

TRANSACTIONS OF THE 10TH INTERNATIONAL CONFERENCE  
ON STRUCTURAL MECHANICS IN REACTOR TECHNOLOGY  
14-18 AUGUST 1989  
ANAHEIM, CALIFORNIA, USA

# Structural Mechanics in Reactor Technology

VOLUME H  
CONCRETE STRUCTURES

*Coordinators*  
JOSEF EIBL  
K.T.S. IYENGAR

*Editor*  
ASADOUR H. HADJIAN



American Association for Structural Mechanics  
in Reactor Technology/Los Angeles/1989

# Recent Studies of Size Effect in Concrete Structures

Z. P. Bažant, M. T. Kazemi, R. Gettu  
*Northwestern University, Evanston, IL USA*

## Abstract

Recent advances in the size effect law for the failure stress of concrete specimens or structures and its application for the determination of fracture energy are reviewed and some extensions of the previous formulations are presented. One extension consists of a modified form of the size effect law which involves only true material parameters, particularly the fracture energy and the effective length of the fracture process zone. These parameters are both uniquely defined on the basis of the extrapolation of specimen size to infinity. This extension makes it possible to define the brittleness number of a structure in terms of an equivalent shape-independent structure size and the limiting length of the fracture process zone. The calculation of the brittleness number requires the value of the nondimensional energy release rate for the equivalent crack in an elastic structure, whose value and derivative take into account the shape of the structure. Another extension is the determination of R-curve from the size effect, and its use to calculate structural response.

## Introduction

The importance of toughening mechanisms which consist of shielding of the crack tip by a nonlinear zone of distributed microcracking is now generally realized. These mechanisms considerably enhance the fracture toughness of brittle heterogeneous materials such as concrete, rock and certain ceramics. The fracture energy or fracture toughness of such materials does not represent the sole material characteristic of fracture response. The size of the nonlinear fracture process zone is another important characteristic. The size of this zone is essentially, although not exclusively, a property of the material, since it is determined by the size of the inhomogeneities in the microstructure. If the size of the zone is negligible compared to dimensions of the structure, the response is close to linear elastic fracture mechanics. If the size of the zone encompasses most of the specimen or structure volume, the failure is determined by strength or yield criteria. If the size of the zone is intermediate, the response is transitional between the strength criterion and the linear elastic fracture mechanics. It is this transitional behavior which is of interest for most concrete structures.

The purpose of the present conference lecture is to briefly review recent advances in the fracture mechanics size effect, with regard to both determination of material fracture parameters and determination of failure

loads of structures. The problem is of particular interest to nuclear concrete structures, for two reasons: (1) the structures are much larger than the specimens and reduced scale models for which the material behavior has been experimentally calculated, and (2) the brittleness of many types of failure of concrete reactor vessels and containments cannot be avoided - e.g. the cryptodome failure, seismic shear fracture of a shell wall, punching shear failure.

The scope of this paper permits only summarizing the results. For detailed derivations and experimental verifications for concrete as well as rock, see recent reports by Bazant and Kazemi (1988), and Bazant, Gettu and Kazemi (1989).

### Size Effect Law

The size effect is described in terms of the nominal stress at failure:

$$\sigma_N = c_n \frac{P_u}{bd} \quad \text{for 2D similarity} \quad (1)$$

$$\sigma_N = c_n \frac{P_u}{d^2} \quad \text{for 3D similarity} \quad (2)$$

in which  $P_u$  is the maximum load,  $b$  = specimen or structure thickness,  $d$  = chosen characteristic dimension of the specimen or structure, and  $c_n$  = a coefficient introduced for convenience. As shown by Bazant (1984), the nominal stress approximately follows the size effect law

$$\sigma_N = B f_u (1 + \beta)^{-1/2} \quad \beta = d/d_0 \quad (3)$$

in which  $B$  and  $d_0$  are empirical coefficients and  $f_u$  represents material strength. See Fig. 1, where  $d_a$  = aggregate size,  $f'_t = f_u$  and  $\lambda_0 = d_0/d_a$ .

