Size Effect and Fracture Characteristics of Composite Laminates

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Measurements of the size effect on the nominal strength of notched specimens of fiber composite laminates are reported. Tests were conducted on graphite/epoxy crossply and quasi-isotropic laminates. The specimens were rectangular strips of widths 6.4, 12.7, 25.4 and 50.8 mm (0.25, 0.50, 1.00 and 2.00 in.) geometrically similar in two dimensions. The gage lengths were 25, 51, 102 and 203 mm (1.0, 2.0, 4.0 and 8.0 in.). One set of specimens had double-edge notches and a $[0/92_2]_s$ crossply layup, and another set had a single-sided edge notch and a $[0/\pm 45/90]_s$ quasi-isotropic layup. It has been found that there is a significant size effect on the nominal strength. It approximately agrees with the size effect law proposed by Bažant, according to which the curve of the logarithm of the nominal strength versus the logarithm of size represents a smooth transition from a horizontal asymptote, corresponding to the strength criterion (plastic limit analysis), to an inclined asymptote of −0.5 slope, corresponding to linear elastic fracture mechanics. Optimum fits of the test results by the size effect law are obtained, and the size effect law parameters are then used to identify the material fracture characteristics, particularly the fracture energy and the effective length of the fracture process zone. Finally, the R-curves are also identified on the basis of the maximum load data. The results show that in design situations with notches or large initial traction-free cracks the size effect on the nominal strength of fiber composite laminates must be taken into account.

1 Introduction

Failure of composite materials has been described in practice by means of failure criteria in terms of stresses or strains, such as the maximum stress, maximum strain, deviatoric strain energy (Tsai-Hill), and the tensor polynomial (Tsai-Wu, 1971) criteria. These criteria are macromechanical and do not account for the various micromechanical failure processes occurring in composites, especially near notches. Damage initiation and development take the form of various interacting failure mechanisms which are sensitive to pre-existing defects and microstructure (micromechanical) anomalies. The damage processes tend to localize and propagate, for which the crucial consideration is energy release. If the material failure criterion involves energy, there are some important consequences. The most important one is the size effect, that is, effect of the characteristic dimension, D, of the structure on the nominal strength σ_N, provided that geometrically similar structures are compared and the cracks at maximum load are also geometrically similar.

The size effect caused by fracture energy release has recently come to the forefront of attention in studies of concrete, rocks, ceramics and other quasi brittle materials, which are characterized by the existence of a sizable fracture process zone at the tip of a macroscopic crack. It has been found (Bažant, 1984, 1993; Bažant and Kazemi, 1990) that in such materials the size effect is transitional between plasticity (for which there is no size effect) and linear elastic fracture mechanics (for which the size effect is the strongest). Thus the plot of log σ_N versus log D is a smooth curve approaching at very small sizes a horizontal asymptote corresponding to plasticity and at very large sizes an inclined asymptote of slope −0.5 corresponding to linear elastic fracture mechanics. Such a size effect must generally occur whenever the load-deflection diagram does not have a yield plateau after the maximum load is reached, provided that the geometrically nonlinear effects of buckling are absent. Therefore, a size effect of this type is expected also for fiber composite laminates. The purpose of this paper is to verify this proposition, describe the size effect quantitatively and utilize measurements of the size effect for determining the material fracture characteristics.

It must be emphasized that this study deals only with the size effect for constant thickness of the laminate and for the same layup (principally the effect of width). If the thickness is varied, a size effect of different type occurs, which is not investigated here. Also, only the size effect for the same notch tip sharpness is studied, however, for sufficiently small notch tip widths, less than about $\frac{1}{3}$ of the spacing of major inhomogeneities, the sharpness effect may be expected to disappear.

Fracture of laminated composites with stress risers has been investigated by using two major approaches. One approach is based on concepts of linear-elastic fracture mechanics carried over from isotropic materials, while the other approach is based on the stress distributions near the notch. The first approach was used by Waddoups et al. (1971) who assumed the existence of Griffith type cracks on the boundary of a hole, and by Cruse (1973). The latter attempted to predict the fracture energy of a multidirectional laminate as the sum of fracture energies of the individual plies. An equivalent summation of the squares of the stress intensity factors has also been proposed by Mandell et al. (1975). These authors studied the damage zone at the crack tip in fiber composite laminates and found that the fracture process zone consists of ply microcracking (matrix cracks parallel to the fibers) and local delaminations of the cracked plies. They found that the intensity of this microcracking is linearly proportional to the square of the stress intensity factor, which means it is proportional to the energy release rate for a given composite layup and ply stacking sequence. Mandell et al. correctly pointed out that the microcracking zone plays the same role as plastic flow in metals, relieving the high local stress concentrations and absorbing the energy released due to fracture

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Table 1 Properties of unidirectional IM7/8551-7A graphite/epoxy

Property	Value
Fiber volume ratio, V_f	0.65
Ply thickness, t, mm (in.)	0.127 (0.005)
Longitudinal modulus, E_1, GPa (Msi)	169 (24.5)
Transverse modulus, E_2, GPa (Msi)	9.4 (1.36)
In-plane shear modulus, G_{12}, GPa (Msi)	6.4 (0.93)
Major Poisson's ratio, ν_{12}	0.30
Longitudinal tensile strength, F_{1t}, MPa (ksi)	2210 (321)
Transverse tensile strength, F_{2t}, MPa (ksi)	65 (9.4)

propagation. Following the second approach, Whitney and Nuismer (1974) proposed two simplified stress-fracture criteria based on the actual stress distribution near the notch, the so-called point stress and average stress criteria. In the case of cracks, they compared results with those based on linear elastic fracture mechanics.

