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Creep and Shrinkage Revisited. Paper by N. J. Gardner and J. W. Zhao

Discussion by Zdeňek P. Bažant and Sandeep Baweja and Authors

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The model proposed by Gardner and Zhao is interesting. But a careful scrutiny reveals that it possesses some theoretically questionable and experimentally unjustified features. They make it inferior to several other existing models and thus unsuitable for practical application.

SHRINKAGE

Lack of final asymptotic value of shrinkage

The author cited the lack of sufficient.experimental evidence as a reason for proposing Eq. (3) in which the shrinkage strain grows without bounds, approaching no final asymptotic value. However, if one considers not only the shrinkage data on large concrete specimens, but also the data on small specimens of mortar and cement paste that shrink faster, it is clear from the experimental evidence alone that a finite shrinkage value does exist in cement past and mortar. This, of course, implies it must exist in concrete, because the shrinkage of cement paste and mortar is the driving force for the shrinkage of concrete as a whole. Anyway, how could one explain unbounded shrinkage of concrete if many tests clearly show the shrinkage of thin cement paste and mortar specimens to be bounded? The impossibility of unbounded final shrinkage can be proven even without the foregoing details. In a specimen exposed to a perfectly dry environment (R.H. = 0 percent), all the processes responsible for drying shrinkage cease after all the evaporable (not chemically bound) water has escaped from the specimen. Therefore, the specimen cannot shrink any further, and so its final shrinkage is bounded. A specimen exposed to some higher relative humidity cannot shrink more than at R.H. = 0 percent. So its shrinkage must be bounded, too.

Besides, there are some data even for concrete that indicate an approach to the final asymptotic value; for instance, the latest readings reported in Bažant, Kim, and Panula (1991) for the tests in Ref. 15 in the paper.

Furthermore, the existing and generally accepted theory of the mechanism of shrinkage requires a finite shrinkage value $\varepsilon_{sh\infty}$ to exist (see, e.g., the state-of-the-art-review in Chapter 1 of *Mathematical Modeling of Creep and Shrinkage of Concrete*[34]). It is generally accepted that shrinkage is caused by increases of capillary tension p_c of liquid water and of surface tension π_s of solids or adsorbed water films covering the solids. The resultants of p_c and π_s in any cross section of the porous material must be balanced by the stress σ_s in the elastic solid particles within the material, and shrinkage strain ε_{sh} is the result of the elastic compression of these particles caused by σ_s. The thermodynamic equilibrium values of p_c and π_s at any temperature are uniquely functionally related to the pore vapor pressure p_v (according to the well-known Maxwell and Kelvin equations); and p_v is, in turn, uniquely functionally related at each age to the specific content of evaporable water w_e, as described by the desorption isotherm (of course, compared to the decrease of environmental humidity h, the shrinkage occurs with a delay because diffusion of water out of the material takes a certain time). Therefore, this functionally determined value of ε_{sh} is the final shrinkage value $\varepsilon_{sh\infty}$. The finiteness of the loss Δw_c of water content of concrete implies the finiteness of $\Delta\sigma_s$, which implies the finiteness of $\varepsilon_{sh\infty}$. Therefore, assuming $\varepsilon_{sh\infty}$ to be unbounded is incorrect.

The impossibility of unbounded final shrinkage can be proven even without the foregoing arguments. In a specimen exposed to a perfectly dry environment (R.H. = 0 percent), all the physicochemical processes responsible for drying shrinkage cease after all the evaporable (not chemically combined) water has escaped from the specimen. Therefore, the specimen cannot shrink any further, and so its final shrinkage is bounded. A specimen exposed to some higher relative humidity cannot shrink more than that at R.H. = 0 percent. So its shrinkage, too, must be bounded.

The diffusion theory further shows that the approach to the final shrinkage value should be an exponential curve. Some tests showing that this is not contradicted exist.

Disagreement of initial shrinkage curve with diffusion theory

If we consider a sudden exposure to constant environment, and if we also assume the delay due to the finiteness of the surface moisture transmission coefficient to be negligible (which is true except for very thin specimens), the initial shape of the shrinkage curve $\varepsilon_{sh}(t)$ must be proportional to $\sqrt{\hat{t}}$ where $\hat{t} = t - t_c$ = drying duration. Moreover, when similar specimens of different thicknesses are considered, the initial curves must be

$$\varepsilon_{sh}(t) = \sqrt{\tau} \times \text{constant} \quad \tau = \hat{t}/D^2 \qquad (A1)$$

This property has been rigorously proven by solutions according to the diffusion theory and is valid even if the nonlinearity of diffusion equation as well as the age dependence is taken into account.[30] As for experimental evidence, many shrinkage tests found in the literature confirm this fundamental property. There also exist test data in the literature which do not conform to this property, but closer scrutiny shows that this is because the proper test conditions were not adequately satisfied. Often the measurements started only some time after the start of drying, which means that a cer-

