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and drainage channels. In addition, there is considerable crack deflection by the grains and crystals, and cavitation and dislocation motion in grain boundary regions.

Structural Size Effect

The fracture process zone is the primary reason why linear elastic fracture mechanics (LEFM) cannot be applied to quasi-brittle materials. In these materials, energy is dissipated throughout the process zone instead of at the crack-tip alone (which is the case for LEFM). This causes the material to fail or fracture in a manner less brittle than that conforming to LEFM. This phenomenon is called crack-tip shielding or toughening, especially in ceramic (cf. Clarke and Faber, 1987; Bažant and Kazemi, 1990b) and concrete literature (cf. Bažant and Kazemi, 1990a). In this brief paper, the size of the process zone is regarded as the measure of material brittleness, the ideal (most) brittle case being a zone of infinitesimal size. In other words, a material with a larger fracture process zone is less brittle.

The toughening mechanisms depend strongly on the properties of the microstructure, both the primary phase (grains, aggregates) and the secondary phases (matrix, mortar, interfaces, boundary layers). Therefore, the maximum size to which the process zone can grow is determined only by material properties (in an unbounded body, for a certain temperature and loading rate). For LEFM to be applicable, the size of the process zone should be negligible compared to the dimensions of the structure or body in which fracture is propagating. This condition is, ideally, always satisfied when the structure is infinitely large. In that case, from LEFM relations (e.g., Tada et al., 1985) and with $K_I = K_{Ic}$, the nominal failure stress $\sigma_N \propto 1/\sqrt{d}$, where $\sigma_N = P_u/bd$, P_u = maximum or peak load, d = characteristic dimension of structure, b = thickness, K_I = stress intensity factor and K_{Ic} = fracture toughness or critical stress intensity factor (a material parameter). The dependence of σ_N on d is called the size effect. The LEFM size effect is the strongest possible size effect in structures of quasi-brittle materials.

When the structure or specimen is small enough for the process zone

Size Effects in the Fracture of Quasi-Brittle Materials

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Abstract

Causes and influences of the structural size effect that is exhibited by materials such as concrete, mortar, ice, rock and ceramics, are discussed. The size effect law, which models this phenomenon and provides reliable fracture parameters, is reviewed.

Introduction

Fracture mechanics provides a firm basis for the analysis of structures that can fail by cracking and fragmentation. However, linear elastic fracture mechanics theory, which has been developed to describe metal behavior, cannot be directly applied. The main reasons for this are inhomogeneities that exist in structural materials and the nonlinear processes that occur prior to failure. This paper reviews a nonlinear fracture mechanics approach that can model the failure response of brittle heterogeneous materials. The model uses the structural size effect to extrapolate laboratory test response to the behavior of structures in the field. Results of studies on concrete, rock, ice and ceramics are discussed. Also, some specific implications for the fracture of ice are pointed out.

Fracture in quasi-brittle materials such as concrete, ice, rock and ceramics is strongly influenced by the nonlinear processes that occur during crack propagation. In concrete, the open crack is preceded by a fracture process zone in which there is considerable crack bridging and crack deflection due to aggregates (grains), and microcracking. These processes are also evident in the fracture of rock and ceramics. In fiber reinforced concrete and ceramics, pullout and debonding are the primary mechanisms. In both saltwater and freshwater ice, cracking is accompanied by microfracturing at gas bubbles, flaws, and brine pockets, platelets

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to occupy a major part, failure is governed by limit or yield stress criteria, i.e., $\sigma_N = \text{constant}$. Therefore, there is no size effect when the size of the structure is very small. In between the yield and LEM criteria there is a transition (where nonlinear fracture mechanics is applicable) illustrated in Fig. 1. The equation of the curve shown is of the form (Bažant, 1984):

$$\sigma_N = \frac{Bf_u}{\sqrt{1+\beta}}, \quad \beta = \frac{d}{d_0} \quad (1)$$

where Bf_u and d_0 = empirical parameters, and β = brittleness number. Eq. 1 is the size effect law and models the change in failure (fracture) mode with structural dimensions. This model has been verified extensively by tests of materials such as concrete, mortar, rock, ice and ceramics (Bažant and Pfeiffer, 1987; Bažant, Gettu and Kazemi, 1989; Bažant and Kim, 1985; Bažant and Kazemi, 1990b).

