



# SCALING OF STRUCTURAL STRENGTH

Second edition

Zdeněk P Bažant



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## Foreword

Up to the 1980s, all the experimentally observed size effects in solid mechanics were generally attributed to material strength randomness. Bažant revisited the scaling theory beginning with his 1984 discovery of the scaling law for the size effect caused by the release of stored energy due to stable growth of large fractures or large damage zones prior to failure. Using asymptotic matching arguments, he derived a deceptively simple law of surprisingly broad applicability, bridging the power scaling laws of classical fracture mechanics and plasticity. With his assistants, he experimentally verified his law for various materials, and showed how to use the scaling law to identify the cohesive fracture characteristics from experiments. Later, using extreme value statistics, he formulated a probabilistic generalization describing the transition to the classical statistical size effect in very large structures failing at fracture initiation. He also extended his size effect law to compression fracture, including kink band propagation in fiber composites. Recently, he used similar asymptotic arguments to show that the currently accepted dislocation-based strain-gradient theory of metal plasticity for micron scale needs a fundamental revision because of unreasonable asymptotic properties on approach to nanoscale.

Scaling of structural strength remains, however, a topic that many researchers in solid mechanics seem to have temporarily set aside. It is indeed striking to see that scaling and dimensional analysis have a tendency to disappear from curricula and from the scientific literature in solid mechanics. Is it because computers are allowing large size calculation today that scale extrapolations have become useless? This topic reflects upon the relationship between the experiments, material characteristics and structural engineering. As in statistical physics, it sheds new lights on the existing theories and helps in building new, consistent engineering models. Above all, I am sure that when reading the conclusion of the book, the reader will be convinced that scaling ought to play a pivotal role into the understanding new problems such as earth dynamics and nanomechanics (to mention just two extremes).

I am very glad that Zdeněk Bažant agreed to take the time to write this volume. It is an excellent and condensed presentation of the author's pioneering

works in this field. Zdeněk decided also to include some new, unpublished results in his manuscript. I am indebted to him for this mark of esteem and trust.

Finally, thanks should also be extended to my graduate student Bruno Zuber who helped in the preparation of the final manuscript.

Nantes, November 2001

Gilles Pijaudier-Cabot

## Author's Preface

In 1973, while browsing in the library, one paper in the *Indian Concrete Journal* caught my eye. P.F. Walsh, a young Australian then unknown to me, was reporting remarkable experiments. They revealed, in concrete specimens, a strong size effect. But that size effect did not conform to a power law and thus was in conflict with the Weibull statistical theory, then reigning supreme and sacrosanct.

At about the same time, luckily, the late Stanley Fistedis invited me to consult his group at Argonne National Laboratory in matters of failure analysis of concrete vessels and containments under various hypothetical scenarios of nuclear accidents in a liquid-metal-cooled breeder reactor. The objective was reliable extrapolation from normal-scale laboratory specimens to these very large (and politically very sensitive) structures. In view of dense distributed reinforcement, it was necessary to somehow take realistically into account the distributed cracking, for which it seemed unavoidable to postulate strain-softening. This phenomenon, as we know today, gives rise to a deterministic size effect.

Then, in the early 1970s, there was the luck of my getting to teach advanced topics in structural stability to some very inquisitive students in our solid mechanics program at Northwestern, who argued about stability of softening structures, and of hearing a great seminar by Jim Rice on the triggering of *localization instability by geometrically nonlinear plastic deformations*, which is analogous to the strain-softening trigger.

Somehow all these stimuli set me at the beginning of the 1970s on an initially controversial but exciting path which has not yet reached its end. It has been struggle and fun—struggle because most solid mechanics sages regarded at that time the strain softening (the cause of deterministic size effect) as a lowly crime of ignorants (fortunately, I was no longer behind the Iron Curtain where the mechanics bosses actually managed to get any funding for strain-softening models proscribed by a ruling of the academy)—and fun because it led at NSF Workshops to all these lively polemics about improperly posed boundary value problems, uniqueness, regularization, mesh sensitivity, material stability, etc.

In my efforts leading to this monograph, I am indebted to many. I wish to thank my friend Jacky Mazars for advising Pijaudier-Cabot to become my doctoral student. Gilles often dropped to my office cheerfully but debated forcefully and gave me hard time. He contributed some key ideas of the nonlocal damage concept and its stability foundations, which finally made the concept of strain softening noncontroversial. Aside from Gilles, I have been blessed in my studies of size effect and localization with the collaboration of a long line of bright and hard-working doctoral students —E. Becq-Giraudon, S. Beissel, M. Brocca, F.C. Caner, G. Cusatis, M. Cyr, R. Desmorat, R. Gettu, Z. Guo, M. Jirásek, M.E. Karr, M.T. Kazemi, J.-J. H. Kim, J.K. Kim, Z. Li, F.-B. Lin, G. de Luzio, B.-H. Oh, P.A. Pfeiffer, P.C. Prat, W.F. Schell, M. Thoma, S. Şener, Y. Xi, K. Xu, Q. Yu, Y. Zhou and G. Zi, as well as postdoctoral associates and visiting scholars—I. Carol, L. Cedolin, J. Červenka, D. Ferretti, Y.-N. Li, P. Kabele, Y.W. Kwon, D. Novák, J. Ožbolt, J. Planas and J. Vitek. Their enormous help to my research leading to this monograph is deeply appreciated.