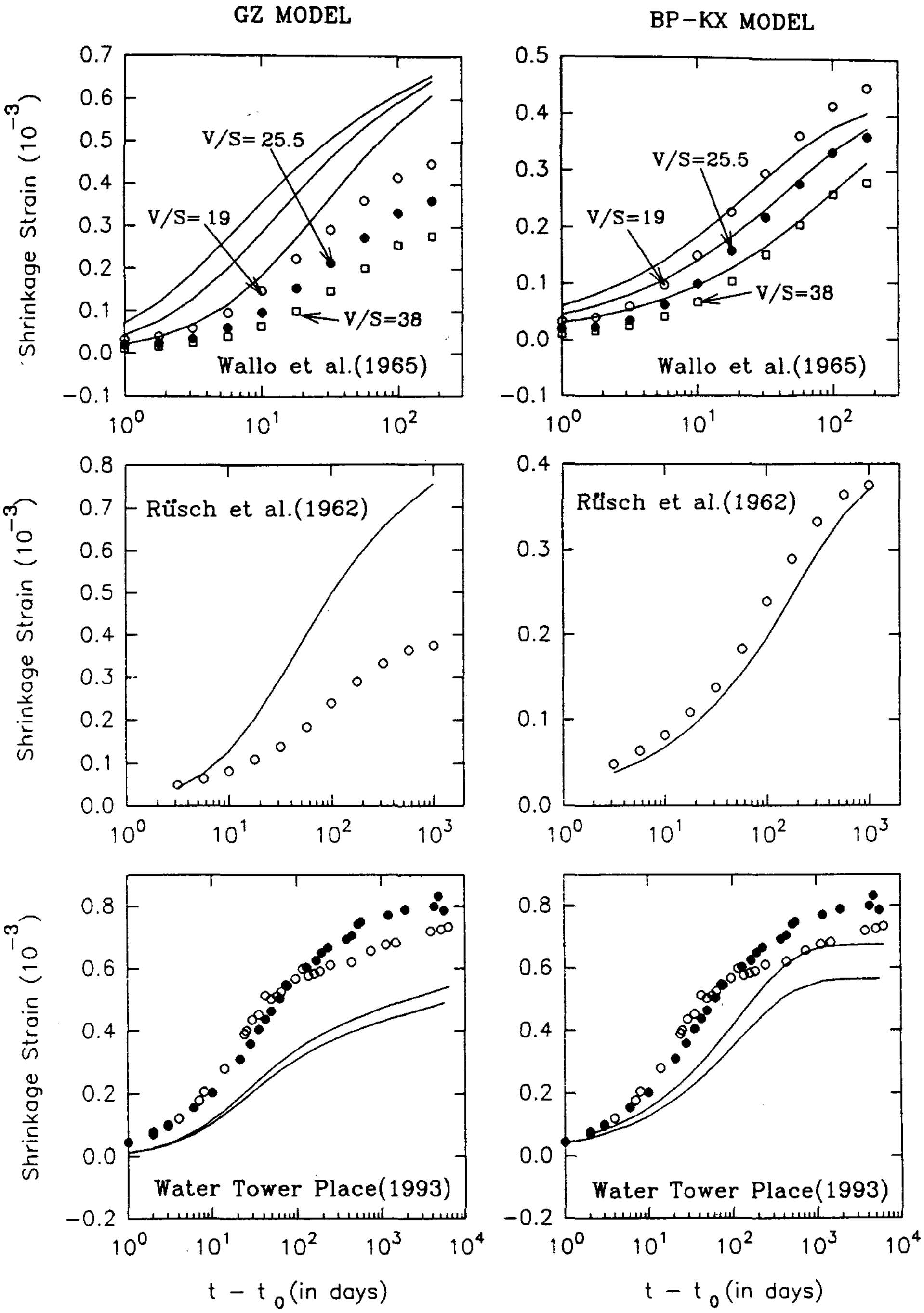

Fig. A — Some important shrinkage test data compared to the predictions by Gardner and Zhao's model (left) and BP-KX model (right)[35]

tain initial shrinkage strain has been missed, and often the seals leaked moisture before exposure to drying environment, which means that the shrinkage actually started earlier. However, when the proper test conditions are carefully implemented (as in the tests of Bazant, Wittman, Kim, and Alou,[15] the initial property $\varepsilon_{sh} = \sqrt{\tau} \times$ constant is satisfied very well, as far as the inevitable scatter of measurements permits it to say. But the authors' Eq. (3) violates this property.

Negative initial shrinkage values

Eq. (3) yields negative shrinkage values for drying durations $\hat{t} < 1$ min. This is incorrect and thermodynamically inadmissible. Although times under 1 min. are of no practical interest to engineers, violation of the basic principles nevertheless indicates that the entire formulation is theoretically unfounded and questionable. Besides, a formula that is free from this problem exists and is not more complicated.