For the experimental calibration of the model in Eq. 1, specimens of different sizes (at least with the size range 1:4) and geometrically similar in two-dimensions (thickness $b = \text{constant}$) need to be tested. From the peak loads P_u the values of σ_N can be determined for each specimen, and parameters Bf_u and d_0 obtained by fitting Eq. 1 to the data. Linear regression analysis may be used by converting Eq. 1 to (Bažant and Pfeiffer, 1987):

$$Y = AX + C, \quad X = d, \quad Y = (f_u/\sigma_N)^2, \quad B = C^{-1/2}, \quad d_0 = C/A \quad (2)$$

where f_u = some arbitrary measure of material strength. Bf_u can also be lumped together as one parameter without changing any of the results. In that case, $Y = \sigma_N^{-2}$ and $Bf_u = C^{-1/2}$. With regard to ice, such an analysis was carried out previously (Bažant and Kim, 1985) for the tests conducted by Butiagin (1966) on ice beams. Actually his 516 tests of lake and river ice at different temperatures would yield many size effect curves, one for each temperature and microstructure. The average of all such lines was obtained by fitting all the data together, and is shown in Fig. 2 (where f'_t = assumed average tensile strength).

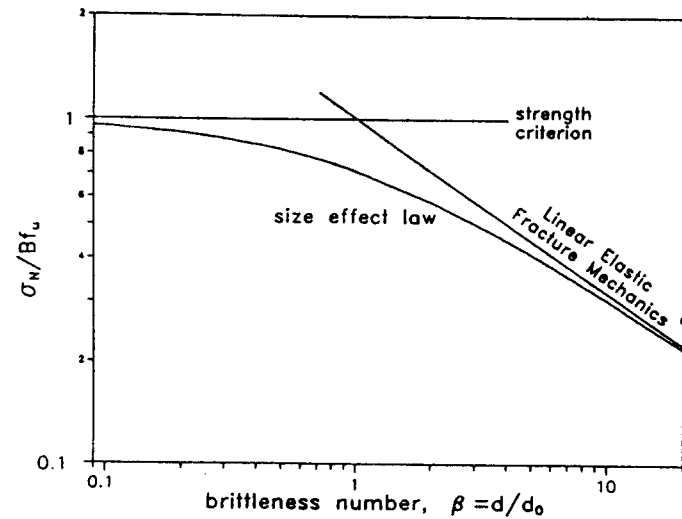


Fig. 1 Structural Size Effect

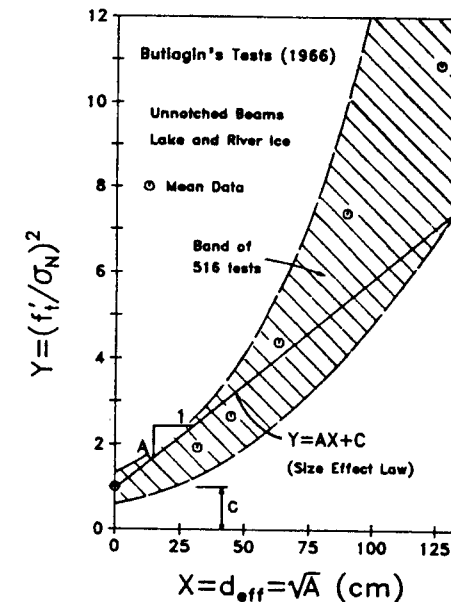


Fig. 2 Size Effect on Failure Stress of Ice

Parameter β (Eq. 1) is a measure of the brittleness of the failure of a structure. For $\beta < 0.1$, yield or maximum stress criteria are valid, and for $\beta > 10$, LEFM governs. In the range of $0.1 < \beta < 10$, nonlinear fracture mechanics should be applied. This definition of structural brittleness is more objective than using the crack density or the peak strain to decide whether the failure is brittle or ductile. It has been shown that β is practically shape-independent, and thus a universal measure of brittleness (Bažant and Pfeiffer, 1987).

