

Creep and Damage in Concrete

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Too much patience can be disastrous for a scientist.

—Solomon H. Snyder

Several recent advances in the mathematical modeling of concrete creep and its interaction with damage or fracture are reviewed and briefly outlined. The solidification theory for concrete creep, in which the effects of aging due to the progress of hydration are modeled by volume growth of a nonaging solidifying constituent, is described. The creep of the nonaging constituent is characterized by a continuous retardation spectrum. Subsequently it is shown how the solidification theory can be generalized to describe the effects of temperature and relative humidity in the pores of concrete on the basic creep, taking into account separately the effect on viscosity and the effect of the rate of aging or hydration. Attention is then focused on the interaction of creep with fracture. A simplified mathematical model for the rate of crack growth, based on the activation theory for bond ruptures, is presented and the derivation outlined. The role of creep in time-dependent fracture is discussed and various experimentally observed phenomena associated with loading at different rates or at changing rates are analyzed. Recent novel experiments that allow separating the apparent drying creep due to microcracking from the drying creep due to stress-induced shrinkage—a phenomenon arising on the level of gel pores—are discussed and the results described. Finally, the effect of the composition of concrete on its creep properties is analyzed mathematically on the basis of a series-parallel model in which the subdivision of the mortar component between the parts coupled in series and in parallel is determined on the basis of the concept of maximum compactness of aggregate. The exposition emphasizes mathematical modeling that is based on the description of physical processes on the microstructural level, which is the key to significant advances in this complex subject.

Introduction

Similar to most quasi-brittle materials such as rocks, ceramics, and fiber composites, portland cement concrete is not capable of plastic or viscoplastic deformation except under very high hydrostatic pressures. The dominant mechanisms of inelastic deformations are creep and cracking damage, which precede fracture. The inelastic properties of concrete are complex and have for a long time been described by essentially empirical models. Recently, however, substantial progress has been made in the understanding of the physical mechanisms of these inelastic processes on the microstructural level. This makes it possible to formulate more realistic mathematical models.

The purpose of the present article is to review some of the recent advances in the mathematical modeling of creep and shrinkage that are based, at least partly, on the physical mechanisms in the microstructure. These include:

1. The solidification theory, which handles the aging aspect of creep, associated with the long-term progress of hydration and other chemical changes in the microstructure of the hardened portland cement paste.
2. The activation energy theory for the effect of temperature on the breakage of bonds that are involved in creep as well as in fracture growth.
3. The role of microcracking and fracture in creep and rate-dependent inelastic deformations.
4. The prediction of the basic creep characteristics of concrete from its composition.

This chapter will explain the basic ideas, the mathematical approaches, and the resulting macroscopic material models, but for the detailed presentations and discussions of some of these recent advances the reader will be referred to other works.

Solidification Theory for the Aging Aspect of Creep

The creep properties of concrete evolve in time and the changes are significant even after many years. This aging (or maturing) behavior, which complicates the mathematical description of creep of concrete, has traditionally been treated simply as a change in the material parameters involved in the constitutive equation for creep. Such an approach is purely phenomenological, and is physically unfounded.

The mechanical properties of chemical substances are generally fixed; they do not change with time. Their change must be regarded as a change in the concentrations of various substances in concrete or hardened portland cement paste. This is the same as in chemical thermodynamics, in which it is implicitly

accepted that properties of a system do not change because of a change in the properties of its constituents but because of a change in the concentrations of some time-invariable constituents.

To describe a system of time-variable properties in the spirit of chemical thermodynamics, the system must be broken down to its time-invariant constituents and the changes in their quantities or volume concentrations must be regarded as the basic variables in the problem. Without such an approach, the thermodynamic theory for concrete viscoelasticity cannot be properly formulated.

In the case of portland cement concrete, the nonaging (or nonmaturing) constituent may be regarded as the hardened portland cement gel (roughly the same as the tobermorite gel), consisting mainly of calcium silicate hydrates. From a more fundamental viewpoint, the hardened cement gel itself might further be broken into certain subconstituents because its structure also undergoes long-term changes. One might conceive of two kinds of processes causing such changes:

1. A sort of a polymerization process that represents a gradual increase of the internal binding (number of bonds) in the hardened cement gel.
2. Gradual long-term relaxation of microstress peaks representing potential creep sites, that is, the sites at which localized deformations that are the source of creep on the macroscale take place.