Insufficient comparisons with experimental evidence

The comparisons with shrinkage test data from the literature that were presented by the authors were very limited. Fig. A shows comparisons with several important and widely used data sets. Obviously, there are serious discrepancies. (The figure also shows the curves predicted by another recent model — the BP-KX model.[31]

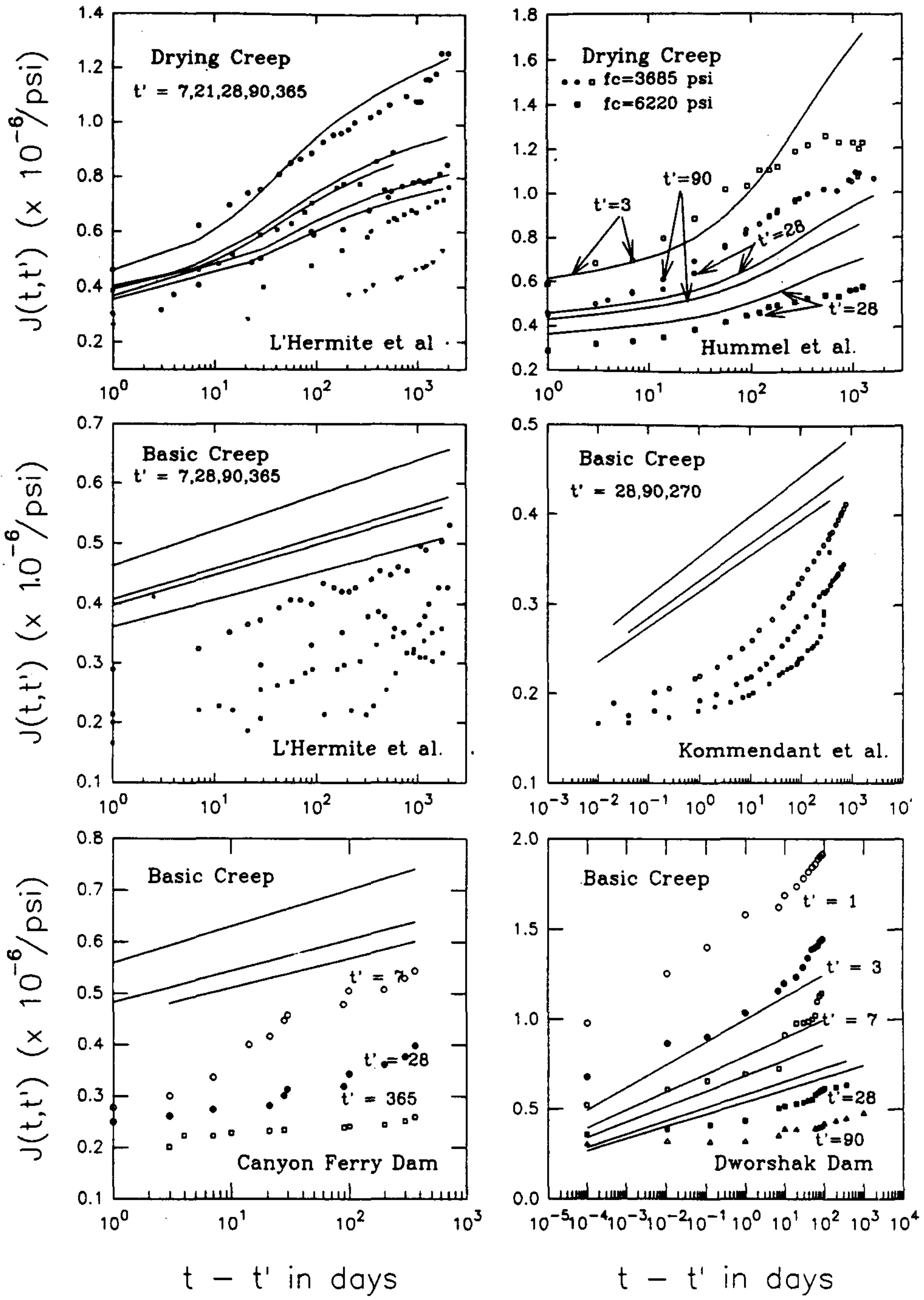

Fig. B(a) — Shape of creep curves and effect of age t′ at loading (in days) on creep as given by Gardner and Zhao's formulas, in comparison with some important test data from various laboratories

CREEP

Lack of age effect on basic and drying creep

Eq. (4) for the creep coefficient ϕ describes the effect of the age at loading t_0 by means of the concrete strength f'_{cmt0} at age t_0. But f'_{cmt0} is involved in such a form that there is no age effect in the case of basic creep, that is, creep of sealed specimens. The only effect of the age at loading on the basic creep compliance function as given by the author's formula is in the elastic part of the compliance because the elastic modulus as defined by Eq. (1) and (2) is dependent on age at loading. The basic creep coefficient, obtained by putting $h = 1$ Eq. (4), is independent of age at loading. This is theoretically incorrect, because the hydration reactions, which cause the age effect on creep, take place in both drying and sealed specimens. In fact, they proceed faster in the latter. The values of the basic creep coefficient given by the author's formula are the same for all ages at loading. The lack of age effect also leads to serious and unacceptable disagreements with test data, some of which are demonstrated in Fig. B(a) through (c) (this figure also shows, for comparison, the curves for the BP-KX model and the existing ACI 209 recommendation[1].

Some engineers might think that the case of basic creep is not important because in most applications concrete structures are exposed to drying environment. Not so, however.

