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Statistical aspects of quasibrittle size effect and lifetime, with consequences for safety and durability of large structures

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ABSTRACT: This paper presents an overview of the statistical aspects of the size effect law on the strength of quasibrittle structures. Two types of size effect law, corresponding to two different failure mechanisms, can be distinguished. The Type 1 size effect law (SEL) applies to the situations in which the maximum load of unnotched structure is attained after the stable formation of a large fracture process zone (FPZ) with distributed cracking. The Type 1 SEL can be explained by size effect on the type of probability distribution of nominal strength based on the weakest-link model with a finite number of links. These links represent the representative volume elements of material whose random strength is derived from atomistic fracture mechanics. The theory is further extended to model the size effect on structural lifetime, which is important for durability of infrastructure. The Type 2 size effect law, which applies to structures that have a deep notch or contain at peak load a large traction-free (i.e., fatigued) crack, has a deterministic mean and the material strength statistics only affects the variance of load, which means that the safe margin may be considered to be uniform for the size range of interest. An example based on experimental data shows that if the Type 2 size effect is ignored, the failure probability may increase from 10^{-6} for small sizes to 10^{-3} for large sizes.

1 INTRODUCTION

The understanding of strength distributions, which is essential for a rational determination of safety factors guarding against the uncertainties of structural strength, is of paramount importance for safe and economic design of engineering structures. For perfectly ductile or perfectly brittle materials, the proper cumulative distribution functions (cdf's) of the nominal strength of structure are known to be either Gaussian or Weibullian, respectively. The type of cdf does not change with structure size and geometry, although the coefficient of variation decreases with size for the former and the mean decreases for the latter.

This study focuses on positive geometry structures consisting of quasibrittle materials, which include, at normal scale, concrete, fiber-polymer composites, tough ceramics, rocks, sea ice, wood, bone, etc., and many more at the scale of MEMS and thin films. Quasibrittle materials are materials that 1) are incapable of purely plastic deformations, and 2) in normal use, have a FPZ which is not negligible compared to the structure size. A salient property of quasibrittle materials is that they obey on a small scale the theory of plasticity characterized by material strength, and on a large scale the linear elastic fracture mechanics (LEFM) characterized by fracture energy. Over the last three decades, extensive studies have shown that the quasibrittle structures exhibit a strong size effect on its nominal strength (Bažant 1976, 1984, 2004, 2005). Two types of simple size effect laws have been distinguished: Type 1 SEL, occurring in structures that fail at crack initiation from a smooth surface, and Type 2 SEL, which occurs in structures with a deep notch or stress-free (e.g., fatigued) crack formed stably before failure. The SEL Type 2 is also called the size-shape effect law, since its fracture mechanics based extension (Bažant & Kazemi 1990) captures the effect of structure geometry through the LEFM energy release function.

The Type 1 SEL applies to quasibrittle structures failing at crack initiation from a smooth surface. Because of material heterogeneity, a finite cracking zone representing the FPZ must develop before the cracking can coalesce into an initial macro-crack of finite depth attached to the surface. Formation of the initial FPZ causes stress redistribution and energy release necessary to drive the macro-crack. Except for the large size limit, the Type 1 SEL can be derived by considering the limiting case of energy release where the energy release approaches zero with a vanishing crack length. For the large size limit, the Type 1 size effect must converge to the classical Weibull theory.

Bažant & Pang (2006, 2007) and Bažant et al. (2009) presented a probabilistic theory for the size effect on strength distribution, by which the Type 1 size effect can be explained alternatively and more fundamentally. For failures at crack initiation, the structure can be statistically modelled as a chain of representative volume elements (RVEs). It is important that the chain is finite, which rules out Weibull distribution The strength distribution of one RVE was derived by relating the free energy loss at dach single-atom crack jump to the energy release from an atomic lattice block, and introducing a multi-scale transition based on a hierarchy of series and parallel coupling, the former accounting for compatibility conditions and the latter for strain localizations. It is found that the strength distribution of one RVE must be Gaussian with a remote power-law (or Weibull) tail grafted to the probability of about 0.001.

The strength distribution of quasibrittle structures modeled as a finite chain depends on the structure size and geometry, and varies gradually from a Gaussian cdf with a remote Weibull tail to a fully Weibull cdf at large sizes. The same theoretical framework also provides a plausible physical explanation for the crack growth rate law. The theory can further be extended to the distribution of lifetime under sustained load (Bažant et al. 2009, Bažant & Le 2009a, Le et al. 2009).