Daniel (1978, 1980, 1981, 1982, 1985) investigated cracks in graphite/epoxy laminates and found the critical size of the damage zone at the tip of the notch or at the boundary of a hole to be about 3 to 5 mm. The studies of Daniel (1982) and others revealed that failure of a fiber composite laminate involves a combination of several microscopic failure mechanisms, including ply microcracking (subcracks), delamination, fiber breakage and fiber pullout. These observations revealed the existence of a characteristic length in this composite material. Noting that the size of the damage zone at failure was roughly independent of the notch length, Daniel achieved a good fit of his experimental results by replacing the actual crack length by an extended equivalent crack length in order to take the damage zone size into account. With the modified crack length, the apparent stress intensity factor was essentially constant for the range of his data, including mixed mode loading. Daniel also translated his test results into an R-curve (resistance curve), describing the dependence of the apparent stress intensity factor on the crack length. A similar approach was also used for concrete. Nallathambi and Karihaloo (1986) found that a constant crack length extension allowed good fits of all their data for different crack lengths. In general, however, such a simplification is likely to be inadequate. Based on analogy with extensive studies of concrete fracture (ACI Committee 446, 1992), a good description of a broad range of test data requires not only replacing the actual crack length with some equivalent extended crack length but also considering the critical energy release rate to depend on this equivalent crack length, that is, introducing an R-curve.

2 Fracture Tests of Composite Laminates

The material used was IM7/8551-7A (Hercules) graphite/epoxy obtained in unidirectional prepreg form. Unidirectional, crossply and quasi-isotropic laminates were prepared from this prepreg. The unidirectional material was used to characterize the lamina following established procedures (ASTM Standards, 1990; Daniel and Ishai, 1994). This characterization was based on testing $[0_6]$, $[90_8]$, and $[10_6]$ coupons under uniaxial tension. The results of this characterization are listed in Table 1.

Two sets of specimens were prepared for the fracture tests: crossply of $[0/90_2]_s$ layup, and quasi-isotropic of $[0/\pm45/90]_s$ layup. Each set consisted of four rectangular specimens of the same thickness but different sizes, geometrically similar in the planes of the laminates, with width x gage length $= 6.4 \times 25$ mm, 12.7×51 mm, 25.4×102 mm, and 50.8×203 mm (0.25×1.0 in., 0.50×2.0 in., 1.00×4.0 in. and 2.00×8.0 in.). Thus the size ratios were 1:2:4:8. The thickness of the crossply specimens was 0.76 mm (0.030 in.) and that of the quasi-isotropic specimens was 1.02 mm (0.040 in.). Two edge notches of length $a = D/16$ were machined in the crossply specimens and a single edge notch of length $a = D/5$ was machined in the quasi-isotropic specimens, where D is the specimen width (Fig. 1). The notches were machined with a 0.2 mm (0.008 in.) diameter diamond-studded wire. Thus, the crack tip radius was 0.1 mm (0.004 in.) in all cases. All specimens were prepared with 38 mm (1.5 in.) long glass/epoxy tabs for gripping purposes. The tab length (grip constraint) was not scaled because it has no appreciable effect on the stored energy and because fracture occurs away from the grips.

The notched laminate specimens were tested under uniaxial tensile loading in a servohydraulic testing machine (Instron). The tests were conducted at a constant crosshead rate (stroke control) for the double edge notched specimens and under crack opening displacement (COD) control for the single-edge-notched specimens. The crosshead rate was adjusted for the different size specimens so as to achieve roughly the same average strain rate of 0.2 percent min. in the gage section for which the peak load is reached within approximately 10 min. in all cases.

Figures 2 and 3 show typical stress-strain curves for the notched crossply and quasi-isotropic specimens of various sizes. For the largest specimen size, these curves are almost linear up to failure, which indicates pronounced brittle behavior. For the smallest specimen sizes, there is a significant nonlinear segment before the peak stress, which indicates hardening inelastic behavior and reduced brittleness (or higher ductility). This behavior represents a transition from ductile response for small sizes to brittle response for large sizes, which conforms with the size effect theory to be discussed further on.

The machine stiffness and controls did not permit a stable test in the post-peak regime of descending load, even when the crack opening displacement was controlled. A stable post-peak test might possibly be obtained for notched bending specimens (Wisnom, 1992a), but such tests are more difficult to carry out for thin laminates. The failures of the specimens were catastrophic (dynamic), and occurred shortly after the peak load. Damage consisting of microcracks in layers and delamination between layers before peak load was observed in the tests (in

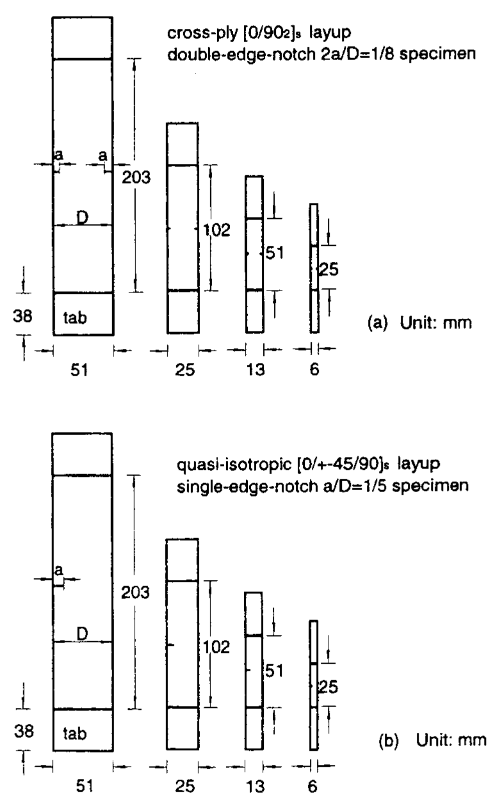

Fig. 1 Geometry of test specimens; (*a*) double-edge notched specimens, (*b*) single-edge notched specimens (units: mm)

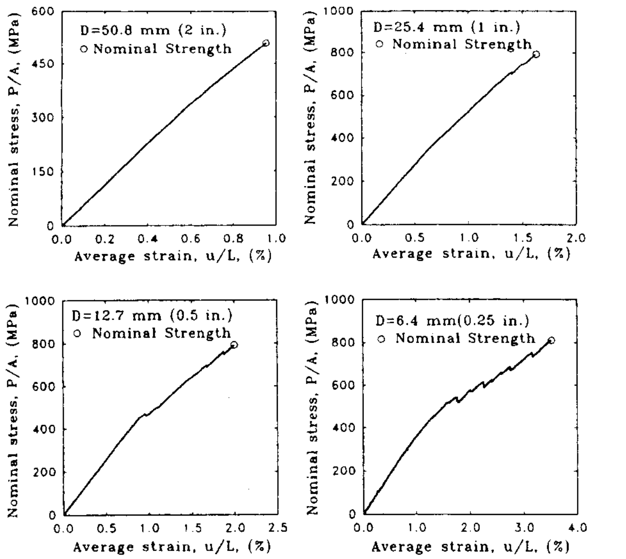

Fig. 2 Typical stress-strain curves of $[0/90_2]_s$ crossply double-edge notched specimens of various sizes, showing increasing nonlinearity with decreasing size

