), the laws of thermodynamics can be written only for systems of substances whose properties do not vary in time.

2. For the differential form of the creep law based on a rheologic model, the implication of aging is that the spring moduli,  $E_{\mu}$ , and dashpot viscosities,  $\eta_{\mu}$ , depend on time. This is a major complication for numerical solutions, and is a questionable approach from the thermodynamic viewpoint.

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3. The existing algorithms for the identification of  $E_\mu$  and  $\eta_\mu$  from given or measured compliance data yield results that are very sensitive to the scatter of data and do not guarantee  $E_\mu$  and  $\eta_\mu$  to be nondecreasing positive functions of time. In fact, the resulting  $E_\mu$  and  $\eta_\mu$  values usually violate this condition for short time periods. While this does not necessarily imply violation of the laws of thermodynamics, compliance with these laws cannot be guaranteed ("Mathematical" 1986; Bažant 1982, 1975).

4. It appears impossible to prevent the creep curves for different ages at loading to exhibit divergence. Thus, it is impossible to ensure the creep recovery curves obtained from the principle of superposition to decline monotonically.

5. The linearity of the existing comprehensive creep models is a source of significant disagreements with test results. For example, tests of step-wise stress histories or creep recovery reveal significant deviations from the principle of superposition. The hypothesis of linearity may also be partly responsible for the other shortcomings listed above.

The objective of the present study is to eliminate these shortcomings with a new theory with a basic mathematical form proposed by Bažant (1986), summarized by Bažant and Prasanna (1986), and presented at a recent conference (Bažant 1987). This theory has a physical basis in micromechanics of the aging process.

#### MICROMECHANICS-BASED CREEP MODEL FOR SOLIDIFYING MATERIAL

The total strain  $\epsilon$  of concrete may be decomposed as:

$$\epsilon = \frac{\sigma}{E_0} + \epsilon^c + \epsilon^0, \quad \epsilon^c = \epsilon^v + \epsilon^f \dots \dots \dots (1)$$

in which,  $\sigma$  = stress;  $E_0$  = elastic modulus;  $\sigma/E_0$  = elastic strain;  $\epsilon^c$  = creep strain;  $\epsilon^v$  = viscoelastic strain;  $\epsilon^f$  = viscous strain (flow); and  $\epsilon^0$  = sum of the hygrothermal strain, such as drying shrinkage, thermal dilatation, chemical strain such as autogeneous shrinkage, and, at high stresses, the cracking strain (which involves strain-softening). At high stresses, the dependence of  $\epsilon^v$  and  $\epsilon^f$  on stress becomes nonlinear, and then  $\epsilon^v$  and  $\epsilon^f$  represent the viscoelastic-plastic strain and the viscoplastic strain.

The elastic strain  $\sigma/E_0$  results from the deformation of the mineral aggregate pieces in concrete and the microscopic elastic particles in the hardened cement paste, including anhydrous cement grains, calcium hydroxide crystals as well as the crystalline particles (sheets and needles) in the cement gel. Due to their physico-chemical nature, the elastic properties of all these microscopic components are constant (nonaging). The elastic strain of concrete, however, has usually been considered as  $\sigma/E(t)$ , where the elastic modulus,  $E(t)$ , is considered to be a function of the age,  $t$ , of concrete. We nevertheless take the view that age-dependence of  $E(t)$  is a complication that is both unnecessary and thermodynamically objectionable. If modulus  $E_0$ , which is a material constant, is defined as the asymptotic modulus, a modulus applicable to vanishingly short-load durations (practically below 1  $\mu$ s), then the age-dependent part of the conventional elastic strain (usually a strain for load duration from 1 min to 1 hr) is recognized to be only an apparent elastic deformation, which, in reality, represents shorttime creep. The use

<sup>1</sup>Yield results that are very sensitive to the scatter of data.