The Type 2 SEL applies to the case where the structure has a deep notch or stress-free (e.g., fatigued) crack formed before the peak load is reached. Due to the stress concentration, there is no chance for the dominant crack to initiate elsewhere in the structure volume, which means that material randomness cannot cause any size effect in the mean. Thus, the size effect on the mean nominal strength of structure is essentially energetic, while material randomness can affect only the standard deviation of structure strength (Bažant & Xi 1991). The Type 2 size effect law can be derived by using asymptotic approximation of the energy release function for the propagating crack based on the equivalent linear elastic fracture mechanics (LEFM), or the J-integral (Bažant 2005). Investigation of a large database collected from various laboratories shows that if Type 2 size effect is ignored, the safety margin for large structures is substantially compromised.

2 TYPE-I SIZE EFFECT DERIVED FROM ATOMISTIC FRACTURE MECHANICS

2.1 *Strength distribution of one RVE*

The fracture at macro-RVE scale originates from the breakage of interatomic bonds at the nano-scale (Henderson 1970, Zhurkov 1965, Zhurkov & Korsukov 1974). Consequently, the statistics of structural failure of an RVE must be related to the statis tics of interatomic bond breakage. Consider a nano-structure, either an atomic lattice block or a disordered nano-structure, with some intrinsic defects

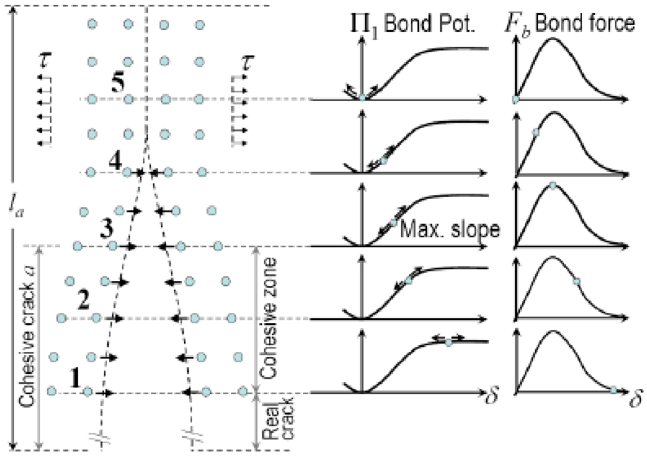

Figure 1. Propagation of nanocrack.

such as nano-cracks. The stress applied at macro-scale causes nano-stress concentrations under which the nano-crack begins to propagate (Fig. 1). When it advances by one atomic spacing in the atomic lattice (or, in a disordered nano-structure, by one nano-bond spacing), the energy release increment must equal the change of activation energy barrier. With the equivalent LEFM, the energy release increment can be expressed as a function of the remote stress applied on the nano-structure (Bažant et al. 2009).

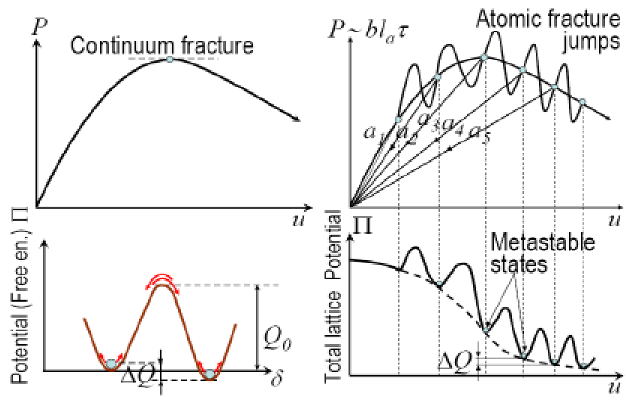

Figure 2. Mechanism of nanocrack jumps.