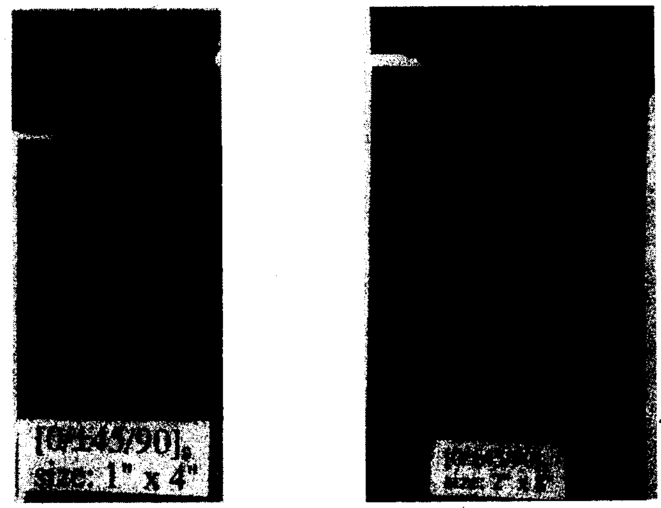

Fig. 4 Typical failure patterns of single-edge notched quasi-isotropic specimens of two sizes

agreement with the observations of Mandell et al., 1975). The typical appearance of the specimens after failure is shown in Fig. 4, where the microcracking damage can be seen. For the quasi-isotropic specimens, some fractures run at 45 deg with the notch and there are 45 deg microcracks (Fig. 4(*b*)), as observed by Daniel (1978).

The test results for the notched specimens are summarized in Table 2, in which the nominal strength is defined as the average stress at failure based on the unnotched cross section, $\sigma_N = P_{\max}/hD$, where D is the specimen width (characteristic dimension) and h the laminate thickness.

Previous studies (Bažant and Tabbara, 1992) have shown that the double-edge notched specimen has one undesirable feature: the response path exhibits a bifurcation after which only one of the two cracks can propagate, and thus the response becomes asymmetric. This property, however, does not invalidate the foregoing procedure because the bifurcation happens only after the peak load. Nevertheless, the post-peak data from such tests are difficult to interpret. It was for this reason that the second series of tests used single-edge notched specimens. In that specimen configuration there is no bifurcation of the response path and the response is asymmetric from the beginning.

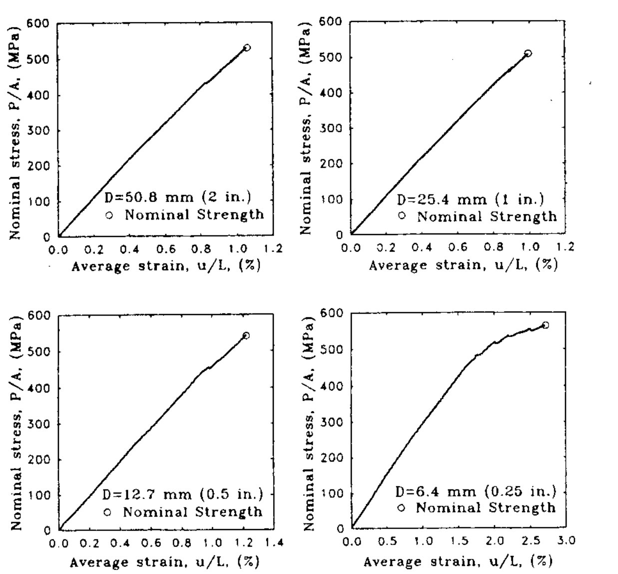

Fig. 3 Typical stress-strain curves of $[0/\pm45/90]_s$ quasi-isotropic single-edge notched specimens of various sizes, showing increasing nonlinearity with decreasing size

3 Observed Size Effect

The effect of structure size on the nominal strength of geometrically similar quasi-brittle structures generally follows the approximate size effect law (Bažant, 1984, 1993):

$$\sigma_N = Bf_u(1+\beta)^{-1/2}, \quad \beta = D/D_0 \tag{1}$$

where β = relative structure size; $\sigma_N = c_N P/bD$ = nominal strength of structure, with P = maximum load, D = characteristic dimension (size) of structure, b = width of structure in the third dimension (only two-dimensional similarity is considered here), c_N = chosen coefficient introduced for convenience (for example to make σ_N coincide with the maximum stress in the specimen calculated by bending theory); D_0 = constant depending on both fracture process zone size and specimen geometry; B = constant characterizing the solution according to plastic limit analysis based on the strength concept; f_u = reference strength of the material (laminate), introduced to make constant B dimensionless. Equation (1) is valid not only for the two-dimensional similarity considered here (h = constant) but also for three-dimensional similarity.

The size effect law (1) has been verified by numerous tests, especially for concrete, rocks, and toughened ceramics. It has been derived, under certain reasonable simplifying assumptions, by energy release analysis coupled with dimensional analysis and similitude arguments. Recently, it has been derived very generally by asymptotic matching of the large-size and small-size asymptotic expansions of size effect (Bažant, 1995). By considering the large size and small size asymptotic expansions of size effect, Eq. (1) has been shown to represent the matched asymptotic. It has also been shown that Eq. (1) represents the limiting case of a nonlocal generalization of the statistical Weibull-type theory for the size effect, in which the material failure probability is considered to depend on the average strain of a certain characteristic volume of the material rather than the stress at the same point (Bažant and Xi, 1991c). It has been shown that the predictions of finite element codes with a nonlinear fracture model (such as the cohesive crack model; Li and Bažant, 1994) or with a nonlocal damage material model (Bažant and Lin, 1988; Bažant and Ozbolt, 1992, Bažant, ed., 1992), agree well with Eq. (1). Furthermore, fracture simula-

Table 2 Results of tensile tests of notched composite laminates of different sizes and different notches

Specimen length cm (in.)	Specimen width cm (in.)	Double-edge notched crossply specimens		Single-edge notched quasi-isotropic specimens	
		Max. load N (lb)	Nominal strength σ_N MPa (ksi)	Max. load N (lb)	Nominal strength σ_N MPa (ksi)
2.54 (1)	0.635 (0.25)	3,871 (870)	800 (116)	3,650 (820)	618 (90)
5.08 (2)	1.27 (0.5)	7,354 (1,653)	760 (110)	7,088 (1,593)	580 (84)
10.16 (4)	2.54 (1.0)	12,980 (2,917)	670 (97)	13,774 (3,095)	534 (77)
20.38 (8)	5.08 (2.0)	21,360 (4,800)	552 (80)	24,715 (5,554)	479 (69)

tions by random particle models also agree with this law; Bažant and Tabbara (1990), Bažant and Jirásek (1995).