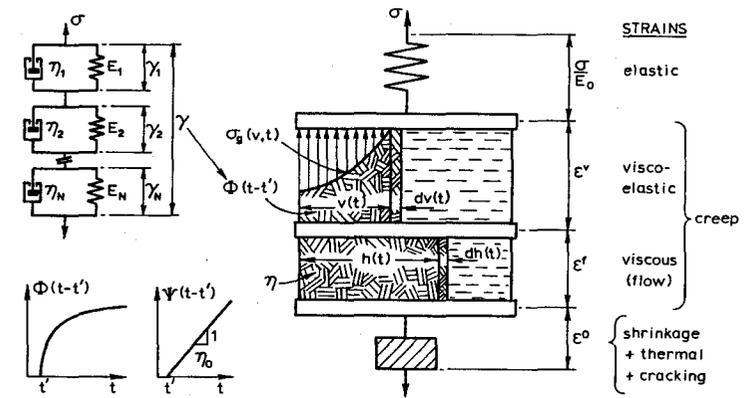


FIG. 1. Model for Role of Solidification in Creep

of constant  $E_0$  instead of variable  $E(t)$ , made in the double power law (Bažant 1975; Bažant and Osman 1976), log-double power law, and triple power law (Bažant and Chern 1985a, b), considerably simplifies the stress-strain relation for creep.

The additivity of the strains in Eq. 1 is reflected in the rheologic model in Fig. 1. This model at the same time portrays aging as a consequence of the growth of the volume fractions  $v$  and  $f$  of the solidified matter (hydrated cement) associated with the viscoelastic and viscous strains, respectively. The growth of  $v(t)$  and  $f(t)$  may be mathematically formulated in the manner proposed by Bažant (1977, 1979).

Let us first analyze the effect of  $v(t)$  associated with  $\epsilon^v$ . The elementary volume,  $dv(t)$ , which solidifies at time  $t$ , is assumed to be represented by a layer deposited on the surface of the material that previously solidified from a solution (a fluid), as shown in Fig. 1. The basic hypothesis illustrated by this model is that the volume elements  $dv(t)$  solidified at various times are all subjected to the same strain, which is equal to the overall (macroscopic) creep strain  $\epsilon^v(t)$ . This hypothesis implies the coupling of all these volume elements to be in parallel (Fig. 1).

In general, of course, some more complicated combination of parallel and series couplings should be more realistic. The parallel coupling hypothesis generally gives upper bounds on the stiffness of composites, and for composites such as concrete, it also gives relatively good estimates. So it should also be acceptable for creep, as a simple approximation.

The laws of thermodynamics can be formulated only for systems of substances that have constant (i.e., time-invariable) properties. In chemical thermodynamics (e.g., Denbigh 1966), aging substances are never considered; they are intractable. Rather, a time-dependent chemical system is obtained as a consequence of varying composition (or varying concentrations) of the substances in the system.

The hydration of cement is a chemical reaction, and so it must be treated similarly, as proposed by Bažant (1977). In this approach, which is adopted as the basis of the present formulation, one introduces the microstress,  $\sigma_g(v,t)$ , in the solidified matter (cement gel) at time  $t$ , defined as the stress at the

location where the solidification occurred when the volume of all the solidified matter was  $v$  (Fig. 1). A layer of thickness,  $dv(\tau)$ , is assumed to solidify and bind with the previously solidified matter at time  $\tau$ , at which the volume of all the solidified matter is  $v(\tau)$ . According to the model in Fig. 1, equilibrium requires that:  $\int \sigma_g(v', t) dv' = \sigma(t)$ , for all  $t > 0$ , the integration being from  $v' = 0$  to  $v' = v(t)$ . This condition may be written as

$$\int_{\tau=0}^t \sigma_g[v(\tau), t] dv(\tau) = \sigma(t) \dots \dots \dots (2)$$

The solidifying material on the microscale is considered to be nonaging and linearly viscoelastic. So its stress-strain relation has the form

$$\epsilon^v(t) - \epsilon^v(\tau) = \int_0^t \Phi(t - t') \sigma_g[v(\tau), dt'] \dots \dots \dots (3)$$

in which we assume that  $\sigma_g[v(\tau), dt'] = 0$  for  $t' < \tau$ ;  $\epsilon^v(t) - \epsilon^v(\tau) =$  viscoelastic strain actually suffered by the element that solidified at time  $\tau$ , and  $\Phi(t - t') =$  microscopic creep compliance function of the solidified matter (hydrated cement), representing the creep strain at time  $t$ , caused by a unit microstress ( $\sigma_g = 1$ ) applied at time  $\tau$ . According to our assumption that, on the microscale, the solidified material does not undergo any aging, function  $\Phi(t - \tau)$  is written as a function of only one variable—the load duration  $t - \tau$  (time lag), rather than two independent variables  $t$  and  $\tau$  as required for the compliance function  $J(t, t')$  on the macroscale.  $\sigma_g(v, dt')$  denotes the change of  $\sigma_g$  when  $t'$  changes by  $dt'$ , and if  $\sigma_g$  is differentiable, then  $\sigma_g(v, dt') = [\partial \sigma_g(v, t') / \partial t'] dt'$ .

