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PROBABILISTIC PREDICTION OF CREEP AND SHRINKAGE IN CONCRETE STRUCTURES: COMBINED SAMPLING AND SPECTRAL APPROACH

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ABSTRACT

The sources of the randomness in creep and shrinkage are basically three: (a) random variation of material parameters; (b) uncertainties of the creep and shrinkage models; and (c) influence of the random variation of environmental humidity and temperature. The purpose of this paper is to solve the random shrinkage stress problem with all three sources of randomness taken into account. The random variables in source (a) are considered as correlated, those of source (b) as independent random variables, and those of source (c) as stochastic processes which can be determined by spectral analysis of real humidity records. For the sake of simplicity, all of the system equations are assumed to be linear. A numerical example is given and a simplified expression for predicting the confidence band of response is developed.

KEYWORDS

Concrete; creep; shrinkage; viscoelasticity; aging, humidity effects; random material properties; random environment; model uncertainty; random stresses; sampling; spectra method.

1. Introduction

The sources of randomness of creep and shrinkage effects in concrete structures are essentially threefold: (1) the randomness of material properties, (2) the uncertainty of the material model for creep and shrinkage, and (3) the random fluctuation of the environmental humidity. Effective methods to analyze each one of them have been developed before (for a review, see Ref. 9). However, a method to analyze the simultaneous effect of all three sources of randomness is lacking. Its development is the objective of this conference contribution. A detailed presentation will be given in a separate journal article [10].

2. SYSTEM MODEL

DIFFUSION EQUATION One complicating characteristic of the heat conduction and moisture transfer in concrete is their coupling. However, couples fluxes can be avoided if the diffusion flux is expressed in terms of the pore vapor pressure (Darcy's Law) and the relative vapor pressure, h (humidity), in the pores of concrete, which depends on both the specific moisture content and temperature. For the sake of simplicity, we consider in the present study only humidity fluctuation, but not the temperature effect. We also neglect the direct effect of hydration on pore humidity and the dependence of diffusivity on h. Thus, we may assume the variation of humidity inside the structure to be governed by the linear diffusion equation $\partial h/\partial t = C(t)\nabla^2 h$, in which h is the relative vapor pressure in the pores at concrete, and C(t) is the diffusivity, which strongly depends on the age, t, and on h(t). An acceptable approximation is to chose c as constant, equal to the mean value of (H) for the general range of humidity variation.

MATERIAL CHARACTERIZATION Pore humidity changes cause local free (unrestrained) shrinkage strain, $\epsilon_{sh}(x,t)$. The shrinkage law as nonlinear [2], and we describe it in the incremental form [4]:

$$d\epsilon_{sh} = \psi\epsilon_s^0 \,[E(t_0)/E(t_0 + \tau_{sh})]\, [df_s(h)/dh]\dot{h}dt \qquad (1)$$

in which ϵ_s^0 = material constant and is taken as $\epsilon_{s\infty}$ in the BP model [3]; ψ is introduced as a random variable reflecting the uncertainty of the shrinkage law per se; τ_{sh} is a function of cross section size taken from BP model; $f_s(h)$ is taken as $(C - h(x,t)) = 0$. In our study the humidity inside concrete is considered as 1.0, in which case, C = 1.0. Equation (1) is an acceptable approximation within a relatively broad humidity range 0.5 - 1.0. The shrinkage stresses are reduced by creep. To take creep into account, one may get the elastic solution of the problem first, and then express the stresses in an incremental form based on (1). Subsequently, according to McHenry's analogy [2], the stresses in the presence of creep may be calculated as: $\sigma(t) = \int_{t_0}^{t} R(t,t')/E(t')d\,\sigma(t')$, where R(t,t') is the relaxation function of concrete.

Random Material Parameters and Model Uncertainty In the present study, only the uncertainty of the shrinkage model is considered and that of the relaxation function is neglected because statistical data are lacking. To simplify the analysis we assume the random material parameters involved in our problem to be governed by a joint normal distribution. The material parameters in consideration are concrete cylinder strength f_c', water-to-cement ration w/c, aggregate-to-cement ratio g/c and cement content c. The 4 x 4 correlation matrix of the joint probability distribution is given in Ref. 8. The model uncertainty is assumed to be governed by an independent normal distribution. Another assumption is that the joint normal distribution of the material parameters is independent of the random variation of environmental humidity.