Since the crack jumps by one atomic spacing or one nano-inhomogeneity are numerous and thus very small, the activation energy barrier for a forward jump differs very little from the activation energy barrier for a backward jump. Therefore, the jumps of the state of the nano-structure, characterized by its free energy potential, must be happening in both directions, albeit with different frequencies (Fig. 2). After a certain number of jumps of the nano-crack tip, the length of the nano-crack reaches a critical value at which the crack loses its stability and propagates dynamically, causing a break of the nano-structure. Since, at nano-scale, it may generally be assumed that each jump is statistically independent, the failure probability of the nano-structure is proportional to the sum of the frequencies of all the jumps that cause its failure. The failure probability of the nano-structure has been shown to follow a power-law function of the remote stress with a zero threshold ((Bažant et al. 2009), i.e.:

$$P_f \propto \tau^2 = (c\sigma)^2 \qquad (1)$$

where τ = micro-stress, σ = macro-stress and c = nano-macro stress concentration factor.

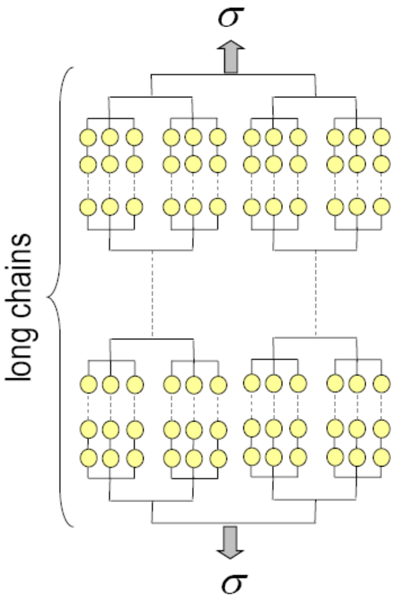

Figure 3. Hierarchical model for statistical multiscale transition.

To relate the strength distributions of a nano-structure and a macro-scale RVE, certain statistical multiscale transition framework is needed. Though various stochastic multi-scale numerical approaches have been proposed to capture the statistics of structural response (Graham-Brady et al. 2006, William & Baxer 2006, Xu 2007), the capability of these approaches is always limited due to the incomplete knowledge of the uncertainties in the information across all the scales. In this study, the multi-scale bridging between the strength cdf at the nano-scale and at the RVE scale is statistically represented by a hierarchical model consisting of parallel and series couplings (Fig. 3). The parallel couplings statistically reflect the load redistribution mechanisms at various scales subject to compatibility conditions. The series couplings, represented by the weakest-link chain model, reflect the localization of sub-scale cracking and slippage (or damage) into larger scale cracks or slips.

For a chain of n elements where all of the elements have a strength cdf with a power-law tail of exponent p, the strength cdf of the entire chain has also a power-law tail and its exponent is also p. If the tail exponents for different elements in the chain are different, then the smallest one is the tail exponent of the cdf of strength of the entire chain.

For parallel coupling of elements of random strength (fiber bundle), the strength distribution of the bundle depends on the mechanical behavior of each element. However, two asymptotic properties of the strength distribution of the bundle are independent of the behavior of each element: 1) If the cdf of strength of each element has a power-law tail of exponent p, then the cdf of strength of a bundle of n elements also has a power-law tail, and its exponent is np (Bažant & Pang 2006, 2007, Bažant & Le 2009b), while the reach of the power-law tail is drastically shortened as the number of elements n increases, 2) The strength cdf of bundle converges to Gaussian distribution for an increasing number of elements (Daniels 1945, Bažant & Pang 2006, 2007, Bažant and Le 2009b, Harlow et al. 1983, Phoenix et al. 1997). The reach of the power-law tail and the rate of convergence to Gaussian distribution depend on the deformation behavior of the element (Bažant & Pang 2006, 2007).

Numerical simulation shows that the strength distribution of one RVE, which is statistically modelled by the hierarchical model (Fig. 3), can be approximately described as Gaussian, with a Weibull tail grafted on the left at the probability of about 10^{-4} – 10^{-3} (Bažant & Le 2009b). Mathematically, one may approximate the strength distribution of one RVE as (Bažant & Pang 2006, 2007):

$$P_1 = 1 - \exp\left[-(\sigma_N / s_0)^m\right] \;\; (\sigma \leq \sigma_{gr}) \qquad (2)$$

$$P_1 = P_{gr} + \frac{r_f}{\delta_G \sqrt{2\pi}} \int_{\sigma_{gr}}^{\sigma_N} e^{-(\sigma' - \mu_G)^2 / 2\delta_G^2} \mathrm{d}\sigma' \;\; (\sigma > \sigma_{gr}) \qquad (3)$$