Now an important point is that, at the time at which layer  $dv(t')$  solidifies it must be free of stress, i.e.,  $\sigma_g[v(\tau), \tau] = 0$ . Indeed, it would be physically incorrect to assume that a layer at the liquid-fluid interface (Fig. 1) would solidify in a stressed state.

It must be admitted that material solidification in a stressed state is possible when solutes diffuse into a solid-solid interface. Such solidification causes pressure across the interface, known as the crystal growth pressure. Consideration of such phenomena, however, is not germane to the age-dependence of creep. Anyhow, it would require a model that is more complex than the simple parallel coupling of elements  $dv(t')$  in Fig. 1. If the material solidified in a stressed state, the stress would have to involve pressure across the layer, but that cannot occur in the model of Fig. 1 because the layer is in contact with an unstressed fluid. (Since the bulk modulus of water at room temperature is about 1/20 that of concrete as a whole, water in the pores cannot receive a significant stress from applied loads even if the pores are saturated.)

The interpretation of volume function,  $v(t)$ , requires further comment. The change of the creep curves with increasing age at loading appears to be significant up to high ages exceeding 10 years. Yet the growth of the volume fraction  $v_h$  of hydrated cement terminates much earlier, essentially at about one month of age. So volume  $v(t)$  cannot be interpreted as  $v_h$ , at least not literally. But modeling of the aging effect by means of a volume fraction growth of a nonaging substance is mathematically inevitable. So what is the precise physical meaning of  $v(t)$ ?

In response to this question, only conjecture is possible at present. As suggested by Bažant (1979), the aforementioned discrepancy between  $v_h$  and  $v(t)$  could be explained by interpreting  $v(t)$  as the effective load-bearing volume fraction of hydrated cement. This fraction can continue to grow significantly up to high ages (many years) as a result of progressive formation of bonds among the solid particles of hydrated cement. That further bonds continue to form in the hydrated solid is evidenced, e.g., by the phenomenon of polymerization of tricalcium silicate hydrates in cement gel. As new bonds continue to form, a higher and higher portion of the hydrated cement becomes capable of carrying load and thus may be counted into volume  $v(t)$ , while the solidified volume that is not yet densely bonded cannot be counted as part of  $v(t)$ .

**CONSTITUTIVE RELATION**

Eqs. 2 and 3 represent a system of two coupled-integral equations relating the variables  $\sigma(t)$ ,  $\epsilon^v(t)$  and  $\sigma_g(v, t)$ . A surprising but useful property of this system is that the microstress  $\sigma_g(v, t)$  can be eliminated if  $\sigma_g[v(\tau), \tau] = 0$  (as shown by Bažant 1977, Eqs. 2–10). The result is

$$\dot{\epsilon}^v(t) = \frac{1}{v(t)} \int_0^t \dot{\Phi}(t - t') d\sigma(t') \dots \dots \dots (4)$$

in which,  $\dot{\Phi}(t - t') = d\Phi(t - t')/dt$ ; and the superimposed dots denote derivatives with respect to time  $t$ . Note that this integral equation is written for the viscoelastic strain rate  $\dot{\epsilon}^v$  rather than the total strain  $\epsilon^v$ . This is a cardinal difference from other formulations.

Eq. 4 has been derived under the assumption of linear viscoelasticity. For high stresses, however, the dependence on stress becomes nonlinear. There are many possible ways of nonlinear generalization of the history integrals for creep, such as Eq. 4. In theory, such a generalization could involve nonlinear functions of  $\sigma(t')$  inside the integral in Eq. 4, as well as multiple integrals over  $d\sigma(t')$ . However, this would be too complicated for practice and, besides, the physical justification would be dubious.

