Preprint, Engineering Foundation Conference on "Advances in Cement Manufacture and Use", held at Trout Lodge, Potosi, Missouri, August 1-5, 1988, Chairman E. Gartner, W. R. Grace and Co., Paper No. 5, pp. 5.1-5.8.

BRITTLENESS AND SIZE EFFECT IN CONCRETE STRUCTURES

by Z. P. Bažant and M. T. Kazemi
Center for Concrete and Geomaterials
Northwestern University
Evanston, Illinois 60208

Abstract

The existing results on the size effect law for the nominal stress at failure of concrete specimens on structures and its application for determination of fracture energy are reviewed and an extension of the previous formulations is presented. The extension consists in a modified form of the size effect law which involves only true material parameters, particularly the fracture energy and the length of the fracture process zone, which are both uniquely defined on the basis of the extrapolation of specimen size to infinity. This extension makes it possible to define the brittleness number of a structure in terms of an equivalent shape-independent structure size and the limiting length of the fracture process zone. The calculation of the brittleness number requires the value of the nondimensional energy release rate for the equivalent crack in an elastic structure, whose value and derivative take into account the shape of the structure.

Introduction

The fracture toughness of concrete, as well as other heterogeneous brittle materials such as rocks and various ceramics, is considerabaly enhanced by a toughening mechanism which consists in shielding of the crack tip by a nonlinear zone of distributed microcracking. The fracture energy or fracture toughness of such materials does not represent the sole material characteristic of fracture response. The size of the nonlinear fracture process zone is another important characteristic. The size of this zone is essentially, although not exclusively, a property of the material, since it is determined by the size of the inhomogeneities in the microstructure. If the size of the zone is negligible compared to structure dimensions, the response is close to linear elastic fracture mechanics. If the size of the zone encompasses most of the specimen or structure volume, the failure is determined by strength or yield criteria. If the size of the zone is inter-mediate, the response is transitional between the strength criterion and the linear elastic fracture mechanics. It is this transitional behavior which is of interest for most concrete structures.

The purpose of the present conference presentation is to briefly review the size effect in failure which is due to the existence of the fracture process zone with distributed microcracking, and to show that Bažant's (1984) size effect law can be used not only to determine the fracture energy of concrete, as demonstrated before (Bažant and Pfeiffer, 1987), but also to determine the limiting fracture process zone length as a second material parameter characterizing nonlinear fracture. The presentation will also include a modified new form of the size effect law in terms of two material parameters, and will then use these parameters to give a size- and shape-independent definition of the brittleness number of a concrete specimen or structure.

The scope of the conference presentation permits only summarizing the results. For detailed derivations and experimental verifications for concrete as well as rock, see a fresh report by Bažant and Kazemi (1988).

<u>Review of Size Effect Law and Its Use to Determine Fracture Energy</u>

The size effect is described in terms of the nominal stress at failure:

$$\sigma_N = c_n \frac{P_u}{bd} \qquad \text{for 2D similarity} \tag{1}$$

$$\sigma_N = c_n \frac{P_u}{d^2} \qquad \text{for 3D similarity} \tag{2}$$

in which P_u is the maximum load, b = specimen of structure thickness, d = chosen characteristic dimension of the specimen or structure, and c_n = a coefficient introduced for convenience. As shown by Bažant (1984), the nominal stress approximately follows the size effect law (Fig. 1, where d_a = aggregate size, $f'_t = f_u$ and $\lambda_0 = d_0/d_a$).

$$\sigma_N = B\, f_u\, (1 + \beta)^{-1/2}, \qquad \beta = d/d_0 \tag{3}$$

in which B and d_0 are empirical coefficients and f_u represents the material strength.

For $d >> d_0$, Eq. 3 gives the size effect of linear elastic fracture mechanics, for $d << d_0$, Eq. 3 gives no size effect, which is characteristic of the failures governed by strength or yield criteria, and for the intermediate range of d, Eq. 3 describes a transitional behavior corresponding to nonlinear fracture mechanics. Eq. 3 is applicable only for the size range of approximately 1:20. For a broader size range, further terms of an asymptotic series expansion need to be included in Eq. 3 (Bažant, 1985 and 1987).