3. INPUT HUMIDITY MODEL

The environmental humidity is a stochastic process in time. To characterize it, we must analyze climatic records over a rather long period of time, and obtain the first- and second-order moments. As an example, we take the climatic records of Chicago area [6], from which the unbiased estimate of sample covariance C_r and the estimate of spectral density Φ may be obtained as shown in Fig. 1a,1b. From this we find that the environmental humidity process is composed of three different parts. The first part, H_1, represents the mean value of H, characterizing the stable horizontal trend; the second part, H_2, is a random phase process corresponding to the harmonic variation of humidity; and the third part H_3, is a random-noise process, which may be simulated by Poisson square-wave process. By combining the statistical functions of H_2 and H_3, the input

humidity model H, its autocorrelation function R_H and it spectral density Φ may be written as

$$H = H_m + H_1\cos(\omega_1 t + \varphi_1) + A_n w(t,\tau_n), \qquad R_H(t) = 0.5H_1^2\cos\omega_1\tau + \mathrm{Var}(A_n)\, e^{-\lambda|\tau|}$$
$$\Phi_H(\omega) = 0.5H_1^2\,\pi(\delta(\omega - \omega_1) + \delta(\omega - \omega_1)) + 2\lambda\,\mathrm{Var}(A_n)/(\lambda^2 + \omega^2) \qquad (2)$$

where $w(t, \tau_n) = 1.0$, for $\tau_{n+1} > t > \tau_n$ and $w(t, \tau_n) = 0.0$, otherwise; n = 1,2,...N(t); N(t) is a Poisson process with intensity λ; τ_n is the arrival time of A_n, and A_n are assumed to be uniformly distributed random variables. The interarrival times τ_1, $\tau_2 - \tau_1$... are statistically independent and identically distributed exponential random variables with the same parameter λ.

The coefficients of the above equations can be determined in the following way: When τ is large enough, $R_H(\tau) = 0.5H_1^2\cos\omega_1\tau_1$ where amplitude H_1 can be determined; when $\tau = 0.0$, $R_H(0) = 0.5H_1^2 + \mathrm{Var}(A_n)$, where $\mathrm{Var}(A_n)$ can be obtained. From the peak point of the spectrum in Fig. 1b, frequency ω_1 can be determined. Furthermore, for $\omega = 0.0$, $\Phi_H(0) = 2\mathrm{Var}(A_n)/\lambda$; therefore λ can be evaluated. With the coefficients identified in this manner, Fig. 1(a,b) shows the theoretical curves compared with the estimated curves of the climatic sample.

There is one point to mention: The period obtained from our humidity sample is 1 day and no long-time periodic variation can be found. A possible explanation might be that only two-year weather records were included in our statistical analysis. That might not be long enough to detect long time periods, say 1 year, although such a period exists, as other observations demonstrated [5]. Therefore, it is reasonable to add a term with the long period of 1 year.

4. RESPONSE ANALYSIS

a) Method of Solution - The latin hypercube sampling method is especially convenient for random initial value problems, such as these of random material parameters and model uncertainties, provided that every sample unit can be kept at equal probabilistic content. A successful procedure has been developed to ensure both the uncorrelated and correlated normal variables to have the same probabilistic content [1]. In the present case, even though the random phase angle and random noise H_3 are included in the sample, the latin hypercube sampling method can still be formulated. The probability of one sampling unit will be $P_u = P(fc', w/c, g/c, C)\, P(\psi)P(\varphi)\, P(A_n) = \mathrm{const}$ in which P(fc', w/c, g/c, c) is the joint probability of material parameters in a specific sample; $P(\psi)$ is the probability of model uncertainty, $P(\varphi)$ is the probability of random phase H_2, $P(A_n)$ is the probability of noise process H_3; $P(A_n) = P(A^*)P(0)$, in which $P(0) = \exp(-\lambda\Delta t)$ is the probability for no state change during Δt; $P(A^*)$ is the probability for H_3 holding a certain value $H_3 = A_n = A^*$; $P(A^*) = \Delta A_n/(b-a)$ = constant, $A^* \epsilon \Delta A_n$; 1/(b-a) is the probability density function for the uniform distribution; Δt is a small time interval at which the input value is given. The foregoing expression for P_u means that the probability content of every sampling unit will be the same, which renders the estimates of structural response to be more efficient than those obtained by a simple sampling method. Although the sampling method seems to be able to solve the entire problem by itself, the spectral method will nevertheless be also introduced in present study because it can further simplify the calculation, especially for a linearly behaving aging structure, such as a concrete structure [7].