For  $d \gg d_0$ , Eq. 3 gives the size effect of linear elastic fracture mechanics, for  $d \ll d_0$ , Eq. 3 gives no size effect, which is characteristic of the failures governed by strength or yield criteria, and for the intermediate range of  $d$ , Eq. 3 describes a transitional behavior corresponding to nonlinear fracture mechanics. Eq. 3 is applicable only for the size range of approximately 1:20. For a broader size range, further terms of an asymptotic series expansion need to be included in Eq. 3 (Bazant, 1985 and 1987).

Eq. 3 has the advantage that it can be transformed to a linear regression plot  $Y = A X + C$ , in which  $X = d$ ,  $Y = (f_u/\sigma_N)^2$ ,  $B = C^{-1/2}$ , and  $d_0 = C/A$ .

For the size effect law in Eq. 3, there exists a wide range of justifications:

1. Some simple energy release solutions.
2. Dimensional analysis and similitude arguments.

3. Experimental results on fracture specimens as well as brittle failures of various concrete structures (e.g. Fig. 1).
4. Finite element results obtained by either blunt fracture models (crack band model, Hillerborg's fictitious crack model) or nonlocal damage models.
5. Random particle simulations of concrete (interface element model).
6. Micromechanics analysis showing that a nonlocal damage model is a proper homogenization of a quasiperiodic crack array.

The experimental justification obtained of Eq. 3 at Northwestern University included:

1) Mode I fracture specimens: a) three-point bend specimens, b) edge-notched tension specimens, c) eccentric compression specimens, and d) compact tension specimens.

2) Mode II specimens (approximately Mode II), with alternating loads at four points on a beam.

3) Mode III specimens: cylinders with a circumferential notch subjected to torsion.

The materials for which the size effect has been experimentally verified at Northwestern University included: 1) concrete, 2) mortar, 3) rocks of various types, 4) certain ceramics (SiC, SiO<sub>2</sub>), and 5) aluminum alloy.

The size effect has further been experimentally verified at Northwestern University under a wide range of conditions, including:

1. Various temperatures, ranging from room temperature to 200°C.
2. Wet specimens and dried specimens.
3. Specimens subjected to various rates of loading, with times to peak load ranging as 1:10<sup>5</sup> (ongoing work of R. Gettu at Northwestern University).
4. Monotonic as well as cyclic loads (work in progress by K. M. Xu at Northwestern University).

Coefficients B and d<sub>0</sub> in Eq. 3 are not material parameters and depend on the specimen shape. Bazant and Kazemi (1988), however, have shown that Eq. 3 can be reformulated in a manner which involves only true material parameters, G<sub>f</sub> (fracture energy), c<sub>f</sub> (effective length of fracture process zone), and E (Young's modulus of elasticity). Such a modified version of the size effect law can be written as:

$$\tau_N = \left( \frac{EG_f}{c_f + D} \right)^{1/2} \quad (4)$$

in which  $\tau_N$ , called the shape-independent nominal strength, is defined as:

$$\tau_N = \sqrt{g'(\alpha_0)} \frac{P_u}{bd} \quad \text{for 2D similarity} \quad (5)$$

$$\tau_N = \sqrt{g'(\alpha_0)} \frac{P_u}{d^2} \quad \text{for 3D similarity} \quad (6)$$

and D represents the shape-independent characteristic dimension of the structure, defined as:

$$D = \frac{g(\alpha_0)}{g'(\alpha_0)} d \quad (7)$$

Here  $\alpha_0 = \frac{a_0}{d}$  (the initial relative crack length),  $a_0$  = notch or initial crack length, and  $g(\alpha)$  is the nondimensional energy release rate for crack length  $a = \alpha d$  calculated according to linear elastic fracture mechanics, the actual energy release rate being  $G(\alpha) = P_u^2 g(\alpha) / Ebd^2$  (for 2D specimens).  $g'(\alpha_0) = dg(\alpha)/d\alpha$  is evaluated at  $\alpha = \alpha_0$ . It can be shown that within a size range of up to about 1:20, in which the approximate size effect law (Eq.3) is applicable, the values of  $g(\alpha_0)$  and  $g'(\alpha_0)$  sufficiently take into account the shape of the structure.