With regard to the statistical approach to size effect, the recent study by Jackson et al. (1992) of the size effect of graphite/epoxy composites under tension and flexure deserves mention. Geometrically similar specimens of size ratios 1:2:3:4 were used and the results were analyzed on the basis of Weibull's statistical theory based on a distribution function of random material strength. Good agreement with the test data was obtained. However, it should be pointed out that the Weibull statistical theory can be applied only to the crack initiation stage of failure. The reason is that, in the classical form of this theory, the failure probability is considered to depend on the local stress, calculated from elasticity. The stress redistributions and stress concentrations caused by prior fracture growth are disregarded. These phenomena make Weibull-type theories inapplicable to failures following large stable crack growth, which is in the present case simulated by the notches of various lengths (see Bažant, Xi and Reid, 1991b). In that case, the nonlocal generalization of the Weibull approach is required, and only then Eq. (1) is obtained as the deterministic limit.

To determine the parameters of the size effect law by regression analysis of experimental data, we define the following

$$D = X$$

$$\sigma_N = Y^{-1/2}$$

$$Bf_u = A^{-1/2}$$

$$D_0 = \frac{A}{C} = \frac{1}{C(Bf_u)^2} \tag{2}$$

Then, the size effect law in Eq. (1) is expressed in the form

$$Y = A + CX \tag{3}$$

A linear regression analysis was conducted by plotting Y versus X in Figs. 5 and 6 for the crossply and quasi-isotropic specimens, respectively. The parameters of the size effect law were obtained from the slope C and the vertical intercept A of these plots as follows:

$$D_0 = 30.9 \text{ mm } (1.216 \text{ in.})$$

$$Bf_u = 892 \text{ MPa } (129 \text{ ksi})$$

for the crossply specimens, and

$$D_0 = 77.5 \text{ mm } (3.05 \text{ in.})$$

$$Bf_u = 611 \text{ MPa } (89 \text{ ksi})$$

for the quasi-isotropic specimens.

Subsequently, the resulting size effect was represented by plotting log (σ_N/Bf_u) versus log (D/D_0) in Figs. 7 and 8 for the two sets of specimens. These size effect plots represent a transition from the strength criterion (plastic limit analysis) characterized by a horizontal asymptote, to an asymptote of slope -0.5, representing linear elastic fracture mechanics (LEFM). The intersection of the two asymptotes corresponds to $D = D_0$, called the transitional size.

The test results in Figs. 7 and 8 show that: (1) the failure of fiber composite laminates containing traction-free cracks (or notches) exhibits a significant size effect, and that (2) the size effect represents a gradual transition with increasing size from the strength criterion (e.g., maximum stress) to linear elastic fracture mechanics, as described by the size effect law, eq. (1). The foregoing conclusions are verified on the average, as the mean statistical trend. These conclusions ought to be taken into account in all design situations and safety evaluations where a large traction-free crack can grow in a stable manner prior to failure. Especially, these conclusions are important for extrapolation from small-scale laboratory tests to real size aerospace or other structures. The strength theory, which does not account for size effect, is inadequate for these applications.

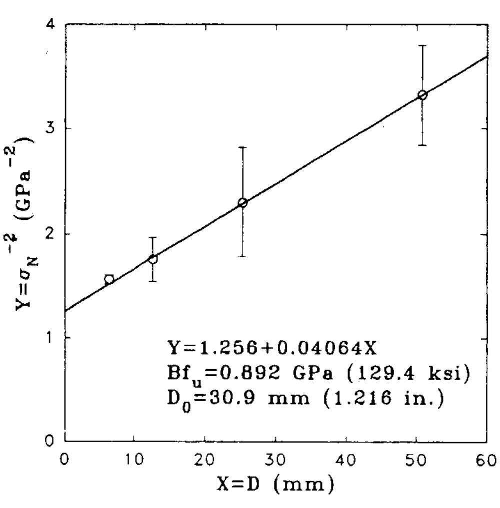

Fig. 5 Linear regression for determination of size effect parameters (crossply specimens)

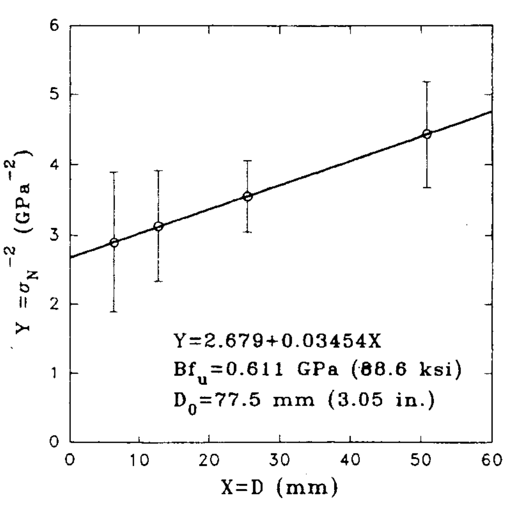

Fig. 6 Linear regression for determination of size effect parameters (quasi-isotropic specimens)