Eq. 3 has the advantage that it can be transformed to a linear regression plot $Y = A X + C$, in which $X = d$, $Y = (f_u/\sigma_N)^2$, $B = C^{-1/2}$, $d_0 = C/A$.

The size effect law in Eq. 3 has received a wide range of justifications:

1. Some simple energy release solutions.
2. Dimensional analysis and similitude arguments.
3. Experimental results on fracture specimens as well as brittle failures of various concrete structures (e.g. Fig. 1).
4. Finite element results obtained by either blunt fracture models (crack band model, Hillerborg's fictitious crack model) or nonlocal damage models.
5. Random particle simulations of concrete (interface element model).
6. Micromechanics analysis showing that a nonlocal damage model is a proper homogenization of a quasiperiodic crack array.

The experimental justification of Eq. 3 at Northwestern University included:

1)Mode I fracture specimens: a) three-point bend specimens, b) edge-notched tension specimens, c) eccentric compression specimens and d) compact tension specimens.

2) Mode II specimens (approximately Mode II), with alternating loads at four points on a beam.

3) Mode III specimens: cylinders with a circumferential notch subjected to torsion.

The materials for which the size effect has been experimentally verified at Northwestern University included: 1) concrete, 2) mortar, 3) rocks of various types, 4) certain ceramics (SiC, SiO_2), and 5) aluminum alloy.

The size effect has further been experimentally verified at Northwestern University under a wide range of conditions, including:

1. Various temperatures, ranging from room temperature to 200°C.
2. Wet specimens and dried specimens.
3. Specimens subjected to various rates of loading, with times to peak load ranging as $1:10^5$ (ongoing work of R. Gettu at Northwestern University).
4. Monotonic as well as cyclic loads (work in progress by K. M. Xu at Northwestern University).

Eq. 3 has also been shown applicable to brittle failures of concrete structures. The reason is that concrete structures are not allowed by the codes to be designed so that they fail at the first crack initiation. Rather, the design must be such that a large cracking zone develops before the ultimate load is reached, and this cracking zone serves as a notch, causing that the structure during failure behaves essentially as a fracture specimen with a notch, the failure being significantly influenced by the rate of energy release and stress distributions due to further extensions of the cracking zone.

The applicability of the size effect law has been experimentally verified at Northwestern University for the following types of failures:

1. Diagonal shear failure of beams with longitudinal reinforcement: a) nonprestressed beams without stirrups and with stirrups, and b) prestressed beams.
2. Punching shear failure of slabs.
3. Torsional failure of concrete beams of rectangular cross section, without or with longitudinal reinforcement.
4. Pullout failure of reinforcing bars embedded in concrete.
5. Ring and beam failures of unreinforced concrete pipes.
6. Compression splitting failure, i.e., the Brazilian test (here the size effect law is found to apply only up to a certain size, beyond which the size effect disappears, apparently due to transition to some type of frictional mechanism or strength-controlled failure).

It may also be pointed out that size effects can also be mathematically explained by a probabilistic mode of Weibull-type, which has been very popular in the literature. However, it seems that this explanation is correct only for the failure of uniformly stressed tensile specimens without notches and is not applicable to the typical failures of concrete structures listed above. The existing statistical theories generally neglect the major stress redistributions which take place after the onset of the first cracking and before the attainment of the maximum load, and thus ignore the energy release aspects on the macroscale. Statistics can be included in the fracture analysis, however, if the Weibull parameters are calibrated from the test results for uniformly stressed tensile specimens and the same material parameter values are used for the zone in which the fracture front at failure can possibly be located in the concrete structure (e.g., in the diagonal shear failure of beam), then the statistical part of the size effect is found to be generally negligible (Bažant, 1987).