#### R- Curve for Size Effect

Based on the size effect law (Bazant and Kazemi, 1988), one can show that the R-curve, which represents the dependence of the energy release rate required for crack growth on the crack extension  $c$  from the notch, can be described by the formula:

$$R(c) = G_f \frac{g'(\alpha)}{g'(\alpha_0)} \frac{c}{c_f}, \quad (8)$$

where  $\alpha$  is given by:

$$\frac{g(\alpha)}{g'(\alpha)} - \alpha = \frac{g(\alpha_0)}{g'(\alpha_0)} \frac{c}{c_f} - \alpha_0. \quad (9)$$

Solving for the load  $P$  vs. load-point displacement from Eq. 8, one obtains the maximum load and the corresponding value of  $c$  at maximum load. The larger the structure, the larger is  $c$ , and for infinite size  $c \rightarrow c_f$ . To calculate the post-peak load-deflection diagram, the value of  $R$  must be frozen as constant, equal to the values of  $R(c)$  (Eq. 8) for the peak load.

Since function  $g(\alpha)$  depends on geometry, the R-curves obtained from Eq. 8 for specimens of various geometries are different. In fact, they are rather different. Fig. 2 shows an example of load-deflection curve calculated by equivalent linear analysis from the R-curve. This curve agrees closely with the recent, yet unpublished measurements of Bazant, Gettu and Kazemi (1989). This procedure was also used to predict the data of earlier work by Labuz, Shah and Dowding (1987) where non-symmetric crack propagation was observed.

### Measurement of Fracture Energy

According to various existing methods of measurement (Knott, 1973), the fracture energy value has been found to be highly variable. However, a unique definition, which is independent of the specimen size as well as shape, can be based on the size effect law. The fracture energy can be uniquely defined as the value of the energy required for crack growth (per unit fracture area) in an infinitely large specimen (Bazant and Pfeiffer, 1987); Fig. 3. According to this definition, the fracture energy is found to be given by the formula:

$$G_f = \frac{f_u^2}{c_n^2 A E} g(\alpha_0). \quad (10)$$

It has been shown that the fracture energy values obtained on the basis of this formula from various types of fracture specimens give relatively constant results.

### Applications to Structures

Eq. 3 has also been shown to be applicable to brittle failures of concrete structures. The reason is that concrete structures are not allowed by the codes to be designed so that they fail at the first crack initiation. Rather, the design must be such that a large cracking zone develops before the ultimate load is reached. This cracking zone serves as a notch, causing the structure, during failure, to behave essentially as a fracture specimen with a notch, the failure being significantly influenced by the rate of energy release and stress distributions due to further extensions of the cracking zone.

The applicability of the size effect law has been experimentally verified at Northwestern University for the following types of failures:

1. Diagonal shear failure of beams with longitudinal reinforcement:  
a) nonprestressed beams without and with stirrups, and b) prestressed beams.
2. Punching shear failure of slabs.
3. Torsional failure of concrete beams of rectangular cross section, without or with longitudinal reinforcement.
4. Pullout failure of reinforcing bars embedded in concrete.
5. Ring and beam failure of unreinforced concrete pipes.
6. Compression splitting failure, i.e., the Brazilian test (here the size effect law is found to apply only up to a certain size, beyond which the size effect disappears, apparently due to transition to some type of frictional mechanism or strength-controlled failure).