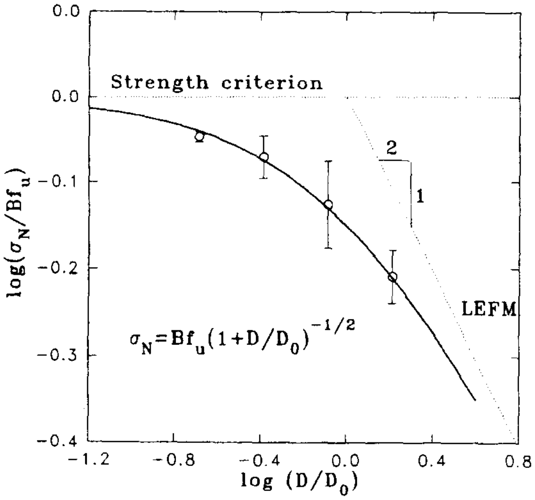

Fig. 7 Size effect measured for crossply specimens with double-edge notches

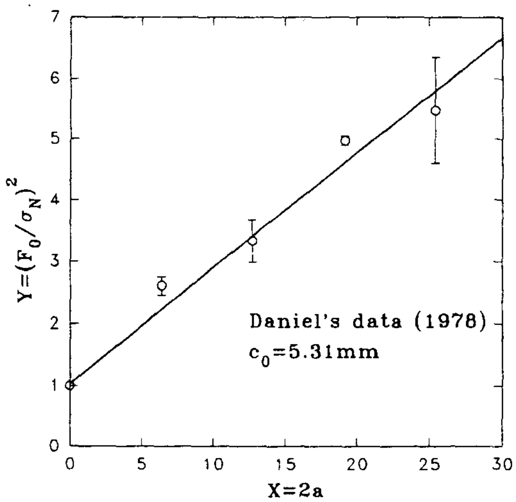

Fig. 9 Linear regression for determination of size effect parameters (quasi-isotropic laminates with central notches, Daniel, 1978)

Furthermore, previously reported experimental results were re-examined from the point of view of the size effect law of Eq. (1). Daniel (1978) reported results on quasi-isotropic graphite/epoxy plates with central notches. The plates were all 8-plies thick (1 mm), 127 mm (5 in.) wide and had central horizontal cracks ranging in length from 6.3 mm (0.25 in.) to 25.4 mm (1.00 in.). He applied the average stress criterion by using Westergaard's solution for stresses near the crack tip and integrating the axial stress over a characteristic distance c_0 from the crack tip. By equating this average stress to the unnotched strength of the material he obtained the following relation

$$\sigma_N = F_0\left[1 + \frac{2a_0}{c_0}\right]^{-1/2} \qquad (4)$$

where,

$2a_0$ = initial crack length
F_0 = strength of unnotched laminate
σ_N = nominal strength of notched laminate
c_0 = characteristic distance from crack tip

The experimental data were fitted to relation (4) by plotting $(F_0/\sigma_N)^2$ versus $2a_0$ in Fig. 9, which yielded the value c_0 = 5.31 mm for the characteristic dimension. The same relation (4) above was plotted in Fig. 10 in the form of the size effect law (1) as log (σ_N/F_0) versus log β where $\beta = 2a_0/c_0$ (solid curve), where, the agreement with the test results is seen to be satisfactory. Relation (4) is equivalent to a special case of Eq. (1) as shown in Appendix I. Since relation (4) is based on the near field solution for the crack, it is valid only when c_0 is much smaller than the crack length $2a_0$.

Additional experimental results reported by Wisnom (1992b) were examined from the point of view of the size effect (1). Wisnom (1992b) tested unidirectional glass/epoxy laminates under uniaxial tension with varying numbers of cut plies at the center. The failure stress, which causes delamination initiation and propagation at the tips of the central notch, was fitted to the number of cut plies **n** as (Wisnom, 1992b)

$$\sigma_N = 1412(1 + 1.111n)^{-1/2} \qquad (5)$$

Denoting

Bf_u = 1412 MPa
β = 1.111n

Wisnom's data were plotted in the form of eq. (1) in Fig. 11. Since **n** can be regarded as proportional to size **D**, the excellent agreement in Fig. 11 represents another confirmation of the size effect law.

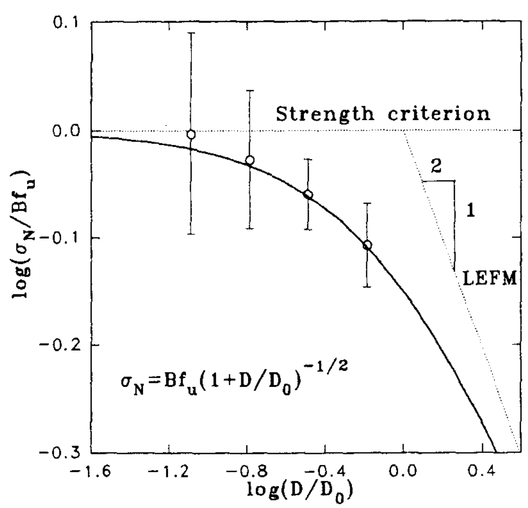

Fig. 8 Size effect measured for quasi-isotropic specimens with single-edge notch

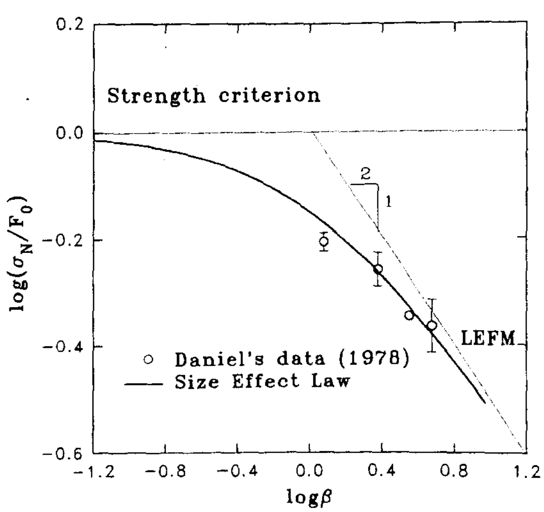

Fig. 10 Size effect measured for quasi-isotropic specimens with central notch (Daniel, 1978)