The former mechanism was postulated some time ago,¹⁻³ but its precise nature has not been pinpointed so far. The latter mechanism, which is here proposed for deeper study, is as yet microscopically unexplored but offers some attractive features that will be explored elsewhere. The stress relaxations of the microstress peaks involved in the latter mechanism would obviously reduce the number of potential sites for the local deformations causing creep. This in turn would cause creep stiffening, which is so significant in concrete even after the lapse of many years.

Consideration of the change of volume of some nonaging constituent is the basic requirement for a sound thermodynamic formulation of the effect of aging on the creep of concrete. Such a description was formulated in Bažant⁴ and was adopted as the foundation of the development of a complete constitutive model in Bažant and Prasannan.⁵ This model is shown in Fig. 1(a), in which it is imagined that layers of a viscoelastic constituent, representing the load-bearing part of the hardened cement gel, gradually precipitate on the surface of the previously solidified constituent. These layers are coupled to the existing solid in parallel (it would hardly be imaginable that they are coupled in series, for reasons explained before). Thus, all the layers have the same strain. Two kinds of processes, one for the viscoelastic part and one for the flow part of creep, are considered separately (Fig. 1(a)). The basic property of the solidification of the

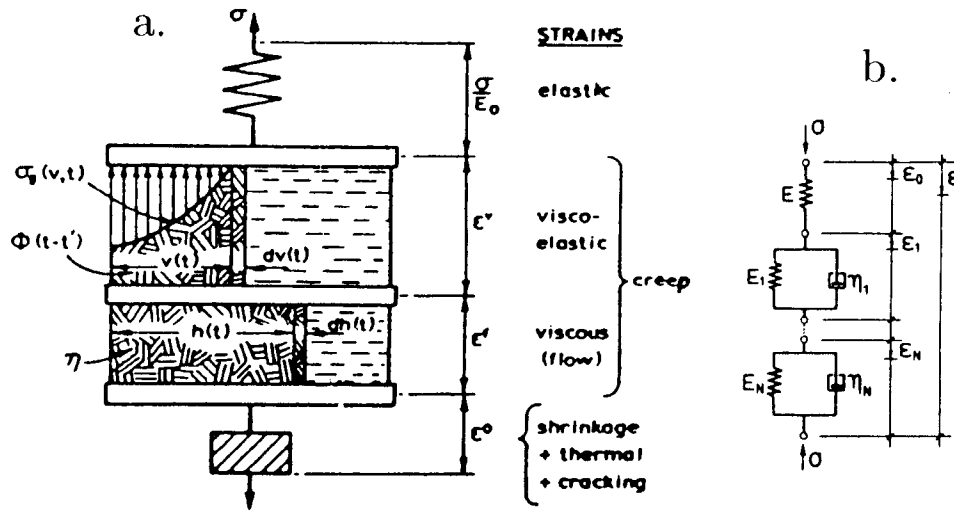


Fig. 1: (a) Schematic model of solidification theory for concrete creep, (b) Kelvin chain model. (Adapted from Ref. 5.)

layers is that they are stress-free at the moment they solidify. This property was crucial in Bažant's derivation⁴ and we will use it now in a somewhat simpler, alternative derivation leading to the same result.

Denote $\Psi(t - t')$ as the relaxation function of the solidifying (nonaging) constituent equal to microstress at time t caused by a unit microstrain imposed since time t' : $v(t')$ is the volume of the constituent solidified up to time t' , and $\epsilon(t')$ is the common strain of all the layers coupled in parallel. A small constant strain introduced at time t' causes at a later time t the macrostress increment $d\sigma(t) = \psi(t - t')v(t')d\epsilon^v(t')$. The stress produced by an arbitrary strain history is thus expressed as

$$\sigma(t) = \int_{t'}^t \Psi(t - t')v(t')d\epsilon^v(t') \quad (1)$$

Denoting the compliance function of the solidifying (nonaging) constituent as $\Phi(t-t')$, which represents the inverse (resolvent) kernel of the creep integral equation based on Ψ , and denoting $\dot{\gamma} = v\dot{\epsilon}$ or $v(t')d\epsilon(t') = d\gamma(t')$, we may write the inverse of the integral equation (1) in the simple form

$$\gamma(t) = \int_{t'}^t \Phi(t - t')d\sigma(t') \quad (2)$$

Differentiating, we obtain

$$\dot{\gamma}(t) = \Phi(0)\dot{\sigma}(t) + \int_{t'}^t \dot{\Phi}(t-t')d\sigma(t') \quad (3)$$