Nonlinear Fracture Parameters

The size effect on the maximum nominal stress also causes the fracture parameters determined from tests to be size dependent. (A typical fracture specimen geometry - three-point bend specimen - is shown in Fig. 3.) Based on Eq. 1, it was proposed (Bažant, 1984) that the parameters be defined for an infinitely large specimen which gives unambiguous and geometry-independent values. The fracture toughness can be related to the size effect parameters as (Bažant and Pfeiffer, 1987):

$$K_{Ic} = Bf_u \sqrt{d_0 g(\alpha_0)} \tag{3}$$

A second parameter has been recently defined for characterizing the nonlinear fracture process. The effective length of the fracture process zone is given by (Bažant and Kazemi, 1990a):

$$c_f = \frac{d_0 g(\alpha_0)}{g'(\alpha_0)} \tag{4}$$

where $g(\alpha_0)$ and $g'(\alpha_0)$ are constants dependent on geometry. Parameter c_f quantifies the effectiveness of all the toughening mechanisms of the material, lumped together. A higher value of c_f signifies a less brittle material as far as fracture is concerned. Therefore, it is a convenient way to compare the brittleness of various materials. Such an analysis was used recently (Gettu, Bažant and Karr, 1989) to compare the failure modes of normal and high strength concretes. It was shown that for an increase of 160% in compressive strength, K_{Ic} increases only by 25%. More importantly, c_f decreases by 60%, and consequently, the brittleness more

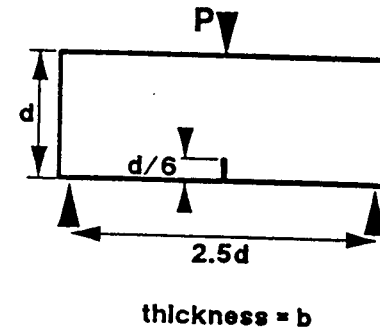


Fig. 3 Three-Point Bend Fracture Specimen

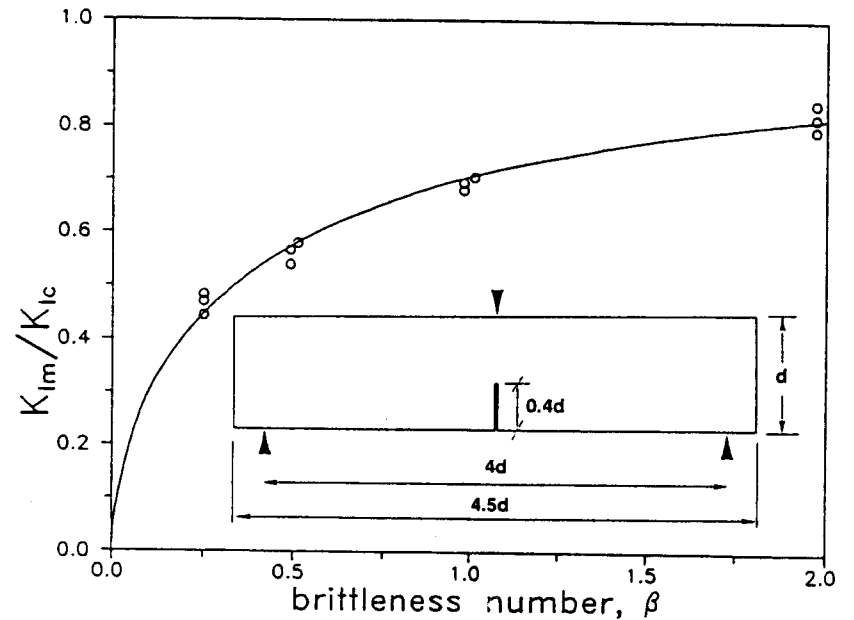


Fig. 4 Size Effect on Apparent Fracture Toughness

than doubles. Similarly, the size effect method can be used to compare the failure characteristics of ice that is, say, obtained from different sources or seeding techniques, or with different grain properties or crystallographic textures.

Eq. 1 can also be rewritten to give the size effect on the apparent fracture toughness (Bažant and Pfeiffer, 1987):

$$K_{Im} = K_{Ic} \sqrt{\frac{\beta}{1+\beta}} \quad (5)$$

where K_{Im} = stress intensity factor calculated from LEFM relations with the crack length as notch length and the load as P_u of the tested specimen. Eq. 5, along with test data of limestone (Bažant, Gettu and Kazemi, 1989), is shown in Fig. 4 (inset shows specimen geometry). Trends similar to Eq. 5 have been observed for ice (Dempsey, 1989), and it appears that the size effect method would yield reliable values for the fracture toughness of ice.