where σ_N = nominal strength, which is a maximum load parameter of the dimension of stress. In general, σ_N = P/bD or P/D^2 for two- or three-dimensional scaling (P = maximum load of the structure or parameter of load system, b = structure thickness in the third dimension, D = characteristic structure dimension or size). Furthermore, m (Weibull modulus) and s_0 are the shape and scale parameters of the Weibull tail, and μ_G and δ_G are the mean and standard deviation of the Gaussian core if considered extended to -∞; r_f is a scaling parameter required to normalize the grafted cdf such that $P_1(\infty)$ = 1, and P_{gr} = grafting probability = 1−exp[-(σ_{gr}/ s_0)^{m}]. Finally, continuity of the probability density function at the grafting point requires that $(\mathrm{d}P_1/\mathrm{d}\sigma_N)|_{\sigma gr+} = (\mathrm{d}P_1/\mathrm{d}\sigma_N)|_{\sigma gr-}$.

2.2 *Size Effect on Mean Structural Strength*

In the context of softening damage and failure of a structure, the RVE cannot be defined by homogenization theory. Rather, it must be defined as the smallest material volume whose failure triggers the failure of a structure. The structure can thus be statistically represented by a chain of RVEs. By virtue of the joint probability theorem, and under the assumption of independence of random strengths of links in a finite weakest-link model, the strength distribution of a structure can be calculated as:

$$P_f(\sigma_N) = 1 - \prod_{i=1}^{n}[1 - P_1(\langle\sigma(x_i)\rangle)] \tag{4}$$

where σ_N = nominal strength of the structure, $\sigma(x_i) = \sigma_N\, s(x_i)$ = maximum principal stress at the center of i^{th} RVE with the coordinate x_i, $s(x_i)$ = dimensionless stress describing the stress distribution in the structure, n = number of RVEs in the structure, and $P_1(\sigma)$ = strength cdf of one RVE. Equation 4 directly indicates the size effect on the type of strength cdf. For small-size structures (small n), the strength cdf is predominantly Gaussian, which corresponds to the case of quasi-plastic behavior. For large size structures, what matters for P_f is only the tail of the strength cdf of one RVE, and it causes that the entire cdf of strength of very large structures follows the Weibull distribution.

Based on the finite weakest link model (Equation 4) and the grafted cdf of strength for one RVE (Equation 2 and 3), the strength cdf of a structure must depend on its size and geometry. The mean strength for a structure with any number of RVEs can be calculated as:

$$\bar{\sigma}_N = \int_0^\infty [1 - P_f(\sigma_N)]\mathrm{d}\sigma_N \tag{5}$$

Clearly, it is impossible to express $\bar{\sigma}_N$ analytically. But its approximate form can be obtained through asymptotic matching. It has been proposed that the size effect on mean strength can be approximated by (Bažant 2004, 2005):

$$\bar{\sigma}_N = \left[\frac{N_a}{D} + \left(\frac{N_b}{D}\right)^{r/m}\right]^{1/r} \tag{6}$$

where parameters N_a, N_b, r and m are to be determined by asymptotic properties of the size effect curve. It has been shown that such a size effect curve agrees well with the predictions by other mechanics models such as the nonlocal Weibull theory (Bažant & Novák 2000) and with the experimental observations on concrete (Bažant et al. 2007). As the large size asymptote, Equation 6 converges to $(N_b/D)^{1/m}$. Calculation of the mean strength from the Weibull distribution shows that m must be equal to the Weibull modulus of strength distribution, which can be determined by the slope of the left tail of strength histogram plotted on the Weibull scale. The other three parameters, N_a, N_b,, and r, can be determined by solving three simultaneous equations based on three asymptotic conditions,

$$[\bar{\sigma}_N]_{D\to l_0},\ [\mathrm{d}\bar{\sigma}_N/\mathrm{d}D]_{D\to l_0},\ [\bar{\sigma}_N D^{1/m}]_{D\to\infty}.$$

3 SIZE EFFECT ON STRUCTURAL LIFETIME

The probability of not achieving the design lifetime of a structure must be tolerable, i.e., sufficiently small. Although the theory just outlined has not yet been extended to fatigue under cyclic loads, the lifetime problem has already been solved for a constant load or stress. The creep crack growth law is needed as a link between the strength and lifetime statistics (Bažant et al. 2009, Bažant & Le 2009a). For decades, extensive experimental evidences showed that the crack growth rate law has a power-law form (Evans 1972, Evans & Fu 1984, Thouless et al. 1983, Munz & Fett 1999):

$$\dot{a} = Ae^{-Q_0/kT}K^n \tag{7}$$

where K = stress intensity factor, Q_0 = activation energy barrier at absence of stress, k = Boltzmann constant, T = absolute temperature, A, n= empirical constants. A recent study showed that the power-law form of the crack growth rate can be physically justified by considering the fracture mechanics of random crack front jumps through the atomic lattice and the condition of equality of the energy dissipation rates calculated on the nano-scale and the macro-scale (Bažant et al. 2009, 2009b, Le et al. 2009).