Division by $\nu(t)$ then yields the following constitutive equation expressing the strain rate associated with cement gel as a function of the stress history:⁵

$$\dot{\epsilon}^{\nu}(t) = \frac{1}{\nu(t)} \left[\Phi(0)\dot{\sigma}(t) + \int_{t'}^t \dot{\Phi}(t-t')d\sigma(t') \right] \quad (4)$$

Here $\Phi(0)$ represents the instantaneous compliance (inverse of the elastic modulus) of the solidifying constituent. Note that the reason we started in our derivation with the relaxation function Ψ rather than the creep function Φ is that the strain of all the solidifying layers is the same. This derivation was shown in Carol and Bazant.⁶ It is possible to base the derivation exclusively on $\Phi(t-t')$, however, the procedure is more complicated.⁴

It turns out that for a good description of concrete creep data it is necessary to consider two distinct solidification processes, corresponding to the viscoelastic strain ϵ^{ν} and the flow strain ϵ^f shown in Fig. 1(a). However, it also appears that for the flow strain there is no delayed elasticity (no memory, no hereditary effect). It suffices to consider the rate of the flow to depend only on the current stress, although the viscosity $\eta(t)$ characterizing this relation is age-dependent. This special form of the constitutive characteristic of ϵ^f greatly simplifies the formulation.

According to the foregoing relations illustrated by Fig. 1, the constitutive law for linear aging creep of concrete may be written as:

$$\dot{\epsilon} = \frac{\dot{\sigma}(t)}{E_0} + \dot{\epsilon}^{\nu} + \dot{\epsilon}^f \quad (5)$$

in which

$$\dot{\epsilon}^{\nu}(t) = \frac{1}{\nu(t)} \dot{\gamma}(t), \quad \dot{\epsilon}^f(t) = \frac{\sigma(t)}{\eta(t)} \quad (6)$$

$$\gamma(t) = \int_{t_0}^t \Phi(t-t')d\sigma(t') \quad (7)$$

Here σ and ϵ are uniaxial stress and strain in concrete, respectively, γ is effective strain in the solidifying constituent on the microscale, ϵ^{ν} is viscoelastic strain, ϵ^f is viscous strain (or flow), $\eta(t)$ is viscosity for flow, which depends on the age

t of concrete, $v(t)$ is volume (or volume concentration) of the solidified constituent at age t , $E_0 = 1/\Phi(0)$ is asymptotic elastic modulus representing the material stiffness for extremely fast deformations (extrapolated to zero load duration, and $\Phi(t - t')$ is nonaging linear viscoelastic microcompliance function for the solidifying constituent (t' is the integration variable). Note that the solidification theory prohibits replacing Eq. (6) by $\epsilon^v(t) = \gamma(t)/v(t)$, because this would violate the second law of thermodynamics (however, some formulations that imply such a relation can be found in the literature).

The aging is described by two functions, $v(t)$ and $\eta(t)$. It was determined that one function would not suffice, except as an approximation when long-term creep of concrete loaded at a young age is not of interest. Note also that the compliance function Φ should include even the very rapid short-term creep that causes the difference between the dynamic elastic modulus and the static elastic modulus; otherwise the elastic modulus E would have to be considered as age-dependent and the formulation would actually become more complex. The conventional value of $1/(E(t))$ at age t , as defined by building codes, is obtained as the strain for $\sigma = 1$ and stress duration of about 0.1 day.

Based on the essential properties of the creep curves observed in experiment, the following expressions were proposed and verified by test data:⁵

$$\Phi(t - t') = q_2 \ln(1 + \xi^n), \quad \text{with } \xi = (t - t_0)/\lambda_0 \quad (\lambda_0 = 1 \text{ day}) \quad (8)$$

$$\frac{1}{\eta(t)} = \frac{q_4}{t}, \quad \frac{1}{v(t)} = \left[\frac{\lambda_0}{t} \right]^m + \alpha \quad (9)$$

in which t_0 is the time of applying a constant unit uniaxial stress; m , n , and λ_0 are constants that can be fixed once for all the concretes; m equals 0.5; n equals 0.1; and q_2 and q_4 are empirical parameters to be identified by fitting of creep test data on each particular concrete or predicted from some approximate creep prediction model based on the composition and strength of concrete.