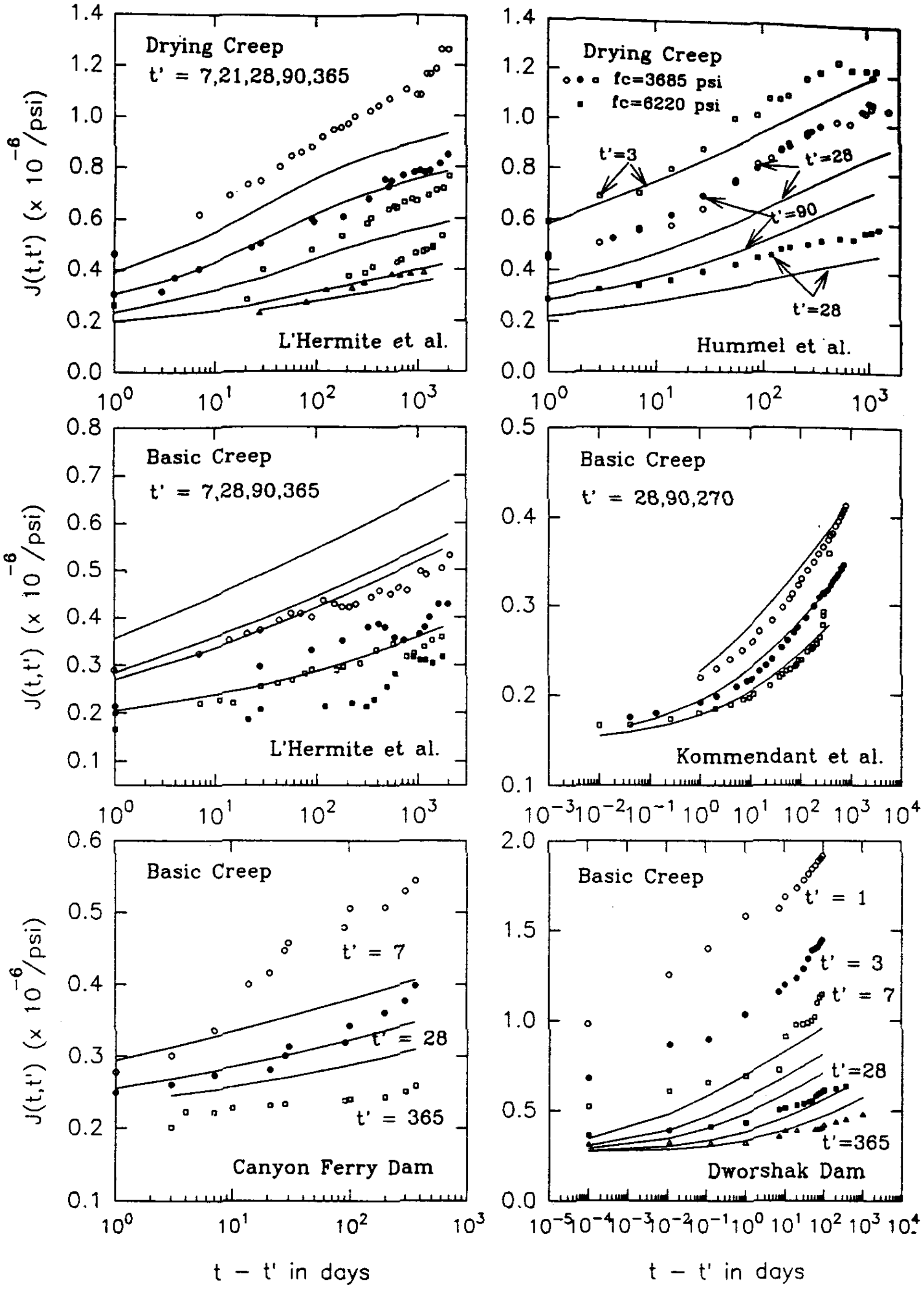

Fig. B(b) — Shape of creep curves and effect of age t′ at loading (in days) on creep as given by BP-KX formulas,[35] in comparison to some important test data from various laboratories

The core of thick members dries so slowly that their creep is closer to the creep of sealed specimens than to the creep of standard 6-in. or 3-in. cylinders in drying environment. For this reason, the formulas for design must agree with the basic creep data well.

Impossibility of characterizing the age effect by strength gain

For creep at drying, the authors' model does exhibit the age effect on creep. But this effect is far too weak and too short lived, as is clear from the examples of comparisons with the test data in Fig. B. For higher ages at loading, the age effect disappears, which contradicts all the relevant test data.

The fact that the effect of age t_0 at loading on the subsequent creep is characterized by means of the strength gain is itself a source of serious error. The authors ignore a large body of experimental evidence that has shown the age effect on creep to be very significant even long after the strength gain due to hydration has terminated, and the volume fraction of hydration products has ceased to grow. This fact, which was at first surprising, has been extensively discussed (see, e.g., Chapter 1 in Reference 34), and has led the research community to conclude that the age effect on creep, in principle, cannot be related to the gain of strength with age, except at early ages. This is contrary to what the authors assumed.