We will now consider a numerical example of an infinitely long cylindrical wall. The external radius of the cylinder b = 21m, the internal radius is a = 20m, (that is, the thickness of the wall is 1m). The initial humidity inside concrete is 1.0, the mean value of the environmental humidity is 0.7. The random phase process of 1-day period is $A_1\cos(2\pi(t - t_0) + \varphi)$, with $A_1 = 0.1$. The random phase process of 1-year period is A_{365}

$\cos(2\pi/365(t - t_0) + \varphi_{365})$, with $A_{365} = 0.08$ [5]; φ_1 and φ_{365} have uniform distributions with the same density function, $1/2\pi$; t_0 is the time the exposure starts, $t_0 = 28$ days. For Poisson square-wave process, A_n is a uniform distribution, the density function is 1/0.24, the intensity of Poisson process is 2.0/day, and so the interarrival time has the exponential distribution of parameter $\lambda = 2.0$. The distribution parameters for model uncertainty are taken from Ref. 8; the mean is 0.944, the variance is 0.0046. The mean values of f'_c, w/c, g/c and c are 45.2MPa, 0.46, 2.07 and 450 kg/m^3, and their coefficients of variation are 0.1, 0.1, 0.1 and 0.1. The components of the input humidity will be analyzed one by one and then superposed. Only the circumferential stress response σ_θ will be analyzed in this study since it gives the largest stress among σ_θ, σ_r and σ_z.

b) Response to Input of 1-Year Period - The sampling results show that the response to an input of 1-year period represents the dominant part of the response compared with other input processes. Although the sampling method can give mean and standard deviation of response, an even simpler solution exists. The humidity response of a very thick wall to an input of 1-year period can simply be approximated by the steady-state response of a semi-infinite solid

$$H(x,t) = A_{365}\, e^{-kx} \cos(\omega_{365} t - kx - \varphi) \tag{3}$$

The thickness of our cylinder wall is about two wavelengths, which is enough for Eq. 3 to hold (wavelength $L = \sqrt{4\pi CT} = 37.09$cm, $K = 2\pi/L$, $c = 0.3$ cm /day, $T = 365$ days). The comparison of the cylinder responses and the simplified half-space solution are shown in Fig. 1c. We can see that they agree perfectly, including the periodic characteristics, the amplitude attenuation and the progressive phase lag at increasing depths.

Based on the same argument, we further may reasonably assume the stresses in the cylinder to have the same spatial distributions as the humidity. The only difference is the amplitude of the stress, A_σ, which will be attenuated at a different rate with increasing depth (A_σ is a random variable);

$$\sigma = S_\sigma \cos[\omega_{365}(t - t_0) - kx - \varphi] \tag{4}$$

If x and t are fixed and A_σ is one of the realizations of the random amplitude, Eq. (4) will become a random phase process. The mean value is zero, the variance is $A_\sigma^2/2$, and the standard deviation is $S_\sigma = A_\sigma/\sqrt{2}$. One should remember that S_σ, same as amplitude A_σ, is not a deterministic value but a random variable; its mean value and standard deviation are $\mu_{s_\sigma} = E(A_\sigma)/\sqrt{2}$, and $S_{s_\sigma} = \mathrm{Var}(A_\sigma)/\sqrt{2}$. On the other hand, the effect of material parameters and model uncertainty on amplitude A_σ may be effectively taken into account by sampling.

Fig. 1d shows the mean value of the stress distribution over the cross section. One can see that the mean stresses are reduced as the depth increases. The deepest influence region of the random process is about 20 cm. Fig.1e shows the envelope of stress amplitude. It may be noticed that A_σ are time dependent and reach their peak value at about 10 years. After that A_σ declines slowly with the increase of time as a result of aging. The mean and standard deviation of A_σ have the same time-dependent behavior. By fitting these sampling results, closed from expressions for the mean and the standard deviation of A_σ can be easily obtained. Finally, substituting $\theta = (t - t_0) - 9.84(x - 1)^9$, the simplified equation to predict the stress confidence band corresponding to the random phase process is

$$\mu_\sigma \pm S_\sigma = \pm\left(-61 + \frac{491}{1 + 0.315x}\right)\left(0.685 + \frac{\theta}{4215 + 0.342\theta^{1.22}}\right) \tag{5}$$

c) Response to an Input of 1-Day Period - The results of analysis indicate that the response to an input of 1-day period is very weak (10^{-4}

for humidity amplitudes, 10^{-1} psi for stress amplitudes) even at a very shallow depth in concrete (3cm). This is mainly because of the very low value of concrete diffusivity. In view of such behavior, one can neglect the input of 1-day period without losing much in accuracy.