The macroscopic uniaxial compliance function for creep that ensues from Eqs. (8) and (9) is then obtained as

$$J(t, t_0) = q_1 + \int_{t_0}^t j(\tau, t_0) d\tau \quad \left(q_1 = \frac{1}{E_0} \right) \quad (10)$$

and the compliance rate is

$$\dot{J}(t, t_0) = \frac{\dot{\Phi}(t - t_0)}{v(t)} + \frac{1}{\eta(t)} = \frac{n(q_2 t^{-m} + q_3)}{(t - t_0) + (t - t_0)^{1-n}} + \frac{q_4}{t} \quad (q_3 = \alpha q_2) \quad (11)$$

Here q_1 and q_3 are additional empirical constants to be identified from test data for each particular concrete or predicted from the composition and strength.

An important advantage of this formulation is that the parameters $q_1 \dots q_4$ are all involved linearly, which means that they can be obtained from test data by linear regression (which is always convergent, unlike nonlinear regression).

The compliance function is given according to Eq. (10) by an integral that, for the expression in Eq. (11), is a binomial integral and is not integrable in a closed form. This is no problem for computer programs solving the creep problem in time steps, because only the compliance rate, J , is needed for that purpose. For simple code-type calculations, the values of J can be obtained by numerical integration of Eq. (11) or can be calculated from a simple explicit approximate formula in Ref. 5, where tables and graphs for the values of J are also given.

Aside from the physical and thermodynamic justification, the solidification theory also has two important practical advantages. First, this theory guarantees avoidance of the so-called divergence of the creep curves for different ages at loading, which further avoids problems with nonmonotonic creep recovery and nonmonotonic relaxation recovery. Second, this theory lends itself to a much simpler rate-type formulation suitable for finite element programs, which we review next.

The integral-type description of linear viscoelastic behavior is a strictly phenomenological concept lacking any physical basis. It implies the notion of memory, yet the material has no memory per se. The memory property arises indirectly. It is embedded in the current values of many variables characterizing the state of the microstructure. These variables are approximately reflected in continuum theories of materials by means of the so-called internal variables (or hidden variables). Their current values take into account the previous stress or strain history of the material.

The internal variables, in linear viscoelasticity, can be regarded as the deformations or stresses in the elements of spring-dashpot rheological models. Although there are infinitely many possible geometric configurations of such models, it was proven by Roscoe,⁷ for nonaging viscoelasticity, that the most general possible viscoelastic behavior can be approximated with any degree of accuracy by either the Kelvin chain model or the Maxwell chain model. This makes it unnecessary to consider any more complicated models. Each unit of such a model is described by a first-order linear differential equation in time, and thus the integral-type creep law is converted to a system of first-order linear

differential equations in time. These differential equations, unlike the integral equation, can be correlated to various physical concepts for the processes in the microstructure, for example, the activation energy-controlled rate process for the breakages of bonds that are the source of creep.

The rate-type form of the creep law can be obtained by expanding the micro-compliance function Φ into Dirichlet series (or Dirichlet-Prony series):^{8,9}

$$\Phi(\xi) = \sum_{\mu=1}^N \frac{1}{E_{\mu}} \left(1 - e^{-\xi/\tau_{\mu}}\right), \quad (\xi = t - t') \quad (12)$$

in which ξ is the elapsed time from the moment of loading, t' ; τ_{μ} is a constant, called the retardation time; and E_{μ} is the associated elastic modulus. It can be shown that these moduli correspond to the spring constants of the Kelvin chain model (Fig. 1(b)) and that τ_{μ} equals η_{μ}/E_{μ} where η_{μ} is the viscosity of the dashpot of the μ th Kelvin unit. The expansion in Eq. (12) makes it possible to replace the integral-type constitutive relation based on the principle of superposition by the following system of first-order linear differential equations:^{8,9}

$$E_{\mu}\dot{\gamma}_{\mu} + \eta_{\mu}\dot{\gamma}_{\mu} = \dot{\sigma}, \quad (\mu = 1 \dots N), \quad \text{with } \gamma = \frac{\sigma}{E_0} + \sum_{\mu} \gamma_{\mu} \quad (13)$$

in which a set of internal variables γ_{μ} , representing the strains of the individual units of the Kelvin chain, is introduced, and $\eta_{\mu} = \tau_{\mu}E_{\mu}$ are viscosities of dashpots.