The simplest way to introduce nonlinearity is to take the history integral for the strain rate (rather than the total strain), such as Eq. 4, and multiply it by a nondimensional function  $F[\sigma(t)]$  of current stress  $\sigma(t)$ . This approach not only has a clear physical meaning (acceleration of creep rate) but also has been shown to yield good results for concrete, as demonstrated by extensive studies and comparisons with test data in Bažant and Kim (1979). So we have

$$\dot{\epsilon}^v(t) = \frac{F[\sigma(t)]}{v(t)} \dot{\gamma}(t), \quad \dot{\gamma}(t) = \int_0^t \dot{\Phi}(t - t') d\sigma(t') \dots \dots \dots (5)$$

where we also introduced variable  $\gamma(t)$  called the viscoelastic microstrain. It represents the strain of the binder, i.e., cement gel, whose volume grows with time.

For the viscous strain  $\epsilon^f(t)$ , the effect of the growth of the volume function  $f(t)$  of the solidified matter (Fig. 1) is mathematically analogous, therefore, the result must be

$$\dot{\epsilon}^f(t) = \frac{F[\sigma(t)]}{h(t)} \int_0^t \dot{\Psi}(t-t') d\sigma(t') \quad (6)$$

in which  $\dot{\Psi}(t-t') = \partial\Psi(t-t')/\partial t$ , and  $\Psi(t-t')$  is the corresponding microscopic compliance function of the solidified matter (hydrated cement), which is nonaging (Fig. 1). Since the strain  $\epsilon^f$  is viscous, the strain rate at time  $t$  caused by microstress  $\sigma_f$  acting at time  $t'$  is  $\dot{\epsilon}^f = \sigma/\eta_0$  (for the range of linear dependence on stress), where  $\eta_0 = \text{constant} = \text{effective viscosity of the solidified matter (hydrated cement)}$ . So we have  $\dot{\Psi}(t-t') = 1/\eta_0$ ,  $\Psi(t-t') = (t-t')/\eta_0$ . Then Eq. 6 may be integrated to yield:

$$\dot{\epsilon}^f(t) = \frac{F[\sigma(t)]\sigma(t)}{\eta(t)} \quad (7)$$

where  $\eta(t) = \eta_0 h(t) = \text{apparent (effective) macroscopic viscosity}$ , which is not constant but increases with age  $t$ .

The nonlinear dependence on stress has been assumed in Eqs. 6 and 7 to be described by the same function,  $F[\sigma(t)]$ . In principle, the functions of stress in Eqs. 5 and 7 could be, and probably are, different. For moderate stress levels, however, no benefit for data fits was found to accrue by considering different functions of stress.

It may be noted that no significant complication in the analysis that follows would arise if the actual times  $t$  and  $t'$  in Eqs. 5 and 6 were replaced with some reduced times  $\theta(t)$  and  $\theta(t')$ , such that  $\theta(t) = t'$ ,  $0 < r < 1$ . This has been tried. Analysis of test data, however, revealed that the optimum fits achievable for  $r < 1$  are not appreciably better than those for  $r = 1$ . Therefore, the use of reduced time appears to be unnecessary.

For the case of constant stress  $\sigma$  applied at the age  $t'$ , Eqs. 1, 5 and 7 yield

$$\dot{\epsilon}(t, t') = F(\sigma)\sigma\dot{C}(t, t') + \dot{\epsilon}^0 \quad (8)$$

where

$$\dot{C}(t, t') = \frac{\dot{\Phi}(t-t')}{v(t)} + \frac{1}{\eta(t)} \quad (9)$$

$\dot{C}(t, t')$  is the creep compliance rate. The compliance function, representing the strain at age  $t$  caused by a unit constant stress applied at age  $t'$ , is then obtained as

$$J(t, t') = \frac{1}{E_0} + \int_0^t \dot{C}(\tau, t') d\tau = \frac{1}{E_0} \int_0^t \left( \frac{\dot{\Phi}(\tau-t')}{v(\tau)} + \frac{1}{\eta(\tau)} \right) d\tau \quad (10)$$

## MATERIAL FUNCTIONS AND PARAMETERS

In view of the fact that the creep curves at constant stress are known to have the shape of power curves  $(t-t')^n$  for short  $(t-t')$  and the shape of logarithmic curves in  $(t-t')$  for long  $(t-t')$ , we introduce

$$\Phi(t-t') = q_2 \ln(1 + \xi^n), \quad \xi = \frac{(t-t')}{\lambda_0} \quad (11)$$

$$\eta(t)^{-1} = q_4 t^{-1} \quad (12)$$

in which  $q_2, q_4, n, \lambda_0 = \text{empirical material constants}$ . Eq. 12 implies that  $\Psi(t-t') = q_4 \ln(t/t')$ . Furthermore, in view of the aging law known from the double power law, we introduce