d) Response to Poisson Process - For present problem, the response of the structure at time t is the sum of the responses to all individual square waves, i.e. $\sigma(t,x) = \Sigma Y_n w(t - \tau_n)$, $n = 1,\ldots,N(t)$; here $w(t - \tau_n)$ is a step function called the shape function; $w(t - \tau_n) = 1$ if $t - \tau_n > 0$, otherwise $w(t - \tau_n) = 0$. N(t) and τ_n have the same meanings as those in Eq. (2); Y_n is the increment of stress at depth x due to a square wave. The mean value μ_σ, covariance K_σ and variance $\mathrm{Var}(\sigma)$ are

$$\mu_\sigma(t) = \int_0^t E(Y[w(t,\tau)])\tau dT = \tau E(Y)t,$$

$$K_\sigma(t_1,t_2) = \int_0^{t_1} E(Y[(t_1,\tau)]Y[w(t_2,\tau)])\tau\, dt = \tau E(Y^2)t_1,\ t_1 < t_2 \tag{6}$$

$$\mathrm{Var}(\sigma(t)) = \int_0^t E(Y^2[w^2(t,\tau)])\tau\, d\tau = \tau E(Y^2)t$$

From Eqs. (6), one can see that the problem of stress deviation is transformed into a problem of stress increment deviation, which can be determined from only one structure. Random stress increment Y_n depends in a complex way on many parameters. Among these influence parameters, the dominant ones are the depth x, and the amplitude and duration of the input square waves. By keeping all of the other variables constant a single response curve arising only from input H_3 at certain depth x will be obtained. The mean values are always zero, the variance is 43.34 at x = 1cm and 1.16at x = 5cm. The standard deviation S_y decrease rapidly with increasing x. This means the noise component of humidity affects only a shallow layer of the wall, up to about 5cm. The deviation of the initial random variables can be taken into account easily by multiplication with the factor $(1 + \rho)$. Then the expression for the stress confidence band is

$$\mu_\sigma \pm S_\sigma = \pm(1 + \rho)\left(-7.58 + \frac{16.82}{1 + 0.189x}\right)\sqrt{2(t - t_0)} \tag{7}$$

where ρ is the coefficient of variation; $\rho = 0.15$, as explained later. As can be seen, from Eq. (6), the standard deviation of stress increases steadily with time t. Therefore, the deviation of the response to random noise grows rapidly and becomes the dominant part in the total stress deviation in a shallow surface layer of concrete. This is the cumulative consequence of the random stress increment deviation.

e) Response to Initial and Mean Environmental Humidities - Fig.1f shows the sampling results for the stress response history. They indicate that although the mean value and standard deviation are functions of x ant t, their coefficient of variation is independent of x and t; $\rho = S_\sigma/\mu_\sigma = 0.15$ at any time and any depth. This is true not only for the the response to the mean environmental humidity and the initial humidity inside concrete but also for any response discussed before. This characteristic has been used to fit the curves of prediction equations (5) and (7). However, ρ is not always a constant; it depends on the statistical properties of the initial random variables because the stress response is a function of those variables.

According to the principle of superposition and linearity of the system, the response's of the structure is the sum of the responses to the initial value of humidity inside the concrete, the mean value of environmental humidity, the harmonic humidity variations of 1-year period, and the random-noise-like Poisson square-wave process. The total width of the stress response confidence band is the sum of Eq. (5), (7) and $\mu_\sigma(1 + \rho)$, where μ_σ is the response to the mean values of the initial random variables. These simplified formulae yield the confidence band at any time and any depth. The calculation is so simple that it can be done by hand.

5. CONCLUSIONS

The random environmental humidity can be decomposed into three stochastic processes; two random phase processes with 1-day periods, and one Poisson square-wave process. Due to the very low value of diffusivity of concrete, the response for the 1-day period is so feeble that it can be neglected. The stress response to phase process influences a surface layer about 20cm thick the deviation reaches its maximum value at about 20 years and then decreases slowly as a result of aging. The stress response to the noise component of environmental humidity is a filtered Poisson process; it only influences a shallow layer of concrete about 5cm thick, due to the low value of diffusivity. The deviation of the stress response is proportional to t, and so in this region the stress response develops much scatter as time increases. The material parameters and the model uncertainty influence the response of the whole structure. At depth over 20cm, the coefficient of variation of stress response is independent of depth and time, and depends almost exclusively on the values of material parameters and on model uncertainty. Diffusion in an infinitely long cylinder with a thick wall and can be approximately analyzed, for a random-phase input of 1-year period, as a half space. Then, if the stress response to Poisson square-wave process is modeled as a filtered Poisson process, simplified but satisfactory formulae to predict the confidence band of stress response can be developed.

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