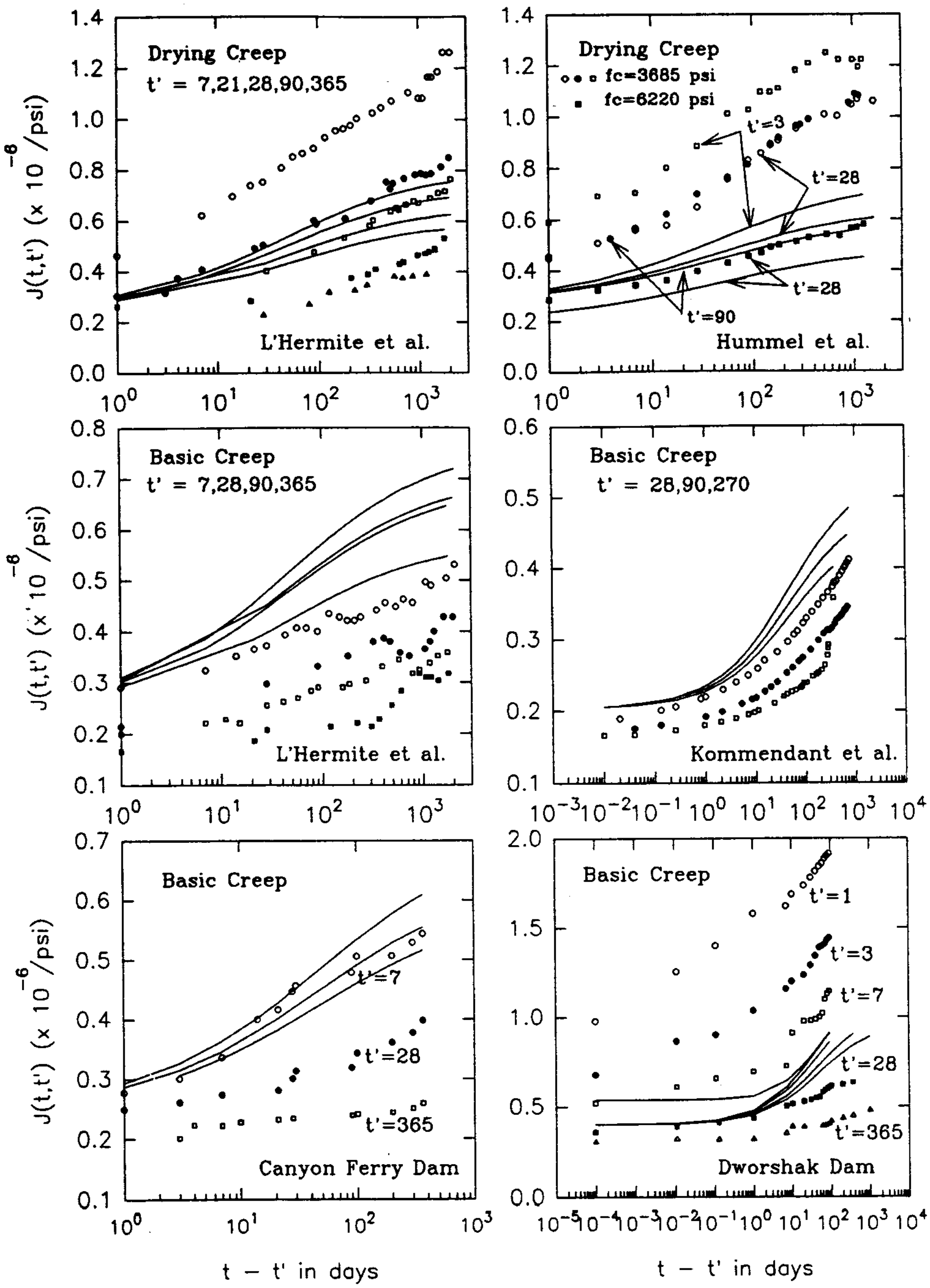

Fig. B(c) — Shape of creep curves and effect of age t′ at loading (in days) on creep as given by ACI 209 formulas,[1] in comparison to some important test data from various laboratories

It may be interesting to add that the previously mentioned experimental fact forced the research community to conclude that the age effect on creep must be caused, at least partly, by some changes in the bond structure of calcium silicate hydrates that are not associated with volume growth of the hydration products.

Lack of final value of drying creep

The creep coefficient for the additional creep due to drying is obtained by writing Eq. (3) for given h and for $h = 1$ and subtracting the equations. In the authors' model, this yields an unbounded drying creep curve that approaches a logarithmic curve for long times. But according to what is known about the mechanism, the drying creep must have a finite asymptotic value. The reason, simply, is that the additional creep due to drying is caused by water loss, and the water loss is bounded. The detailed explanation is similar to that already described for shrinkage. Furthermore, the data on specimens so thin that they can dry up completely before the moment of loading also clearly show that there is no drying creep after the termination of drying (Reference 32).