$$\frac{1}{v(t)} = \left( \frac{\lambda_0}{t} \right)^m + \alpha \quad (13)$$

where  $m, \alpha = \text{empirical constants}$ . The nonlinear dependence of the creep on stress may be approximately introduced as

$$F[\sigma(t)] = \frac{1+s^2}{1-\Omega}, \quad s = \frac{\sigma(t)}{f'_c} \quad (14)$$

where  $\Omega$  represents damage at high stress and is taken as  $\Omega = s^{10}$ . (Then  $\Omega = 0$  for  $s < 0.7$ .) A more realistic but more complicated definition of  $\Omega$  could be made through some damage-evolution law, such as  $\dot{\Omega} = \text{function of } \Omega \text{ and } \sigma$ .

According to Eqs. 8-15, the compliance rate is

$$\dot{C}(t, t') = \frac{\partial J(t, t')}{\partial t} = \left[ q_2 \left( \frac{\lambda_0}{t} \right)^m + q_3 \right] \frac{n\xi^{n-1}}{\lambda_0(1+\xi^n)} + \frac{q_4}{t} \quad (15)$$

where  $q_3 = \alpha q_2$ . Furthermore, integrating with the initial condition  $\dot{\epsilon}(t') = \sigma/E_0$ , we obtain for constant stress applied at age  $t'$ :

$$J(t, t', \sigma) = q_1 + q_2 F(\sigma) Q(t, t') + q_3 F(\sigma) \ln \left[ 1 + \left( \frac{t-t'}{\lambda_0} \right) \right] + q_4 F(\sigma) \ln \left( \frac{t}{t'} \right) \quad (16)$$

in which  $J(t, t', \sigma) = \epsilon(t)/\sigma = \text{compliance function}$ ;  $q_1 = 1/E_0$ ; and

$$Q(t, t') = \int_{t'}^t \left( \frac{\lambda_0}{\tau} \right)^m \frac{n(\tau-t')^{n-1}}{\lambda_0^n + (\tau-t')^n} d\tau \quad (17)$$

The functions multiplying  $q_2, q_3$ , and  $q_4$  represent the nondimensionalized forms of the aging viscoelastic compliance, the nonaging viscoelastic compliance, and the flow compliance, respectively. The dimensions of constants  $q_1, q_2, q_3$ , and  $q_4$  are  $(\text{stress})^{-1}$ . The dimension of  $\lambda_0$  is time. Constants  $n$  and  $m$  are nondimensional. Experience with data fitting indicates that three material constants may be fixed for all concretes, once for all

$$n = 0.1, \quad m = 0.5, \quad \lambda_0 = 1 \text{ day} \quad (18)$$

Thus, there remain only four material constants  $q_1, q_2, q_3$ , and  $q_4$  that need to be identified from test data. They characterize all basic creep, aging and elastic behavior.

From Eq. 16 we see that the compliance depends on  $q_1, \dots, q_4$  linearly. This means that the parameters can be determined from the given compliance data by linear regression, which is a great advantage compared to the previously used creep laws for concrete.

TABLE 1. Exact Values of Function  $Q(t, t')$ , and Deviations of Approximate Formula from Exact Value