The most efficient method of step-by-step integration of these differential equations, which leads to an algorithm that is stable for every value of the time step, is obtained by assuming that σ , $\dot{\gamma}$, $\dot{\gamma}_{\mu}$, E_{μ} , and η_{μ} are constant within each time step and change only by jumps between the time steps. Then the differential equation (13) can be integrated for each time step exactly. This yields the so-called exponential algorithm, which was formulated for nonaging materials in Zienkiewicz, Watson, and King¹⁰ and Taylor, Pister, and Goudreau,¹¹ and for aging materials in Bažant¹² and Bažant and Wu.^{13,14} To the solidification theory, this algorithm was adapted in Bažant and Prasanna.⁶

Continuous Retardation Spectrum

The diagram of the values of $1/E_{\mu}$ vs $\log \tau_{\mu}$ (Eq. (12)) is called the retardation spectrum (Fig. 2(a)). For a Kelvin chain model with a finite number of Kelvin units, this spectrum is discrete, consisting of a set of points. However, it is advantageous to consider a generalization of Eq. (12) in which the spectrum

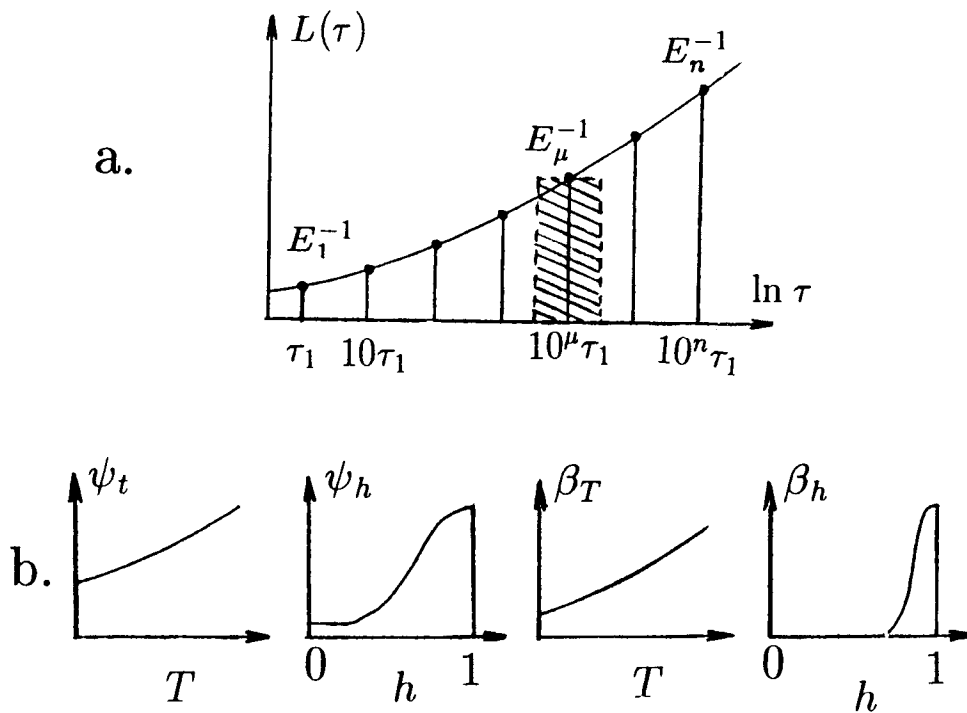


Fig. 2. (a) Discrete and continuous retardation spectra, (b) functions defining dependence of creep viscosities and hydration rate on temperature and humidity.

is continuous (Fig. 2(a)), that is, there are infinitely many Kelvin chain units with retardation times τ distributed infinitely closely. According to this generalization, well known from classical viscoelasticity,¹⁵ one has, as the limit case of Eq. (12),

$$\Phi(\xi) = \int_{-\infty}^{\infty} L(\tau) (1 - e^{-\xi/\tau}) d(\ln \tau) \quad (14)$$

in which the function $L(\tau)$ characterizes the continuous spectrum and ξ is the duration of unit uniaxial stress. The solidification theory has the major advantage that this spectrum is age-independent.