Now consider both the strength and lifetime tests for an RVE. (1) In the strength test, the load is rapidly increased till the RVE fails. The maximum load registered corresponds to the strength of the RVE, which may be chosen to be equal to σ_N. (2) In the lifetime test, the load is rapidly increased to a certain level σ_0 and then is kept constant till the RVE fails. The load duration up to failure represents the lifetime λ of the RVE at stress σ_0. By applying Equation 7 to both of these tests, one finds that the structural strength and the lifetime are related through the following simple equation:

$$\sigma_N = \beta\sigma_0^{n/(n+1)}\lambda^{1/(n+1)} \tag{8}$$

where κ = loading rate for the strength test, and $\beta = [\kappa(n+1)]^{1/(n+1)}$ = constant. Christensen (2008) used the same approach to study the damage accumulation rules and showed that Equation 8 represents a nonlinear damage accumulation rule, which is more physical compared to the widely adopted linear damage accumulation rule such as the Palmgren-Miner rule (Palmgren 1924, Miner 1945). By substituting Equation 8 into Equations. 2 and 3, one obtains the lifetime distribution of one RVE. Similar to strength statistics, one can calculate the lifetime distribution of a structure of any size (Equation 4) and the mean structural lifetime. A simple asymptotic matching formula for the size effect on the mean structural lifetime is:

$$\bar{\lambda} = \left[\frac{C_a}{D} + \left(\frac{C_b}{D} \right)^{r/m} \right]^{(n+1)/r} \tag{9}$$

where m = Weibull modulus of strength distribution, n = exponent of the power law crack growth rate, and $m/(n+1)$ = Weibull modulus of lifetime distribution. Parameters C_a; C_b; r can be determined from three known asymptotic conditions for $[\bar{\lambda}]_{D\to l_0}$, $[\mathrm{d}\bar{\lambda}/\mathrm{d}D]_{D\to l_0}$, $[\bar{\lambda}D^{(n+1)/m}]_{D\to\infty}$.

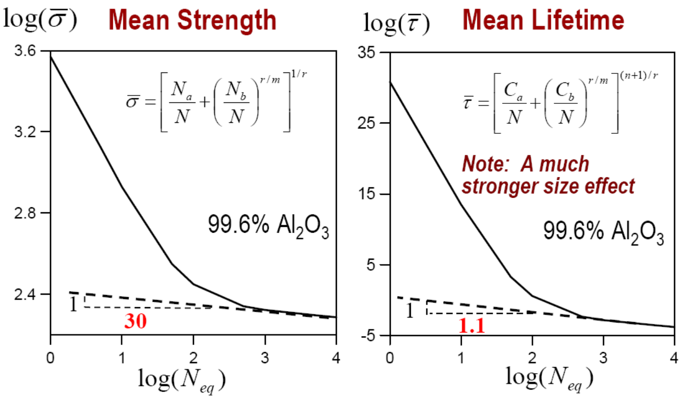

Figure 4. Size effects on structural strength and lifetime.

Figure 4 shows the calculated size effect on the mean structural strength and lifetime of 99.9% Al_2O_3, based on the strength and lifetime cdf's calibrated by Fett and Munz's histogram testing (Fett & Munz 1991). It can be seen that the size effect on mean structural lifetime is much stronger than that on mean structure strength.

This is physically plausible. Consider two geometrically similar beams, with size ratio, say, 1:8. Let the nominal strength of the small beam be μ. Due to the size effect on the mean strength, the nominal strength of the large beam is about $\mu/2$. If a nominal load $\mu/2$ were applied on both beams, the large beam will fail within the standard laboratory testing period (i.e. about 5 minutes) while the small beam is expected to survive at that load for years if not forever. An important consequence is that, for a given tolerable probability, a slightly larger structure would have a much shorter lifespan.