It may be pointed out that size effects can also be mathematically explained by a probabilistic mode of Weibull-type, which has been very popular in the literature. However, it seems that this explanation is correct only for the failure of uniformly stressed tensile specimens without notches and is not applicable to the typical failures of concrete structures

listed above. The existing statistical theories generally neglect the major stress redistributions which take place after the onset of the first cracking and before attainment of the maximum load, and thus ignore the energy release aspects on the macroscale. Statistics can, of course, be included in the fracture analysis. However, if the Weibull parameters are calibrated from the test results for uniformly stressed tensile specimens and the same material parameter values are used for the zone in which the fracture front at failure can possibly be located in the concrete structure (e.g., in the diagonal shear failure of beam), then the statistical part of the size effect is found to be generally negligible (Bazant, 1987).

### Brittleness Number

As proposed by Bazant (1987), see also Bazant and Pfeiffer (1987), the nature of the specimen or structure response at failure can be characterized by the brittleness number,  $\beta$ , as already introduced in Eq. 3. Depending on the value of the brittleness number, three different regimes may be distinguished:

1. For  $\beta < 0.1$ , the fracture is governed by strength or yield criteria, and fracture mechanics need not be used.
2. For  $0.1 \leq \beta \leq 10$ , the failure is governed by nonlinear fracture mechanics, and the finite size of the fracture process zone must be taken into account.
3. For  $\beta \geq 10$ , the failure is governed by linear elastic fracture mechanics, and nonlinear analysis is not necessary.

It has been shown that the foregoing definition of the brittleness number is independent of the specimen or structure shape. On the other hand, some other competing definitions of the brittleness number due to Hillerborg (1985) and Carpinteri (1982) are not independent of the specimen shape, and do not make it possible to compare, in terms of brittleness, specimens or structures of different shapes. For the definition of  $\beta$  according to Eq. 3, such comparison is made possible.

In the original definition of the brittleness number  $\beta$  according to Eq. 3,  $d_0$  is not a true material parameter. However, Bazant and Kazemi (1988) came up with a modified expression for  $\beta$  which is based on the size effect law according to Eq. 4. In this definition, the brittleness number of a specimen or structure may be calculated as:

$$\beta = \frac{d}{d_0} = \frac{g(\alpha_0)}{g'(\alpha_0)} \frac{d}{c_f} = \frac{D}{c_f} \quad (11)$$

This definition is easy for practical applications whenever the energy release rate at failure can be calculated. This can, of course, be done for fracture specimens. For brittle failures of concrete structures it is necessary to know the approximate shape and length of the cracking zone at failure (e.g., at failure of a beam in diagonal shear) and to approximate it by a perfect crack, for which the function  $g(\alpha)$  can then be obtained.

Acknowledgement - Partial financial support has been obtained under a collaborative project with Technical University, Madrid, sponsored under U.S. - Spain treaty (Grant CCA-830971), and (2) for the study of R-curve behavior also from NSF S & T Center for Advanced Cement-Based Materials at Northwestern University (NSF grant DMR-8808432).