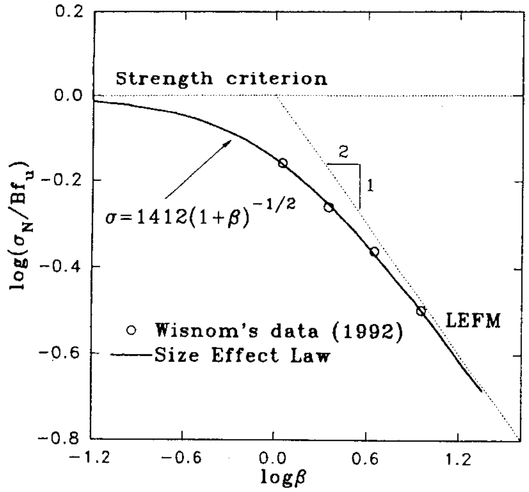

Fig. 11 Size effect measured for unidirectional specimens with central notch (Wisnom, 1992b)

4 Determination of Material Fracture Characteristics from Measured Size Effect

The size effect relation (1), for quasi-brittle fracture can also be expressed in terms of the nondimensionalized energy release rate $g(\alpha)$ (Bažant and Kazemi, 1990; Bažant and Cedolin, 1991, Ch. 12);

$$\sigma_N = c_N\left(\frac{EG_f}{g'(\alpha_0)c_f + g(\alpha_0)}\right)^{1/2}. \tag{6}$$

Here G_f is the fracture energy of the material; c_f is a mechanical constant representing the effective length of the fracture process zone defined by extrapolation to infinite size; $\alpha_0 = a_0/D =$ initial value of $\alpha = a/D$, $a_0 =$ length of notch or initial traction-free crack, $a =$ crack length; $g(\alpha) = ED^2G(\alpha)/P^2b =$ dimensionless energy release rate, where $G(\alpha) =$ energy release rate per unit width of crack front edge calculated by linear elastic fracture mechanics (LEFM); $g'(\alpha) = dg(\alpha)/d\alpha$; $E =$ Young's modulus, $b =$ thickness, $D =$ characteristic structure size taken as the specimen width for a single-edge-notch specimen or half-width for a double-edge-notch specimen (see Bažant et al., 1991). By matching Eqs. (6) and (1), one obtains (Bažant and Kazemi, 1990, Bažant et al., 1991):

$$G_f = \frac{(Bf_u)^2}{c_N^2E}D_0g(\alpha_0), \quad c_f = \frac{g(\alpha_0)}{g'(\alpha_0)}D_0 \tag{7}$$

where $G_f =$ fracture energy of the material. As the only possible unambiguous definition, the fracture energy is defined as the energy required for fracture propagation in a specimen of theoretically infinite size (Bažant and Pfeiffer, 1987). According to this definition, the fracture energy is independent of both the shape and size of the specimen because in a specimen of infinite size the fracture process zone occupies an infinitely small portion of the specimen volume and can be considered as a point, which means that linear elastic fracture mechanics applies.

To determine the material fracture characteristics on the basis of Eq. (7), the expressions for the stress intensity factor K_I available for isotropic specimens (e.g., Tada, 1985) have been used. The assumption of isotropy is quite good for the quasi-isotropic laminates, but may involve a larger error for the cross-ply laminates (this should be checked in subsequent studies). According to LEFM,

$$K_I = \sigma\sqrt{\pi D\alpha}F(\alpha), \quad \alpha = a/D \tag{8}$$

where $\sigma = \sigma_N =$ average stress in the laminate specimen and F is a function of the variable α.

The crossply laminate is not isotropic but orthotropic. The energy release rate and the stress intensity factor for orthotropic specimens of the present geometry have recently been solved by Bao et al. (1991). Their solution uses elastic parameters defined as:

$$\rho = \frac{\sqrt{E_xE_y}}{2G_{xy}} - \sqrt{\nu_{xy}\nu_{yx}}, \quad \lambda = \frac{E_y}{E_x} \tag{9}$$

where E_x, E_y, G_{xy}, ν_{xy}, ν_{yx}, are the elastic constants of the orthotropic material referred to its principal material axes x and y, which can be calculated from the lamina properties (Daniel and Ishai, 1994). The stress intensity factor can be written as:

$$K_I = \sigma\sqrt{\pi D\alpha}Y(\rho)F(\alpha) \tag{10}$$

where (Bao et al., 1991)

$$Y(\rho) = [1 + 0.1(\rho - 1) - 0.015(\rho - 1)^2 + 0.002(\rho - 1)^3]\left(\frac{1+\rho}{2}\right)^{-1/4}$$

$F(\alpha)$ is the same function of the relative crack length $\alpha = a/D$ as for isotropic materials. $Y(\rho)$ is a material parameter depending on the orthotropy parameter ρ. The energy release rate for an orthotropic material is:

$$G(\alpha) = \sqrt{\frac{1+\rho}{2E_xE_y\sqrt{\lambda}}}K_I^2 \tag{11}$$

Bringing (10) into (11), one can write $G(\alpha)$ in the same form as for the isotropic materials:

$$G(\alpha) = \frac{K_I^2}{E^*} = \frac{\sigma^2D\pi\alpha}{E^*}F^2(\alpha) = \left(\frac{P}{bD}\right)^2\frac{D}{E^*}g(\alpha) \tag{12}$$

where

$$E^* = \frac{1}{[Y(\rho)]^2}\sqrt{\frac{2E_xE_y\sqrt{\lambda}}{1+\rho}}$$

and $g(\alpha)$ is the nondimensionalized energy release rate defined before. By virtue of (9), we can treat the orthotropic material fracture characteristics in the same way as the isotropic ones if we replace E by the equivalent Young's modulus E^*($E^* = 6.98 \times 10^6$ psi = 48.2 GPa for the crossply specimen tested).