According to various existing methods of fracture energy measurement (Knott, 1973), its value has been found to be highly variable. However, a unique definition, which is independent of the specimen size as well as shape, can be based on the size effect law. The fracture energy can be uniquely defined as the value of the energy required for crack growth (per unit fracture area) in an infinitely large specimen (Bažant and Pfeiffer, 1987); Fig. 2. According to this definition, the fracture energy is found to be given by the formula:

$$G_f = \frac{f_u^2}{c_n^2 \, A \, E} \, g(\alpha_0) \qquad (4)$$

in which A = regression slope of Y vs. X as mentioned before, E = elastic modulus of concrete, and $g(\alpha_0)$ = nondimensional energy release rate of the specimen, in which $\alpha_0 = a_0/d$ = relative notch length, a_0 = notch length. It has been shown that the fracture energy values obtained on the basis of this formula from various types of fracture specimens give relatively constant results.

<u>Size Effect Law in Terms of Material Parameters and Brittless Number</u>

Coefficients B and d_0 in Eq. 3 are not material parameters and depend on the specimen shape. Bažant and Kazemi (1988), however, have shown that Eq. 3 can be reformulated in a manner which involves only true material parameters. Such a modified version of the size effect law can be written as:

$$\tau_N = \sqrt{\frac{EG_f}{c_f + D}} \qquad (5)$$

in which τ_N, called the shape-independent nominal strength, is defined as:

$$\tau_N = \sqrt{g'(\alpha_0)} \, \frac{P_u}{bd} \qquad \text{for 2D similarity} \qquad (6)$$

$$\tau_N = \sqrt{g'(\alpha_0)} \, \frac{P_u}{d^2} \qquad \text{for 3D similarity} \qquad (7)$$

and D represents the shape-independent characteristic dimension of the structure, defined as

$$D = \frac{g(\alpha_0)}{g'(\alpha_0)} d \tag{8}$$

Here $g'(\alpha_0) = dg(\alpha)/d\alpha$ evaluated at $\alpha = \alpha_0$. It can be shown that within a size range of up to about 1:20, in which the approximate size effect law (Eq. 3) is applicable, the values of $g(\alpha_0)$ and $g'(\alpha_0)$ sufficiently take into account the shape of the structure.

As proposed by Bažant (1987), see also Bazant and Pfeiffer (1987), the nature of the specimen or structure response at failure can be characterized by the brittleness number, β, as already introduced in Eq. 3. Depending on the value of the brittleness number, three different regimes may be distinguished:

1. For $\beta < 0.1$, the failure is governed by strength or yield criteria, and fracture mechanics need not be used.

2. For $0.1 \le \beta \le 10$, the failure is governed by nonlinear fracture mechanics, and the finite size of the fracture process zone must be taken into account.

3. For $\beta \ge 10$, the failure is governed by linear elastic fracture mechanics, and nonlinear analysis is not necessary.

It has been shown that the foregoing definition of the brittleness number is independent of the specimen or structure geometrical shape. On the other hand, some other competing definitions of the brittleness number due to Hillerborg (1985) and Carpinteri (1982) are not independent of the specimen shape, and do not make it possible to compare, in terms of brittleness, specimens of structures of different shapes. For the definition of β according to Eq. 3, such comparison is made possible.

In the original definition of the brittleness number β according to Eq. 3, d_0 is not a true material parameter. However, Bažant and Kazemi (1988) came up with a modified expression for β which is based on the size effect law according to Eq. 5. In this definition, the brittleness number of a specimen or structure may be calculated as:

$$\beta = \frac{d}{d_0} = \frac{g(\alpha_0)}{g'(\alpha_0)} \frac{d}{c_f} = \frac{D}{c_f} \tag{9}$$

This definition is easy for practical applications whenever the energy release rate at failure can be calculated. This can of course be done for fracture specimens. For brittle failures of concrete structures it is necessary to know the approximate shape and length of the cracking zone at failure, e.g., at failure of a beam in diagonal shear, and to approximate it by a perfect crack, for which then the function $g(\alpha)$ can be obtained.

Conclusion

Numerous studies which have recently been devoted to the size effect law in concrete structures and its application for determination of material fracture parameters indicate a broad experimental support and varied applications. As a novel result, a modified form of the size effect law which involves only true material parameters has been shown. These material parametes include not only the fracture energy, but also the length of the fracture process zone, both applicable for the extrapolation to an infinite specimen size. This modified form of size effect law makes it possible to redefine the brittleness number of a structure in terms of true material parameters and the nondimensionl energy release rate which takes into account the structure shape.