## References

1. Z. P. Bazant, "Size Effect in Blunt Fracture: Concrete, Rock, Metals," *Journal of Engineering Mechanics*, ASCE, Vol. 110, No. 4 (1984), 518-535.
2. Z. P. Bazant, "Fracture Mechanics and Strain-Softening of Concrete," *Proceedings, U. S. - Japan Seminar on Finite Element Analysis of Reinforced Concrete Structures* (held in Tokyo, May, 1985), American Society of Civil Engineers, New York (1986), 121-150.
3. Z. P. Bazant, "Fracture Energy of Heterogenous Materials and Similitude," *Preprints, SEM/RILEM International Conference on Fracture of Concrete and Rock* (Houston, June 1987), Society for Experimental Mechanics (1987), 390-402.
4. Z. P. Bazant, R. Gettu, and M. T. Kazemi: "Prediction of Material Fracture Parameters and Structure Response from Size Effect," Report 89-3/498p, Center for Advanced Cement-Based Composites, Northwestern University, Evanston, IL, 1989.
5. Z. P. Bazant and M. T. Kazemi, "Determination of Fracture Energy, Process Zone Length and Brittleness Number from Size Effect, with Application to Rock and Concrete," Report 88-7/498d, Center for Concrete and Geomaterials, Northwestern University, Evanston, IL, July 1988; also *Intern. J. of Fracture*, in press.
6. Z. P. Bazant and P. A. Pfeiffer, "Determination of Fracture Energy from Size Effect and Brittleness Number," *ACI Materials Journal*, Vol. 84, No. 6 (1987), 463-480.
7. A. Carpinteri, "Notch Sensitivity in Fracture Testing of Aggregative Materials," *Engineering Fracture Mechanics*, Vol. 16, No. 4 (1982), 467-481.
8. A. Hillerborg, "The Theoretical Basis of a Method to Determine the Fracture Energy  $G_f$  of Concrete," *Materials and Structures, Research and Testing (RILEM, Paris)*, Vol. 18, No. 106, July-Aug. 1985, pp. 291-296.
9. J. F. Knott, *Fundamentals of Fracture Mechanics*, Butterworth, London, 1973, 273 pp.
10. J. F. Labuz, S. P. Shah and C. H. Dowding, "The Fracture Process Zone in Granite: Evidence and Effect", *Intern. J. of Mechanics, Mineral Science and Geomechanics Abstracts*, Vol. 24 (No. 4), 1987, 235-246.