For the double-edge-notched specimen (Tada, 1985):

$$F(\alpha) = \left(1 + 0.122\cos^4\frac{\pi\alpha}{2}\right)\sqrt{\frac{2}{\pi\alpha}\tan\frac{\pi\alpha}{2}} \tag{13}$$

and for the single-edge notched specimen:

$$F(\alpha) = 1.122 - 0.231\alpha + 10.55\alpha^2 - 21.71\alpha^3 + 30.38\alpha^4 \tag{14}$$

Noting that $K_I^2 = GE$ where $G =$ energy release rate and $E =$ Young's modulus, we have $g(\alpha) = \pi\alpha[F(\alpha)]^2$. Thus, for the double-edge-notched specimen

$$g(\alpha) = 2\left(1 + 0.122\cos^4\frac{\pi\alpha}{2}\right)\tan\frac{\pi\alpha}{2}$$

$$g'(\alpha) = \pi\left[\left(1 + \tan^2\frac{\pi\alpha}{2}\right)\left(1 + 0.122\cos^4\frac{\pi\alpha}{2}\right) - 0.244\sin^2\pi\alpha\left(1 + 0.122\cos^4\frac{\pi\alpha}{2}\right)\right] \tag{15}$$

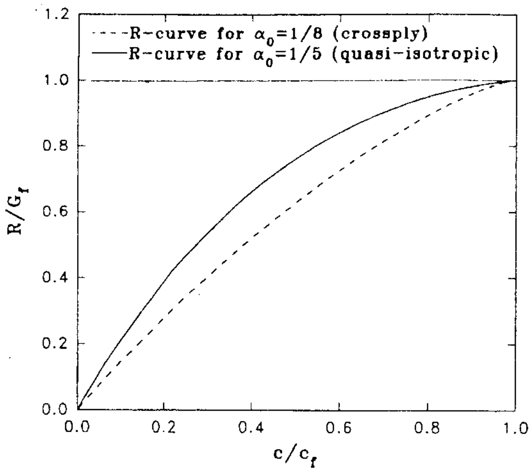

Fig. 12 Normalized *R*-curves for crossply and quasi-isotropic laminates, deduced from size effect measurements

where, for $\alpha = 0.125$, $g(0.125) = 0.492716$, and $g'(0.125) = 2.49552$. For the single-edge notched specimens:

$$g(\alpha) = \pi\alpha[1.122 - 0.231\alpha + 10.55\alpha^2 - 21.71\alpha^3 + 30.38\alpha^4]^2$$

$$g'(\alpha) = \pi[1.259 - 1.037\alpha + 71.18\alpha^2 - 214.4\alpha^3 + 947.5\alpha^4] \quad (16)$$

where, for $\alpha = 0.2$, we have $g(0.2) = 1.184$ and $g'(0.2) = 9.984$. Thus, for double-edge notched crossply laminates, we obtain from eq. (5) the effective fracture characteristics:

$$G_f = 0.252 \text{ MJ/m}^2 \text{ (1.43 ksi} \times \text{in.)},$$
$$c_f = 6.09 \text{ mm (0.24 in.)} \quad (17)$$

Because of orthotropy, these values are valid only for fracture along the principal x-direction of orthotropy. For the single-edge notched quasi-isotropic laminates we obtain:

$$G_f = 0.521 \text{ MJ/m}^2 \text{ (2.97 ksi} \times \text{in.)},$$
$$c_f = 9.19 \text{ mm (0.362 in.)} \quad (18)$$

It is noteworthy that the effective length of the process zone, c_f, found from these size effect measurements is quite close to the experimentally obtained one by Daniel (1985) (the present values are a little larger, which is not surprising considering that Daniel's procedure did not consider extrapolation to infinite size).

Based on the size effect law, the R-curve can be determined as the envelope of the fracture equilibrium curves for geometrically similar specimens of different sizes. This leads to the equations

$$R(c) = G_f \frac{g'(\alpha)c}{g'(\alpha_0)c_f},$$

$$\frac{c}{c_f} = \frac{g'(\alpha_0)}{g(\alpha_0)} \left(\frac{g(\alpha)}{g'(\alpha)} - \alpha + \alpha_0 \right) \quad (19)$$

in which $R(c)$ represents the R-curve. These two equations define the R-curve parametrically; for any chosen value of relative crack length, α, one first evaluates the crack extension from the notch, c, and then the R-value. Obviously, the R-curve depends on the geometry of the specimen. The R-curve calculated from the present test results is shown in Fig. 12 for both crossply and quasi-isotropic laminates.

5 Conclusions

1. The present tests show that the nominal strength of composite laminate specimens that are similar and have similar notches or initial traction-free cracks exhibits a significant size effect.

2. The size effect observed agrees with the size effect law proposed by Bažant, according to which the curve of the logarithm of the nominal strength versus the logarithm of the characteristic dimension (size) exhibits a smooth transition from a horizontal asymptote corresponding to the strength criterion (plastic limit analysis) to an inclined asymptote of slope -0.5, corresponding to linear elastic fracture mechanics.

3. Measurements of the size effect on the nominal strength can be used for determining the fracture characteristics of notched fiber composite laminates, including their fracture energy and the effective length of the fracture process zone. From these characteristics, the R-curve can also be calculated. The size effect method of measuring the fracture characteristics is easier to implement than other methods because only peak load measurements are necessary (the post-peak behavior, crack tip displacement measurement and optical measurement of crack tip location are not needed, and even a soft testing machine without servo-control can be used).

4. The orthotropic properties of fiber composite laminates can and must be taken into consideration while analyzing the fracture characteristics. Replacing Young's modulus by the equivalent Young's modulus proposed by Bao et al. (1992), the formulas of the size effect method previously derived for isotropic materials can be generalized for orthotropic materials. This makes it possible to determine size and shape independent values of the fracture energy, effective fracture process zone length, and R-curve for multi-directional laminates.

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References

ACI Committee 446, 1992, "State-of-Art-Report on Fracture Mechanics of Concrete: Concepts, Models and Determination of Material Properties," *Fracture Mechanics of Concrete Structures*, by Z. P. Bažant, ed. Elsevier Applied Science, London, New York, pp. 4–144.

ASTM, 1990, *Standards and Literature References for Composite Materials, 2nd ed.*, American Society for Testing and Materials, Philadelphia.

Bao, G., Ho, S., Suo, Z., and Fan, B., 1992, "The Role of Material Orthotropy in Fracture Specimens for Composites," *Int. J. Solid Structures*, Vol. 29(9), pp. 1105–1116.

Bažant, Z. P., 1984, "Size Effect in Blunt Fracture: Concrete, Rock, Metal." *J. of Engrg. Mech.*, ASCE, Vol. 110, pp. 518–535.

Bažant, Z. P., Pfeiffer, P. A., 1987, "Determination of Fracture Energy from Size Effect and Brittleness Number." *ACI Materials J.*, Vol. 84, pp. 463–480.