Nondivergence and nonmonotonic recovery

Fig. C shows an example of applying principle of superposition to the authors' model to predict creep recovery (age at loading = 14 days, age at unloading = 90 days). The recovery curve obtained is not monotonic but exhibits a reversal. This is an objectionable feature. It is a consequence of the fact that the compliance function $J(t, t_0) = [1 + \phi(t, t_0)]/E(t_0)$ implied by Eq. (3) violates the condition that always $\partial^2 J(t, t_0)/\partial t \partial t_0 \geq 0$, called the nondivergence condition. The violation means that there exists a time interval in which the slopes $\partial J/\partial t$ of two creep curves for adjacent ages t_0 diverge apart. This then inevitably leads to nonmonotonic creep recovery after unloading. The divergence was extensively discussed in the research community between 1975 and 1985 (see Chapter 2 of Reference 34), and it was concluded that, although the divergence is not prohibited by thermodynamics, it is not supported by test data and ought to be avoided in formulating $J(t, t_0)$ or $\phi(t, t_0)$.

Unsuitability for computer analysis of structures

In this age of computers, it is important that the proposed model be applied easily in computer analysis of creep effects in structures. In this respect, it is a major advantage if $J(t, t_0)$ can be converted easily into a rate-type constitutive relation based on the Maxwell or Kelvin chain (see the principles stated in Reference 33, and the conclusions in Reference 34). The BP-KX model has been formulated in a manner that makes this easy — explicit formulas exist to accomplish this conversion. The authors' model requires cumbersome, nonlinear fitting of $J(t, t_0)$ by Dirichlet series.

Unsuitability of defining creep by creep coefficient

The use of creep coefficient ϕ, Eq. (3) is unsuitable. As RILEM Committee TC-69[32] and TC-107[33] recommend, creep should be characterized in codes by the compliance function $J(t, t_0)$ rather than the creep coefficient $\phi(t, t_0)$, but the latter is adopted by the authors. The use of $J(t, t_0)$ prevents the user from incorrectly combining the creep coefficient value with a noncorresponding value of elastic modulus E. This is a frequent source of error in practice; for example, the E-value defined by ACI is rather different from that which corresponds to the initial stain in creep tests.

It is true that for structural analysis it is often more convenient to use $\phi(t, t_0)$. But the designer can always calculate $\phi(t, t_0) = EJ(t, t_0) - 1$ using any reasonable definition of E. Different values of E and ϕ can be used, but the calculations yield about the same creep effects in structures for $t - t_0 \geq 1$ day as long as the $J(t, t_0)$-values for the different E-values are the same.

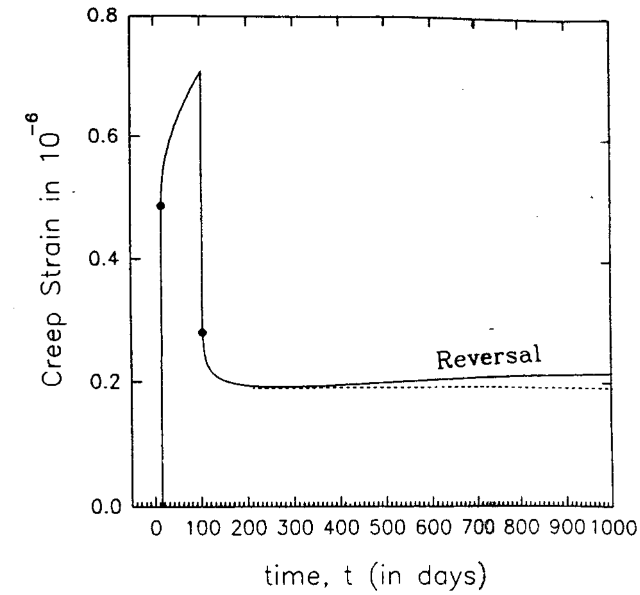

Fig. C — Example of nonmonototic creep recovery when principle of superposition is applied to Gardner and Zhao's model

Table A — Coefficient of variation of the deviations of test data points from the values of shrinkage strain predicted by various methods

Shrinkage				
	BP-KX	GZ	ACI	CEB
Test data	$\bar{\omega}$	$\bar{\omega}$	$\bar{\omega}$	$\bar{\omega}$
Hummel et al.	30.1	86.4	30.0	58.7
Rüsch et al.[1]	31.6	101.9	35.2	44.8
Wesche et al.	33.4	31.4	24.0	36.1
Rüsch et al.[2]	39.9	64.4	13.7	27.8
Wischers and Dahms	28.6	48.8	27.3	35.9
Hansen and Mattock	25.5	29.8	52.9	81.5
Keeton	56.5	76.4	120.6	48.3
Troxell et al.	73.5	25.5	36.8	47.4
Aschl and Stökl	53.9	89.4	61.3	44.2
Stökl	35.1	96.3	19.5	29.6
L'Hermite et al.	88.6	140.7	123.1	69.4
York et al.	33.0	44.9	42.8	8.9
Hilsdorf	27.7	43.6	24.7	29.6
L'Hermite and Mamillan	58.4	28.3	58.7	45.5
Wallo et al.	19.7	76.4	33.0	55.6
Lambotte and Mommens	37.3	44.9	30.7	31.3
Weigler and Karl	34.1	68.3	29.6	21.3
Wittman et al.	17.5	27.0	65.4	40.0
Ngab et al.	12.1	44.8	45.3	64.6
McDonald	14.3	78.4	68.8	21.4
Russel and Burg (Water Tower Place)	30.9	48.7	51.0	58.1
$\bar{\omega}_{all}$	41.7	68.4	55.3	46.3