4 CONSEQUENCES OF IGNORING TYPE-2 SIZE EFFECT

In many common failure types, such as shear, torsion and compression crushing, reinforced concrete structures exhibit a strong deterministic size effect of Type 2. Due to its energetic nature, the randomness of material properties affects only the standard deviation of strength, but not its mean. Nevertheless, statistical analysis of a large database presents other statistical problems. Using a large database, one should note that 1) the major source of scatter in the database is the differences among different concretes tested in different laboratories and among the subjective selections of different experimenters according to their research interests; 2) the entire database cannot be treated as one statistical population; rather, because of the size effect, it should be separated into size intervals and the statistics should be treated as a statistical regression based on the Type 2 size effect.

Figure 5 shows a database of 398 data points collected from various research groups to investigate the conservativeness of the current ACI shear design formula for concrete beams, which assumes a size-independent shear strength $v_c = 2\sqrt{f_c'}$ (f'_c is the specific compressive strength of concrete, which typically equals about 70% of the average compressive strength f'_{cr}). Although obscured by enormous scatter, it can still be noticed that the data cloud of shear strength values in the database displays a downward trend with respect to the beam depth d, which rules out applying population statistics to the database as a whole.

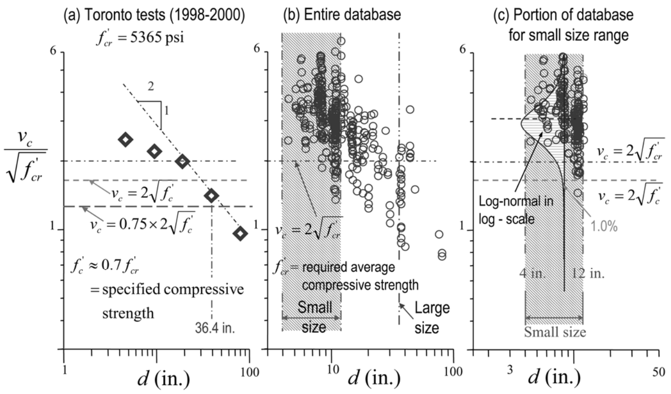

Figure 5. a) Toronto size effect tests; b) shear database of 398 data points; c) isolated data points in the small size range.

However, if data points in small size range (100 to 300 mm) are isolated from the database, the size effect is week enough for treating the data as a population with no statistical trend. The mean and coefficient of variation (C.o.V) are found to be $\bar{y} = v_c/\sqrt{f'_{cr}} = 3.2$ and ω = 27%; see Figure 5. The relatively high value of ω indicates that the scatter band of the isolated points is wider than what is observed in individual test series.

For example, the size effect tests of shear beam at University of Toronto (Podgorniak-Stanik 1998, Lubell et al. 2004) showed ω = 6.9%, and those at Northwestern University ω = 12% (Bažant & Kazemi 1991). The reason is that the database covers a wide range of secondary characteristics such as the steel ratio, shear-span ratio and concrete type, which all vary throughout the database and have a significant effect on the shear strength of beam. The non-uniform distribution of these secondary characteristics is the result of the subjective selection in different laboratories, and its contribution to the scatter of shear strength dominates. Therefore, the choice of the probability density function (pdf) to be calibrated by the test data in each interval must be empirical. Among the normal distribution, log-normal distribution and Weibull distribution, the log-normal distribution is found to give the best fit for the first size interval, in which many data exist (Bažant & Yu 2009). It should be noted that a log-normal distribution would be physically inadmissible for the scatter due purely to material randomness; but this randomness plays a negligible role in the database scatter.

For large-size beams, the type of pdf of shear strength cannot be obtained by the same process because of the scarcity of data points in this size range. However, in view of the origin of scatter, it would be surprising that the type of pdf changed with the structure size. Therefore, it is logical to assume the log-normal pdf will apply for each size interval.

In Figure 5c, the log-normal pdf obtained from small size range is plotted in the double-logarithmic scale. Figure 6a shows the same pdf superposed on size effect tests made at the University of Toronto. The strength value for the test of the single beam 0.925 m deep was just at the limit of the ACI shear design formula $v_c = 2\sqrt{f_c'}$ (Fig. 6), and the test of the single beam 1.89 m deep was below this limit but nevertheless exceeded the value $\phi 2\sqrt{f_c'}$ where ϕ = 0.75 = required capacity reduction factor (understrength factor). Some engineers deemed it to be acceptable, and thus to justify the disregard of size effect. However, a probabilistic analysis demonstrates the opposite.