Influence of Rate, Temperature and Grain-Size on Size Effect

The structural size effect modeled by Eqs. 1 and 5 can be significantly influenced by the loading rate. From quasi-static tests of concrete, it was recently shown that creep effects increase the size effect (and brittleness) as the time to failure (t_p) increases (Bažant and Gettu, 1990). Fig. 5 illustrates the shift towards LEFM for slower rates (the specimen geometry is shown in Fig. 3). Also, K_{Ic} and c_f decrease considerably as t_p increases. This explains the higher dynamic tensile strength observed in several materials including ice (Schulson, 1988). However, an opposite trend is sometimes seen in quasi-static tests of ice (Dempsey, 1989), where the fracture toughness seems to decrease for faster loading or to be unaffected. Two possible reasons for this effect are (Schulson et al., 1989): crack blunting due to creep and crack healing at the crack-tip, which decrease the stress concentration.

Another important effect is that of temperature. Tests of concrete

at various temperatures (Bažant and Prat, 1988; Planas et al., 1989) based on the size effect method showed that K_{Ic} increases with decrease in temperature. This trend is seen to a certain extent in ice (Schulson, 1988). Also, it appears that for concrete c_f decreases with increasing temperature, implying stronger size effect. For ice, Weeks and Assur (1968) concluded that the crack density decreases with increasing temperature, and consequently, according to the aforementioned definitions, brittleness and size effect should increase.

The foregoing discussion concentrates on structural size effect, which is the effect of structural dimensions, for a certain material or microstructure. Another important size effect arises due to change in the physical properties of the primary phase, which is called the grain-size effect. Obviously, there is a strong interaction between the two size effects. However, the size effect method (reviewed here) can be easily used to study also grain-size effects on fracture by extrapolating the response to large specimen size, and thereby eliminating the structural size effect (Dempsey, 1989, 1990). Studies by Cole (1986, 1987) have shown a strong grain-size effect on the brittleness, creep and fracture strength of ice. The strength seems to increase with decreasing grain size (following the Hall-Petch relation) at low loading rates, but at higher rates the trend reverses. Also, the crack density appears to increase with increasing grain size, implying a decrease in brittleness. This supports the trend expected from Eq. 1 (or Fig. 1) when the process zone size is assumed to depend on the grain size.

Conclusions

Structural size effect is inevitable in quasi-brittle materials, and has to be accounted for in analysis. A method has been reviewed which models this phenomenon, and utilizes it to give size- and shape-independent material fracture parameters. Other factors that influence the size effect are also discussed.

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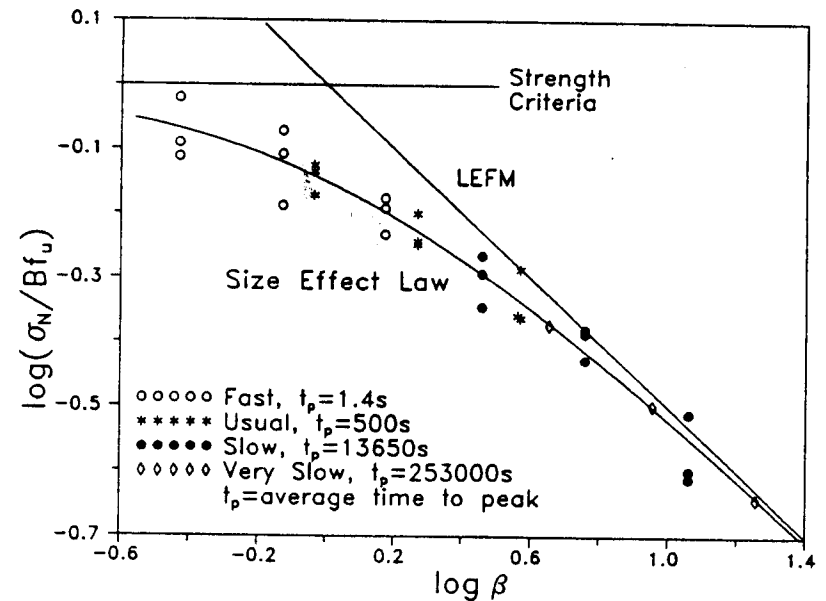


Fig. 5 Influence of Loading Rate on Failure Mode