STATISTICAL COMPARISON TO EXISTING DATA

High coefficients of variation of errors

No matter how simple a creep or shrinkage model for a code is intended to be, it should be statistically compared to all the relevant test data that exist in the literature. Their number is vast, but in the age of computers their statistical analysis is no longer difficult. The existing data have been organized in a data bank by Bažant and Panula[7] which was improved and extended during 1982 to 1988 by a joint ACI-CEB task committee headed by L. Panula and H. Müller, and more recently by an ACI 209 subcommittee headed by Bažant and a RILEM TC-107 subcommittee headed by H. Müller. It is not clear why the authors have not used this data bank, made available to ACI Committee 209.

Computer comparisons of the author's model with all the relevant test data in the existing data bank have now been run by the writers. The coefficients of variation ω_i of the deviations Δ_{ij} of data points $j = 1,2,3...$ of various data sets numbered as $i = 1,2,...$ from the prediction formulas have been calculated for each data set, as well as for all the data sets combined. The results are listed in the columns labeled GZ in Tables A (for shrinkage) and B (for creep). For compar-

Table B(a) — Coefficient of variation of the deviations of test data points from the values of creep compliance function predicted by various models

Basic creep					
	BP-KX		GZ	ACI	CEB
Test data	$\bar{\omega}$	$\tilde{\omega}$	$\bar{\omega}$	$\bar{\omega}$	$\bar{\omega}$
Keeton	24.0	15.0	29.8	37.5	42.8
Kommendant et al	5.1	7.0	54.3	31.8	8.1
L'Hermite et al.	48.3	52.8	143.1	133.4	66.2
Rostasy et al.	9.1	11.0	37.6	47.6	5.0
Troxell et al.	9.3	16.0	86.1	13.9	6.2
York et al.	11.7	12.2	63.3	37.7	12.8
McDonald	23.0	24.1	73.8	48.4	22.2
Maity and Meyers	16.5	25.7	65.4	30.0	15.7
Mossiossian and Gamble	18.0	17.0	38.9	51.5	47.3
Hansen et al. (Ross dam)	17.5	34.6	58.1	51.2	31.1
Browne et al. (Wylfa vessel)	32.0	31.2	46.4	47.3	53.3
Hansen et al. (Shasta dam)	27.9	18.4	150.2	107.8	43.1
Brooks and Wainwright	8.7	20.0	16.3	14.9	15.4
Pirtz (Dworshak dam)	26.7	37.1	40.2	58.2	32.5
Hansen et al. (Canyon ferry dam)	31.6	24.1	54.9	70.2	56.9
Russel and Burg (Water tower place)	17.4	15.5	26.5	19.3	31.5
$\bar{\omega}_{all}$ or $\tilde{\omega}_{all}$	23.1	25.3	71.2	57.8	36.0

Table B(b) — Coefficient of variation of the deviations of test data points from the values of creep compliance function predicted by various models

Drying creep					
	BP-KX		GZ	ACI	CEB
Test data	$\bar{\omega}$	$\tilde{\omega}$	$\bar{\omega}$	$\bar{\omega}$	$\bar{\omega}$
Hansen and Mattock	65.6	99.8	13.5	32.1	19.1
Keeton	18.4	10.3	28.1	46.3	37.9
Troxell et al.	8.4	10.9	10.5	33.0	7.9
L'Hermite et al	17.9	11.3	58.9	55.8	43.9
Rostasy et al.	16.0	21.1	41.3	20.9	14.8
York et al.	31.0	48.9	72.4	42.1	45.1
McDonald	37.0	50.5	68.3	40.4	38.9
Hummel	25.1	30.5	27.6	46.2	24.6
L'Hermite and Mamillan	37.7	27.3	57.9	62.5	45.6
Mossiossian and Gamble	8.0	9.1	36.7	71.7	30.8
Maity and Meyers	8.5	64.5	68.3	45.9	83.7
Russell and Burg (Water tower place)	18.7	16.2	20.3	41.2	19.1
ω_{all} or $\tilde{\omega}_{all}$	29.1	42.7	47.2	46.8	39.1

$\tilde{\omega}$ = Part 6; $\bar{\omega}$ = Part 7

ison, the ω_i values are also listed for the recently proposed BP-KX model,[31] the simplified BP-KX model,[35] the ACI 209 model[1], and the new CEB-FIP model.[2] As we see, all these models, including the current ACI 209 model[1] (which is based on the work of Branson and coworkers completed around 1965), are overall superior to the model presented in the paper. The method of calculation of ω_i was the same as that described in Bažant and Panula (Reference 7, Part VI).

Effect of selective use of existing test data on statistics

The comparisons with test data in the paper included only 10 data sets, compared to 41 data sets in the literature (and in the data bank). In this regard, it must be emphasized that selective use of tests has been shown to be misleading. For example, by selecting 8 data sets out of 12 available data sets for shrinkage available to Bažant and Panula,[36] ω could be reduced from 52 to 20 percent; or by selecting 8 out of 25 data sets for creep, ω could be reduced from 23 to 9 percent. If a selection must be made, it should be made randomly (e.g., by casting dice).