Acknowledgment. - Partial financial support under AFOSR Contract No. F49620-87-C-0030DEF with Northwestern University, monitored by Dr. Spencer T. Wu, is gratefully acknowledged. Some additional financial support has also been obtained for the studies of the temperature effect on fracture energy under a collaborative project with Technical Univeristy, Madrid (Prof. M. Elices) sponsored under U.S.-Spain treaty (Grant CCA-830971).

References

1. Z. P. Bažant, "Size Effect in Blunt Fracture: Concrete, Rock, Metal," Journal of Engineering Mechanics, ASCE, Vol. 110, No. 4, (1984), 518-535.

2. Z. P. Bažant, "Fracture Mechanics and Strain-Softening of Concrete," Proceedings, U. S.-Japan Seminar on Finite Element Analysis of Reinforced Concrete Structures, (held in Tokyo, May, 1985), American Society of Civil Engineers, New York, (1986), 121-150.

3. Z. P. Bažant, "Fracture Energy of Heterogeneous Materials and Similitude," Preprints, SEM/RILEM International Conference on Fracture of Concrete and Rock (Houston, June 1987), Society for Experimental Mechanics, (1987), 390-402.

4. Z. P. Bažant and M. T. Kazemi, "Determination of Fracture Energy, Process Zone Length and Brittleness Number from Size Effect, with Application to Rock and Concrete," Report 88-7/498d, Center for Concrete and Geomaterials, Northwestern University, Evanston, IL, July 1988; submitted to Intern. J. of Fracure.

5. Z. P. Bažant and P. A. Pfeiffer, "Determination of Fracture Energy from Size Effect and Brittleness Number," ACI Materials Journal, Vol. 84, No. 6, (1987), 463-480.

6. A. Carpinteri, "Notch Sensitivity in Fracture Testing of Aggregative Materials," Engineering Fracture Mechanics, Vol. 16, No. 4 (1982), 467-481.

7. A. Hillerborg, "The Theoretical Basis of a Method to Determine the Fracture Energy G_f of Concrete," Materials and Structures, Research and Testing (RILEM, Paris), Vol. 18, No. 106, July-Aug. 1985, pp. 291-296.