$\log(t-t')$ (1)	0 (2)	0.5 (3)	1 (4)	1.5 (5)	2 (6)	2.5 (7)	3 (8)	3.5 (9)	4 (10)
-2	0.4890 -4	0.2750 -1	0.1547 -2	0.08677 -1	0.04892 0	0.02751 0	0.01547 0	0.008699 0	0.004892 0
-1.5	0.5347 9	0.3009 -3	0.1693 -1	0.09519 -1	0.05353 0	0.03010 0	0.01693 0	0.009519 0	0.005353 0
-1	0.5286 -18	0.3284 -6	0.1848 -2	0.1040 0	0.05846 -1	0.03288 0	0.01849 0	0.01040 0	0.005846 0
-0.5	0.6309 -30	0.3571 -11	0.2013 -3	0.1133 -1	0.06372 -1	0.03583 0	0.02015 0	0.01133 0	0.006372 0
0	0.6754 -33	0.3860 -18	0.2185 -7	0.1231 -2	0.06929 -5	0.03897 -1	0.02192 0	0.01233 0	0.006931 0
0.5	0.7108 -16	0.4125 -20	0.2357 -11	0.1334 -4	0.07516 -11	0.04229 -3	0.02379 0	0.01338 0	0.007524 0
1	0.7352 10	0.4335 -10	0.2514 -12	0.1436 -6	0.08123 -21	0.04578 -6	0.02576 0	0.01449 0	0.008149 0
1.5	0.7505 27	0.4480 5	0.2638 -6	0.1529 -7	0.08727 -35	0.04937 -12	0.02782 -3	0.01566 -1	0.008806 -1
2	0.7597 29	0.4570 15	0.2724 3	0.1602 -3	0.09276 -38	0.05293 -19	0.02994 -6	0.01687 -2	0.009494 -3
2.5	0.7652 24	0.4624 17	0.2777 8	0.1652 2	0.09708 -18	0.05616 -20	0.03284 -10	0.01812 -3	0.01021 -1
3	0.7684 17	0.4656 14	0.2808 9	0.1683 5	0.1000 1	0.05869 -8	0.03393 -10	0.01935 -2	0.01094 -2
3.5	0.7703 10	0.4675 10	0.2827 8	0.1702 6	0.1018 3	0.06041 9	0.03541 -3	0.02045 -5	0.01166 -2
4	0.7714 6	0.4686 6	0.2838 6	0.1713 5	0.1029 3	0.06147 9	0.03641 7	0.02131 0	0.01230 -2
4.5	0.7720 3	0.4692 4	0.2844 4	0.1719 3	0.1036 3	0.06210 20	0.03702 -12	0.02190 5	0.01280 1
5	0.7724 1	0.4696 2	0.2848 2	0.1723 2	0.1038 2	0.06247 16	0.03739 -12	0.02225 8	0.01314 4
$\infty$	0.7729 -2	0.4701 -1	0.2853 0	0.1728 0	0.1044 0	0.06297 -2	0.03789 4	0.02275 0	0.01363 -2

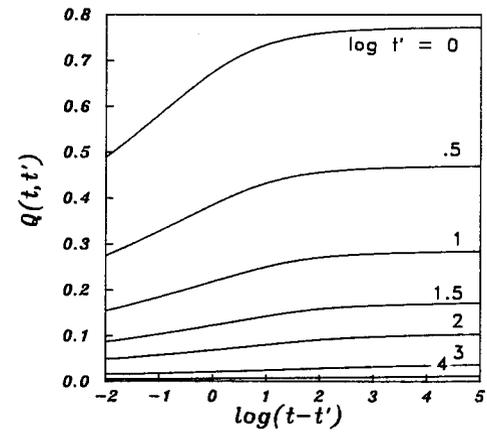


FIG. 2. Function  $Q(t, t')$  that Characterizes Aging Viscoelastic Strain ( $t$  = Current Time,  $t'$  = Age at Loading)

The integral  $Q(t, t')$  from Eq. 17 cannot be expressed in a closed form (this is clear upon noticing that, if the denominator expression is replaced by 1, Eq. 17 becomes a binomial integral that cannot be expressed in a closed form). An asymptotic approximation, valid for  $t - t' \ll t'$ , is obtained by replacing  $\lambda_0/\tau$  in Eq. 17 with  $\lambda_0/t'$  and integrating

$$Q(t, t') \approx Z(t, t') = \left(\frac{\lambda_0}{t'}\right)^m \ln \left[ 1 + \left(\frac{t-t'}{\lambda_0}\right)^m \right] \quad \text{if } t - t' \ll t' \dots \dots \dots (19)$$

Always,  $Q(t, t') < Z(t, t')$ . Also,  $\partial Q(t, t')/\partial t > 0$ ,  $Q(t, t') > 0$ . Furthermore, from the fact the integrand of Eq. 17 is always less than  $n\lambda_0^m \tau^{-m-1}$ , it follows that  $Q(\infty, t') < \infty$ . Taking a small initial value such as  $t_1 - t' = 10^{-9} t'$  from Eq. 19 and transforming the integral in Eq. 19 by substitution of new variable  $\xi = \log(\tau - t')$ , one may use the trapezoidal rule to obtain the numerical values of  $Q(t, t')$ ; see Table 1. A plot of  $Q(t, t')$  is shown in Fig. 2. Furthermore, a good approximate formula has also been found:

$$Q(t, t') \approx Q_f \left[ 1 + \left(\frac{Q_f}{Z(t, t')}\right)^r \right]^{-1/r} \dots \dots \dots (20)$$

with

$$\log Q_f = -[0.1120 + 0.4308 \log t' + 0.0019 (\log t')^2] \dots \dots \dots (21)$$

$$r = 1.7t'^{0.12} + 8 \dots \dots \dots (22)$$

The errors of this formula are indicated by the small numbers in Table 1. Within the range  $1 \leq t' \leq 10,000$  days and  $0.01 \leq t - t' \leq 10,000$  days, the errors of  $Q$  are generally less than  $\pm 0.5\%$  of  $Q$ , with coefficient of variation 0.2%. The final values  $Q_f$  (for  $t \rightarrow \infty$ ) have errors less than  $\pm 0.09\%$ , with coefficient of variation 0.01%. Such accuracy is normally sufficient for practice.

**PROPERTIES OF NEW CONSTITUTIVE MODEL**

Based on extensive experimental evidence (Hanson 1953; Hanson and Harboe 1958; Ross 1958; L'Hermite et al. 1965, 1968; Rostasy et al. 1971; Browne and Bamforth 1975; Kimishima and Kitahara 1965; Mullick 1972; Polivka et al. 1964; Kommendant et al. 1976; Mamillan 1959), the constitutive model should have certain simple asymptotic properties. For small stresses and  $t - t' \ll t'$ , where  $t - t' =$  load duration and  $t' =$  age at loading, the compliance function should asymptotically approach the double power law (Bažant 1975; Bažant and Osman 1976; Bažant and Panula 1978, 1980):

$$J(t, t') = \frac{1}{E_0} + \frac{\phi}{E_0} (t'^{-m} + \alpha)(t - t')^n \dots (23)$$

where  $t_0, \phi_1, n, m, \alpha =$  positive parameters. Indeed, we have  $1 + \xi^n \approx 1, t \approx t', \ln(t/t') \approx 0$ , and  $q_4/t$  is negligible compared to the terms with  $q_2$  and  $q_3$ . Consequently, Eq. 15, for  $t - t' \ll t'$  and  $F(\sigma) \approx \sigma$ , becomes  $J(t, t') = n\lambda_0^{-n}(q_2\lambda_0^m t'^{-m} + q_3)(t - t')^{n-1}$ . Integrating and setting  $q_2 = \phi_1/E_0, q_3/q_2 = \alpha, \lambda_0 = 1$ , we recover Eq. 23. Thus, the optimum values of  $n, m$ , and  $\lambda_0$  should be about the same as found before for the double power law, and Eq. 18 confirms that this in fact is the case.

Furthermore, from the third term of Eq. 16 we may notice that the log-double power law (Bažant and Chern 1985a) is also a special case of the present formulation.

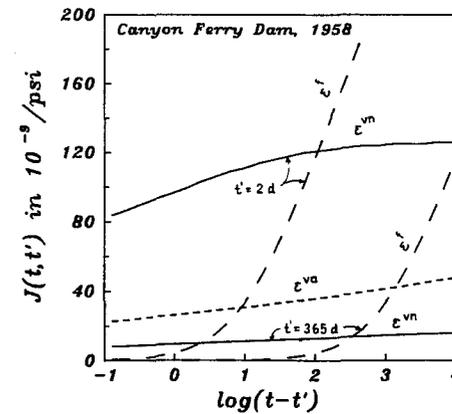
For small stress and very long creep durations, i.e.,  $t - t' \gg t'$ , the compliance function should asymptotically approach the logarithmic law (Hanson 1953; Hanson and Harboe 1958):

$$J(t, t') = A_1 \ln(t - t') + A_2(t') \dots (24)$$

where  $A_1 =$  positive constant and  $A_2(t') =$  function of  $t'$ . For  $t - t' \gg t'$ , we have  $\lambda_0^{-1} n \xi^{n-1} / (1 + \xi^n) \approx n / (t - t') \approx n/t$  and  $q_2(\lambda_0/t)^m + q_3 \approx q_3$ . So Eq. 15, for  $F(\sigma) = 1$ , becomes  $\partial J(t, t') / \partial t \approx b / (t - t')$  with  $b = nq_3 + q_4$ . Thus, integration indeed yields Eq. 24. This indicates that, in the plot of  $J(t, t')$  versus either  $\log t$  or  $\log(t - t')$ , the creep curves for  $\sigma =$  constant and various  $t'$  should at long times approach straight lines of the same slope,  $b$ . The plots of creep-test results agree with this property quite well (see Figs. 1-5, Part II).