I cannot thank enough my esteemed colleague Isaac Daniel for his invaluable advise and help in fracture testing of fiber composites, sandwiches and foams. To John Dempsey, aside from provocative discussions, I am indebted for the truly unique experience of taking part of his 'expedition' to the Arctic Ocean in which size effect tests of sea ice specimens, up to the record-breaking size of 80 m, were successfully carried out. The great research environment that we have at Northwestern University has been a big plus, but my escapes to the calm atmospheres of hotels Maria in Sils, Le Calette in Cefalù, Paraiso del Mar in Nerja, Parador Aiguablava on Costa Brava and others were conducive to thinking through some more challenging sections of this monograph.

Thanks are further due to E.-P. Chen for funding, from his applied mechanics program at Sandia National Laboratories, my work on a review of scaling on which much of this book is based. The present monograph would not have happened had Gilles not pressed me gently but persistently. It certainly would not have happened without generous financial support for my research at Northwestern, which was initially granted by the National Science Foundation and Air Force Office of Scientific Research, and during the 1990s came mainly from the solid mechanics program directed at the Office of Naval Research by Yapa D.S. Rajapakse. I am grateful to Yapa for inducing me to take more fundamental viewpoints and pushing me to shift my focus from the scaling problems of concrete and geomaterials to those of sea ice and, more recently, fiber composites, rigid foams and sandwich structures for ships.

## About the Author

Born and educated in Prague, Dr. Zdeněk P. Bažant joined Northwestern University faculty in 1969, became Professor in 1973, was named to the distinguished W.P. Murphy Chair in 1990, and served during 1981-87 as Director of the Center for Concrete and Geomaterials. In 1996 he was elected to the *National Academy of Engineering*, Washington. He has published over 400 refereed journal articles; authored books on *Stability of Structures* (1991, with L. Cedolin), *Fracture and Size Effect*, (1997, with J. Planas), *Concrete at High Temperatures* (1996, with M. Kaplan), *Inelastic Structural Analysis* (2001, with M. Jirásek) and *Creep of Concrete* (1966, in Czech); and edited 14 books. He served as *Editor* (in chief) of *ASCE J. of Engrg. Mech.* (1988-94) and is *Regional Editor of Int. J. of Fracture*, and member of editorial boards of 16 other journals. He was founding president of IA-FraMCoS, president of Soc. of Engrg. Science (SES), and chairman of IA-SMiRT Div. H. An Illinois Registered Structural Engineer, he has served as consultant to Argonne National Laboratory and many firms. He chaired various committees in ASCE, RILEM and ACI. His honors include: *honorary doctorates* from Czech Techn. Univ., Prague, Universität Karlsruhe, Germany, University of Colorado, Boulder, and Politecnico di Milano. He is a *Foreign Member* of Austrian Academy of Sciences, and of Engrg. Academy of Czech Rep. He received *Prager Medal* from SES; *Warner Medal* from ASME; *Newmark Medal*, *Crocs Medal*, *Huber Prize* and *T.Y. Lin Award* from ASCE; *L'Hermite Medal* from RILEM; *Torroja Gold Medal* (Madrid); *D.M. Roy Lecture Award* from the American Ceramic Society; *ICOSSAR Lecture Award* from the International Association for Structural Safety and Reliability; *Humboldt Award*; *Guggenheim*, *NATO*, *Humboldt*, *JSPS*, *Kajima* and *Ford Science Fellowships*; *Meritorious Publ. Award* from SEAQI; *Stodola Gold Medal* from Slovak Acad. of Sci.; *Best Engrg. Book of the Year Award* from SAP; *Medal of Merit* from Czech Soc. of Mech.; and *China-Taiwan Gov. Lectureship Award*; etc. He was elected *Fellow* of Am. Soc. of Civil Engrs., Am. Soc. of Mechanical Engrs., Am. Concrete Inst. and Am. Academy of Mechanics; and *Honorary Member* of the Czech Soc. of Civil Engrs. and of Building Res. Inst., Spain. He is a *Fellow* of Am. Academy of Mech., SES, ASME, ASCE, ACI and RILEM.