Furthermore, it appears that only some of the originally reported data points have been used in comparisons with the proposed model. Although L'Hermite, Mamillan, and Le Fèvre[4] published data for creep durations $t - t_0$ ranging from 1 day to 2500 days, the authors apparently considered only $t - t_0 = 2500$ days. Similar limited selections of data points have been made from Hansen and Mattock,[14] and from Bažant et al.[15] It is not clear why the authors omitted the important data of Rostasy et al. (basic creep), Rüsch et al. (shrinkage), and Russel and Corley[20] (shrinkage and creep; Water Tower Place) (even though these data sets satisfy the authors' crite-

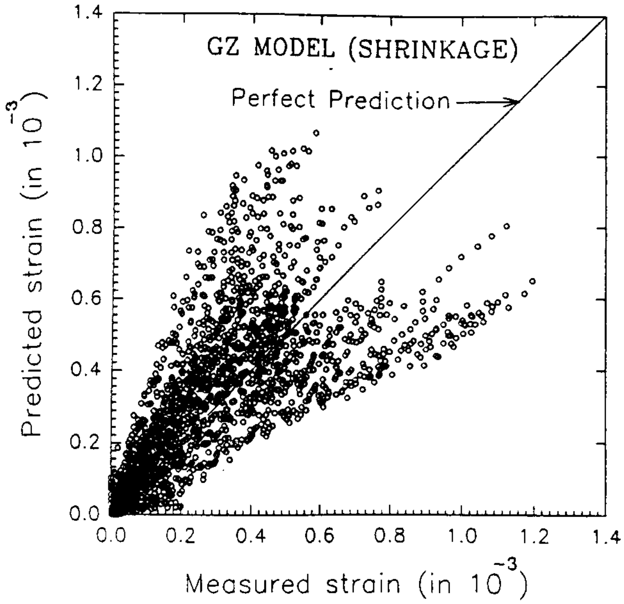

Fig. D(a) — Comparison of calculated and measured shrinkage strains (all data points from the ACI-CEB data bank included)

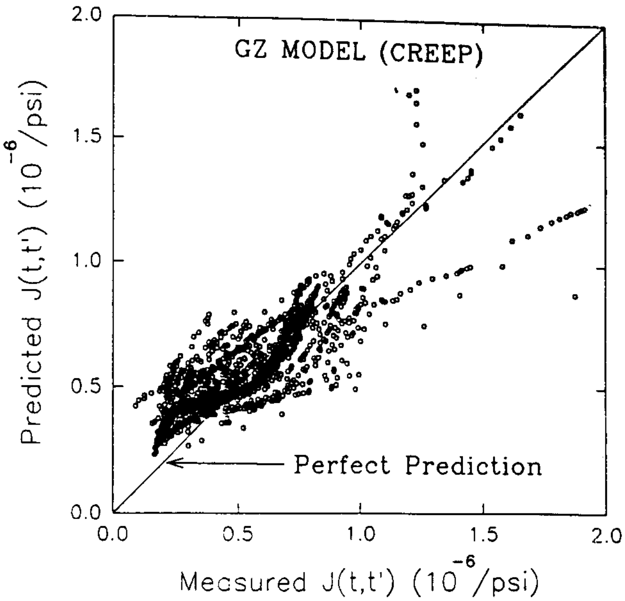

Fig. D(b) — Comparison of calculated and measured creep compliance function (all data points from the ACI-CEB data bank included)

rion of lasting three years or more). If the missing data were also considered, the graphical comparisons of the data to the authors' model would look considerably worse.

Use of one-variable statistics

The authors tried to justify their model by showing the plots of calculated-versus-measured values Y_m (Fig. 6, 8, and 9), graphically representing the scatter of one random variable Y_c/Y_m. Such plots are less useful than multivariable regressions because they hide incorrect trends with respect to some variables of the model. They also hide improper weighting; for example, the errors for higher strength concretes appear in such plots to be less because the creep of high-strength concrete is smaller, but it is the higher strength concretes that are of main interest.

Fig. D(a) and (b) shows the plots of calculated-versus-measured values of shrinkage and creep similar to the authors' Fig. 6, 8, and 9. Here, however, all the data points from the data bank are included in the comparison, and the total number of points is about six times larger than that in the authors' model than that seen in the paper.

RILEM COMMITTEE PRINCIPLES

RILEM Committee TC-107[33] formulated the basic characteristics which every good prediction model for creep and shrinkage should satisfy, and RILEM Committee TC-107[33] expanded these characteristics in the form of 21 principles. The recently proposed BP-KX model is an example of a model that satisfies all these principles. The model proposed in the paper, however, violates many of them, in particular principles No. 2, 6, 7, 8, 9, 11, 12, 13, 15, 16, and 17. It is surprising that this previous result of a careful study by a large group of experts was ignored in the paper.

CONCLUSION

It cannot be claimed that the proposed model is superior to the other existing models. To the contrary, it is inferior to several of them, including the current ACI 209.

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