It is interesting to note that the present formulation represents a compromise between the double power law and the improved Dischinger model used by CEB-FIP (1978). The basic aspect of the Dischinger model is a flow term that corresponds to the aging viscous term  $\epsilon^f$  (Fig. 1), and has the general form  $\phi(t) - \phi(t')$ , and a delayed elastic term, which has the general form  $f(t - t')$ . Obviously, the flow term coincides with the last term of Eq. 16, if one sets  $\phi(t) = \ln t$ . The delayed elastic term coincides with the third term of Eq. 16, if one sets  $f(t - t') = \ln [1 + (t - t')^n / \lambda_0^n]$ . In contrast to the CEB-FIP model, however, this term has no final asymptotic value, and so it does not really represent delayed elasticity. The second term has no counterpart in the CEB-FIP model, but its use has been found to be essential for a good representation of test results.

Another desirable property is nondivergence of the creep curves at constant  $\sigma$  for various ages at loading ("Mathematical" 1986; "Conclusions"



**FIG. 3. Comparison of Typical Curves of Viscous Strain  $\epsilon^f$  (Flow) with the Aging and Nonaging Viscoelastic Strains ( $\epsilon^{va}, \epsilon^{vn}$ ): The Material Parameter Values are the Same as In Fig. 2 of Part II**

1987; Bažant and Kim 1979; Bažant 1982). If this condition is violated, which has been true of all the previous formulations that described the early creep well, then the use of the principle of superposition yields nonmonotonic creep recovery curves, in contradiction to most recovery data. The condition of nondivergence requires that  $\dot{\epsilon}$  for a fixed  $t$  must not decrease as the age at loading increases, i.e.

$$\frac{\partial J(t, t')}{\partial t'} \geq 0 \dots (21)$$

Substitution of Eq. 15 and differentiation shows that this condition is always verified if  $n < 1$ . On the other hand, for the double power law, as well as the log-double power law and other existing formulas that closely describe the creep properties at small  $t'$  and small  $t - t'$ , the condition of nondivergence is violated for a certain time period. The essential property that avoids divergence of the creep curves is the fact that the expression for the creep rate (Eqs. 9 and 15) involves only the current age  $t$  and the load duration  $t - t'$ , but not the age at loading  $t'$ . This property has been obtained as a consequence of micromechanics analysis of the solidification process (Eq. 4).

The nonlinearity of the creep law with respect to stress is introduced by function  $F[\sigma(t)]$  in Eq. 7. The fact that the nonlinearity of creep rate is characterized by the current stress  $\sigma(t)$ , and is independent of the previous stress values, represents a simplification compared to some other nonlinear creep laws in which the history integrals involve nonlinear functions of  $\sigma(t')$  behind the integral sign. Due to the way this function is introduced, it not only describes the increase of the creep rate at high stress but also the reduction of creep recovery compared to the recovery prediction from the principle of superposition (see Part II).

The nonlinear response for short-time loading is obtained due to nonlinearity of the rapid short-time creep (no loading is truly instantaneous). How-

ever, for large short-time strains, the model would have to be generalized by adding strains due to microcracking.

The viscous (flow) strain,  $\epsilon^f$ , is important for long-time creep of concrete loading at a young age. This is illustrated in Fig. 13 of Part II, where fitting of the test data for Canyon Ferry dam was attempted by the present model without the flow term. On the other hand, when  $(t - t')$  is not too large, this term is small compared to the viscoelastic strain,  $\epsilon^v$ ; see Fig. 3, which shows a comparison of the typical magnitudes of  $\epsilon^f$  and the aging and non-aging parts of  $\epsilon^v$ .

Note that the viscous strain,  $\epsilon^f$ , makes no contribution to creep recovery, i.e., the creep recovery is entirely due to viscoelastic strain  $\epsilon^v$ . However, a significant part of the viscoelastic strain is irrecoverable, too.

## CONCLUSION

By separating the phenomenon of aging (solidification) from creep, the present model bears considerable promise. However, it remains to be verified that it is capable of a good representation of the existing test data and is more efficient in structural analysis. To do this will be the objective of the second part of this study, which follows in the next paper.

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