8. J. F. Knott, Fundamentals of Fracture mechanics, Butterworth, London, 1973, 273 pp.

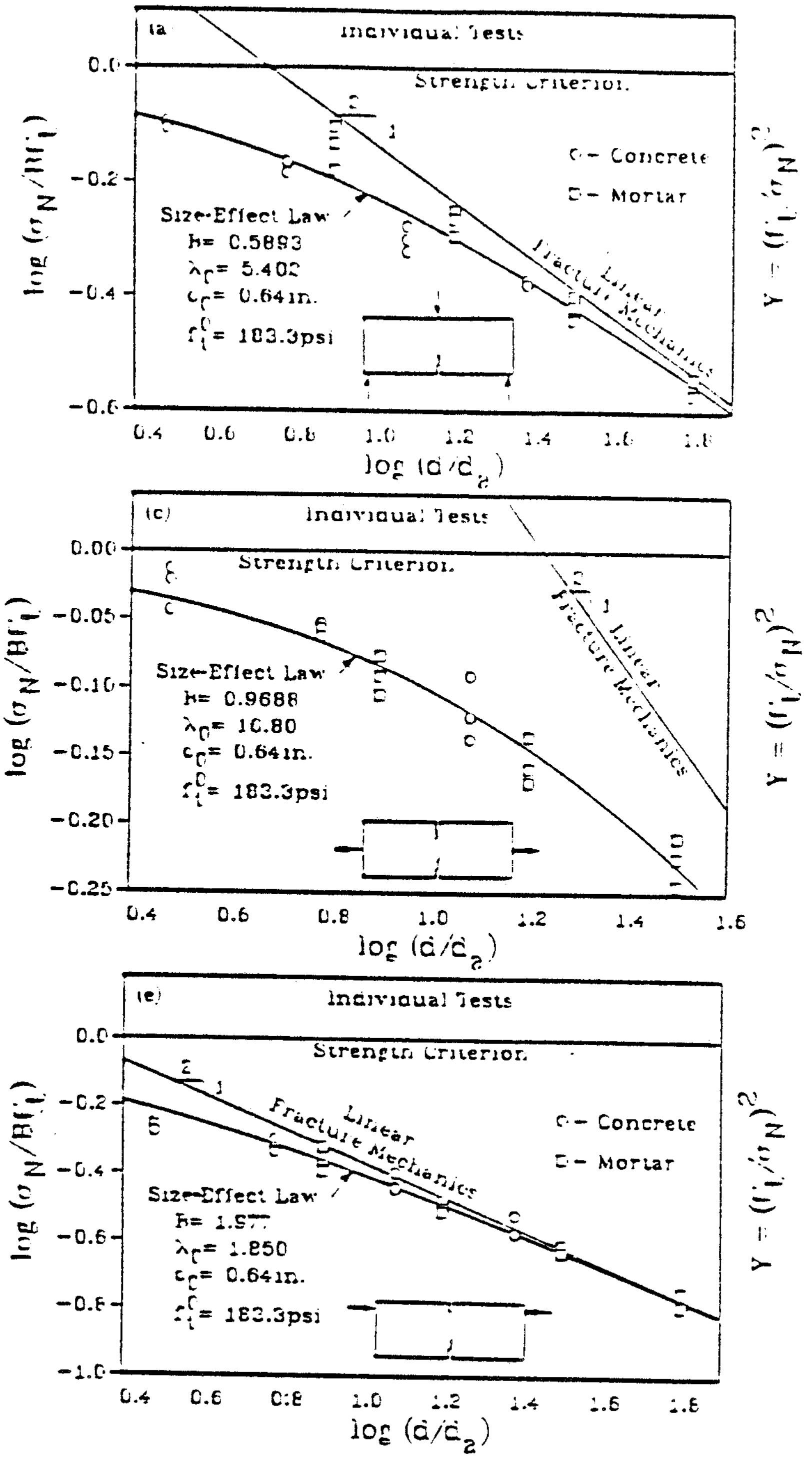

Fig. 1 Size Effect Law Compared with Fracture Test Results of Bažant and Pfeiffer (1987).

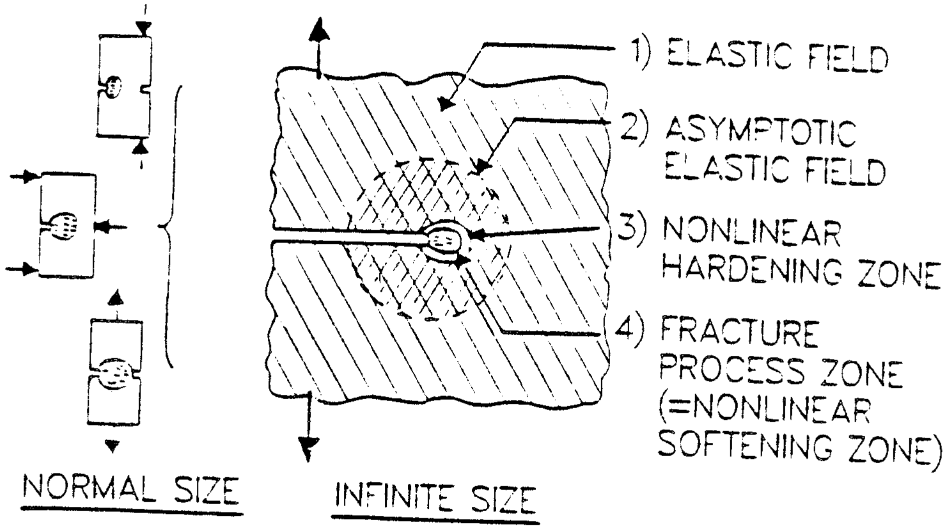

Fig. 2 Fracture Process Zone for Normal Size Laboratory Specimens and Its Surrounding Field for Extrapolation to an Infinitely Large Specimen.