Bažant, Z. P., and Kazemi, M. T., 1990a, "Determination of Fracture Energy, Process Zone Length and Brittleness Number From Size Effect, With Application to Rock and Concrete." *Int. J. of Fracture*, Vol. 44, pp. 111–131.

Bažant, Z. P., and Kazemi, M. T., 1990b, "Size Effect in Fracture of Ceramics and Its Use to Determine the Fracture Energy and Effective Process Zone Length," *J. of American Ceramic Society*, Vol. 73(7), pp. 1841–1853.

Bažant, Z. P., Tabbara, M. R., Kazemi, M. T., and Pijaudier-Cabot, G., 1990, "Random Particle Model for Fracture of Aggregate or Fiber Composites." *J. of Engrg. Mechanics*, ASCE, Vol. 116(8), pp. 1686–1705.

Bažant, Z. P., Gettu, R., and Kazemi, M. T., 1991a, "Identification of Nonlinear Fracture Properties From Size Effect Tests and Structural Analysis Based on Geometry-Dependent R-Curves." *Int. J. of Rock Mechanics, Mining Science & Geomechanical Abstracts*, Vol. 28(1), pp. 43–51.

Bažant, Z. P., and Xi, Y., 1991c, "Statistical Size Effect in Quasi-Brittle Structures: II. Nonlocal Theory," *J. of Engrg. Mech.*, ASCE, Vol. 117(11), pp. 2623–2640.

Bažant, Z. P., and Cedolin, L., 1991, *Stability of Structures: Elastic, Inelastic, Fracture and Damage Theories*, Oxford University Press, New York.

Bažant, Z. P., and Ozbolt, J., 1992, "Compression failure of quasi-brittle material: Nonlocal microplane model." *J. of Engrg. Mechanics*, ASCE, Vol. 118(3), pp. 540–556.

Bažant, Z. P., and Tabbara, M. R., 1992, "Bifurcation and Stability of Structures with Interacting Propagating Cracks." *Int. J. of Fracture*, Vol. 53, pp. 273–289.

Bažant, Z. P., and Jirásek, M., 1995, "Particle Model for Quasi Brittle Fracture and Application to Sea Ice." *J. of Engrg. Mechanics*, ASCE, Vol. 121(9), pp. 1016–1025.

Bažant, Z. P., 1993, "Scaling Laws in Mechanics of Failure." *J. of Engrg. Mech.*, ASCE, Vol. 119(9), 1828–1844.

Bažant, Z. P., 1995, "Scaling Theories for Quasi-Brittle Fracture: Recent Advances and New Directions," *Proc. 2nd Int. Conf. on Fracture Mechanics of Concrete and Concrete Structures* (held at E.T.H., Zürich), F. H. Wittmann, ed. AFICIONADO Publishers, Freiburg, Germany, pp. 515–534.

Cruse, T. A., 1973, "Tensile Strength of Notched Composites." *J. of Composite Materials*, Vol. 7, pp. 218–228.

Daniel, I. M., 1978, "Strain and Failure Analysis of Graphite/Epoxy Plate With Cracks," *Experimental Mechanics*, Vol. 18, July, pp. 246–252.

Daniel, I. M., 1980, "Behavior of Graphite/Epoxy Plates with Holes Under Biaxial Loading." *Exper. Mechanics*, Vol. 20(1), pp. 1–8.

Daniel, I. M., 1981, "Biaxial Testing of Graphite/Epoxy Laminates with Cracks," ASTM STP 734, American Soc. for Testing and Materials, 109–128.

Daniel, I. M., 1982, "Failure Mechanisms and Fracture of Composite Laminates With Stress Concentrations." VIIth International Conference on Experimental Stress Analysis, Haifa, Israel, Aug. 23–27, pp. 1–20.

Daniel, I. M., 1985, "Mixed-Mode Failure of Composite Laminates with Cracks." *Experimental Mechanics*, Vol. 25, Dec., pp. 413–420.

Daniel, I. M., and Ishai, O., 1994, *Engineering Mechanics of Composite Materials*, Oxford University Press, New York.

Jackson, K. E., Kellas, S., and Morton, J., 1992, "Scale Effects in the Response and Failure of Fiber Reinforced Composite Laminates Loaded in Tension and in Flexure." *J. of Composite Materials*, Vol. 26 (18), pp. 2674–2705.

Mandell, J. F., Wang, S.-S., and McGarry, F. J., 1975, "The Extension of Crack Tip Damage Zone in Fiber Reinforced Plastic Laminates," *J. of Composite Materials*, Vol. 9, July, pp. 266–287.

Nallathambi, P., and Karihaloo, B. L., 1986, "Determination of Specimen-Size Independent Fracture Toughness of Plain Concrete," *Mag. of Concrete Res.*, Vol. 38(135), pp. 67–76.

Pijaudier-Cabot, G., and Bažant, Z. P., 1991, "Crack Interacting with Particles or Fibers in Composite Materials." *J. of Engrg. Mech.*, ASCE, Vol. 117(7), July, pp. 1611–1630.

Tada, H., Paris, P. C., and Irwin, G. R., 1985, *The Stress Analysis of Cracks Handbook*, Second Edition, Paris Productions Inc., MO.

Tsai, S. W., and Wu, E. M., 1971, "A General Theory of Strength For Anisotropic Materials." *J. of Composite Materials* Vol. 5, January, pp. 58–80.

Waddoups, M. E., Eisenmann, J. R., and Kaminski, B. E., 1971, "Microscopic Fracture Mechanisms of Advanced Composite Materials," *J. of Composite Materials*, Vol. 5, pp. 446–454.

Whitney, J. M., and Nuismer, R. J., 1974, "Stress Fracture Criteria for Laminated Composites Containing Stress Concentrations." *J. of Composite Materials*, Vol. 8, July pp. 253–264.

Wisnom, M. R., 1992a, "The Relationship Between Tensile and Flexural Strength of Unidirectional Composite." *J. of Composite Materials*, Vol. 26(8), pp. 1173–1180.

Wisnom, M. R., 1992b, "On The Increase In Fracture Energy With Thickness in Delamination of Unidirectional Glass Fiber-Epoxy With Cut Central Plies," *J. of Reinforced Plastics and Composites*, Vol. 11(8), pp. 897–909.