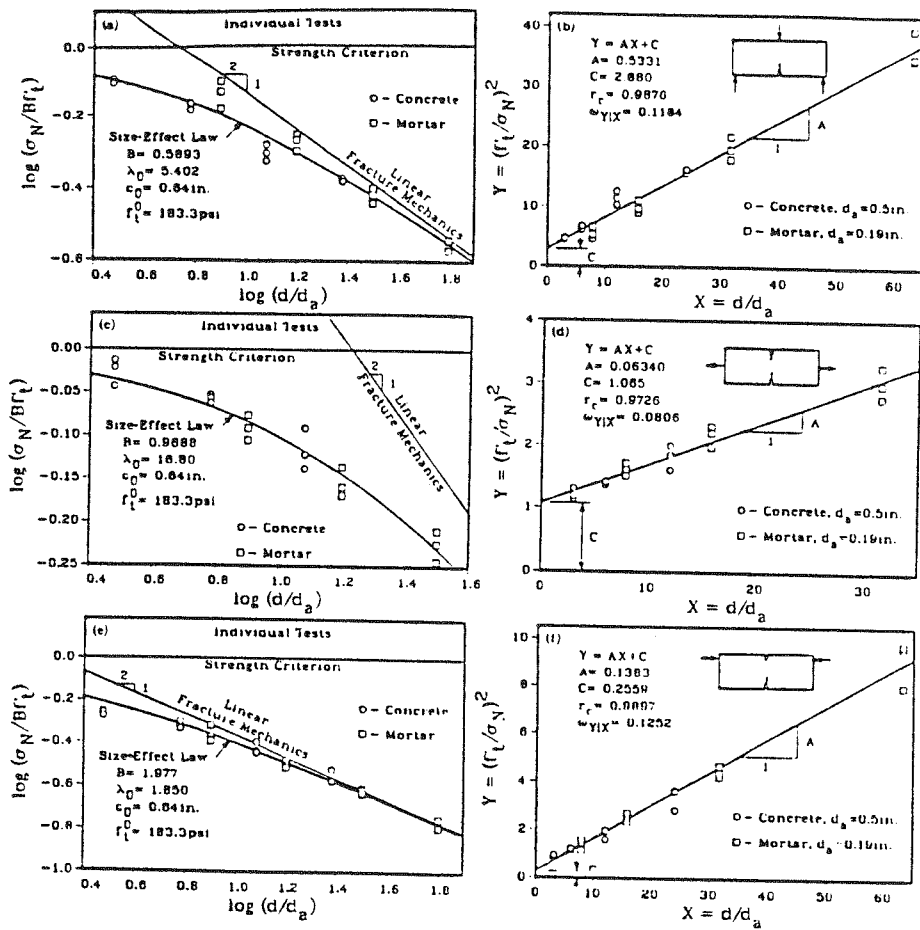


Fig. 1 Size Effect Law Compared with Fracture Test Results of Bazant and Pfeiffer (1987).

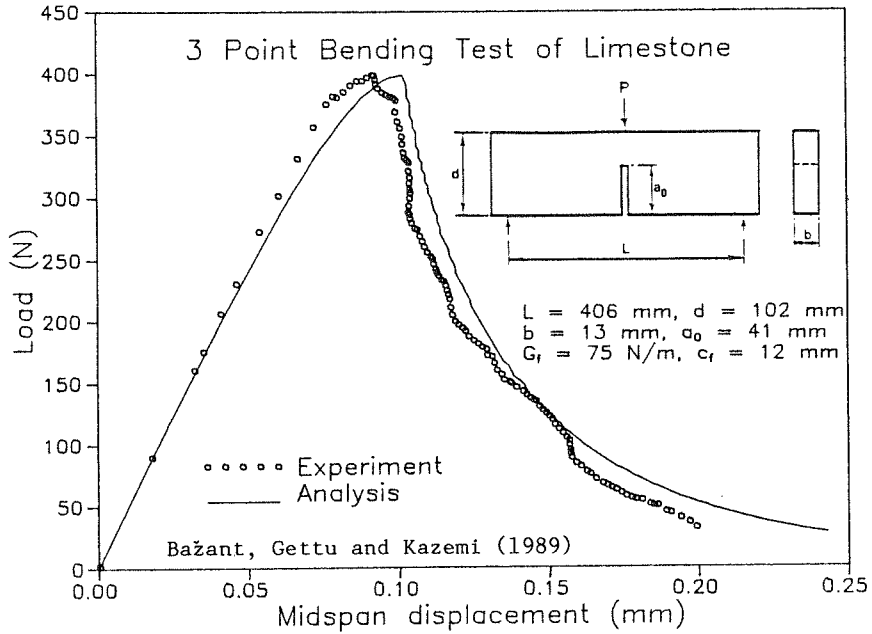


Fig. 2: Comparison of Structure Response Predicted by R-curve Analysis on the Basis of Rock Fracture Tests at Northwestern University

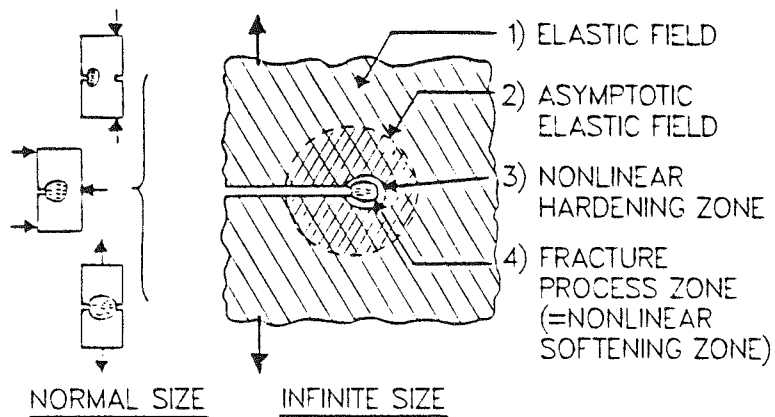


Fig. 3 Fracture Process Zone for Normal Size Laboratory Specimens and Its Surrounding Field for Extrapolation to an Infinitely Large Specimen.