To explain, note that, for the particular secondary characteristics (such as the shear span, steel ratio or concrete type) used in the Toronto tests, the shear strength value (the first bold diamond point) of the Toronto test lies (in the logarithmic scale) at certain distance a below the mean of the pdf of the database interval (Fig. 6a). Since the width of the scatter band in the logarithmic scale does not vary appreciably with the beam size, the same pdf and the same distance a between the pdf mean and the Toronto test result must be expected for every beam size d, including the sizes of d = 0.925 m and 1.89 m. In other words, if the Toronto test for d = 925 mm were repeated for the same secondary characteristics as displayed in the small size range, one would have to expect the same pdf but shifted downwards in the logarithmic scale by distance a as shown in Figure 6a. By assuming the value of a to be the same for the small and large size ranges, we simply imply that the probability, or frequency, of beams having shear strength below the value characterized by a will be the same for these size ranges.

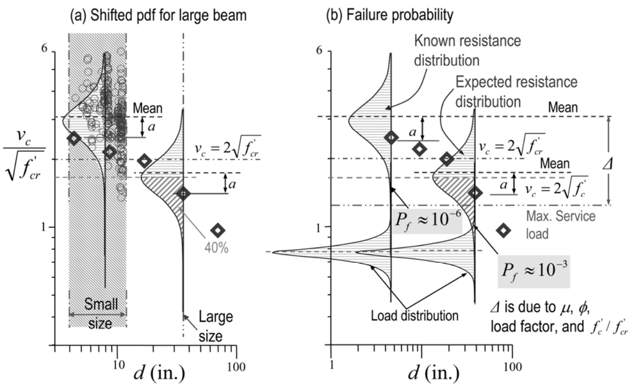

Figure 6. a) Log-normal pdf of shear strength in small size range and its shift to d = 1 m; b) failure probability of beams in different size ranges according to the corresponding pdf.

Now note that the pdf of shear strength in the small size range lies almost entirely above the ACI shear design formula $v_c = 2\sqrt{f_c'}$, but extends well below it for size range centered at $d \approx 1$ m. This means that if the Toronto large size test could be repeated for many different concretes, shear spans, steel ratios, etc., a large portion of the test results would likely fall below the ACI shear strength limit. According to the log-normal pdf obtained here, the percentage of the unsafe large beams would be 40%. This is unacceptable.

Consider now the consequences for the failure probability, P_f, of a structure exposed to the actual service loads. To this end, one must consider the randomness of these loads, reflected in design in the load factor μ. Let $p(y)$ be the pdf of the extreme service loads y. Determining the type of pdf is a demanding task, but for the purpose of comparing small and large structures it will suffice to assume that $p(y)$ is log-normal and that its coefficient of variation is 10%.

Evaluating the classical Freudenthal's reliability integral (Ang & Tang 1984, Madsen et al. 1986, Haldar & Mahadevan 1999) $P_f = \int_0^\infty f(y)R(y)dy$ (where $R(y)$ is the cdf of structure resistance, Fig. 5), one obtains $P_f \approx 10^{-6}$ for beams of $d \approx 200$ mm (centroid of the chosen small size interval in Fig. 6). This is safe since, compared to the inevitable risks that people face, 10^{-6} is generally considered the maximum tolerable. However, for beams of $d = 1$ m, the failure probability increase to $P_f = 10^{-3}$, which is far beyond what the risk analysis experts generally accept as safe. For $d = 1.89$ m, the P_f value is still higher. Therefore, Type 2 size effect must be incorporated in the design code. Otherwise, a uniform safety margin is unachievable.

5 CONCLUSION

The gradual transition of the type of distribution of structural strength and lifetime from Gaussian to Weibullian has serious implications for the safety factors and minimum lifetime guarantees of quasibrittle structures. This needs to be taken into account in the design and safety assessments of large concrete structures, as well as large composite aircraft frames and ship hulls, microelectronic devices, bone implants, etc. By contrast with perfectly ductile and perfectly brittle structures, their safety factors cannot be decided empirically, but must be calculated.

ACKNOWLEDGMENT

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