The continuous spectrum has one important advantage over the discrete spectrum: it eliminates the aforementioned ill-posedness and ambiguity in the determination of the retardation spectrum from the given compliance data. For the discrete spectrum corresponding to Eq. (12), the problem of determining the unknown material parameters τ_μ and E_μ from the given test data is ill-posed,¹⁶ that is, very different sets of E_μ values can give nearly equally good fits of the given creep data on $\Phi(\xi)$. If both τ_μ and E_μ are being determined from test data

(e.g., by the method of least squares), the resulting system of equations is ill-conditioned. Now, for the continuous retardation spectrum, the values of τ need not be determined because τ is a continuous variable, and so the problem of determining the function $L(\tau)$ cannot be ill-conditioned. This means that Eq. (14) can be unambiguously fitted to the given creep compliance data. The discrete spectrum for Eq. (12) and the corresponding discrete Kelvin chain model can then be obtained by a numerical quadrature of $L(\tau)$.

Introducing a new variable ζ by setting $\tau = 1/\zeta$, we get from Eq. (14):¹⁷

$$\begin{aligned}\Phi(\xi) &= - \int_{-\infty}^{\infty} L \left(\frac{1}{\zeta} \right) (1 - e^{-\zeta\xi}) d(\ln \zeta) \\ &= \int_0^{\infty} \frac{1}{\zeta} L \left(\frac{1}{\zeta} \right) e^{-\zeta\xi} d\zeta - \int_0^{\infty} \frac{1}{\zeta} L \left(\frac{1}{\zeta} \right) d\zeta = f(\xi) - f(0)\end{aligned}\quad (15)$$

where $f(0)$ is a constant. We now recognize that function $f(\xi)$ represents the Laplace transform of the function $\zeta^{-1}L(\zeta^{-1})$. So the continuous spectrum can be obtained from the measured compliance function by means of an inversion of Laplace transform. For the inversion, it is convenient to use Widder's formula known from linear viscoelasticity:^{15,18}

$$L(\tau) = \lim_{k \rightarrow \infty} L_k(\tau), \quad L_k(\tau) = \frac{(-k\tau)^k}{(k-1)!} f^{(k)}(k\tau) = - \frac{(-k\tau)^k}{(k-1)!} \Phi^{(k)}(k\tau) \quad (16)$$

(for $k \geq 1$). This inversion formula can be applied only if the compliance function $\Phi(\xi)$ is sufficiently smooth, that is, it has derivatives of a sufficiently high order and no jumps. Therefore, experimental data must be fitted by a smooth function before Eq. (13) is applied. In Xi and Bažant,¹⁷ where the foregoing formulation of a continuous spectrum is described in more detail, $\Phi(\xi)$ is approximated by the log-power law, which is defined as

$$\Phi(\xi) = q_2 \ln [1 + (\xi/\lambda_0)^n], \quad \text{with } \lambda_0 = 1 \text{ day} \quad (17)$$

in which λ_0 , n , and q_2 are three empirical constants. Applying Widder's formula (Eq. (16)) of order $k = 3$, we obtain

$$L(\tau) = \left[\frac{-2n^2(3\tau)^{2n-3} [n-1 - (3\tau)^n]}{[1 + (3\tau)^n]^3} + \frac{n(n-2)(3\tau)^{n-3} [n-1 - (3\tau)^n] - n^2(3\tau)^{2n-3}}{[1 + (3\tau)^n]^2} \right] \frac{(3\tau)^3}{2} q^2 \quad (18)$$

Because the value of n is quite small ($n = 0.1$), the foregoing expression may be approximated by:¹⁷

$$L(\tau) \simeq n(1-n) \frac{(3\tau)^n}{1 + (3\tau)^n} q_2 \quad (19)$$

For computational purposes, the continuous spectrum may be approximated by a discrete spectrum (Fig. 2(a)) corresponding to a Kelvin chain with a finite number of units and $\tau_\mu = \tau_1 10^{\mu-1}$. According to the trapezoidal integration rule,

$$\frac{1}{E_\mu} = L(\tau_\mu) \ln 10 \log \tau_\mu \quad (20)$$

The reason for first determining the continuous spectrum is twofold: a smooth continuous spectrum has no ambiguity and no ill-posedness in the identification of its parameters, and it can be obtained by explicit formulas.

Effect of Temperature and Humidity on Basic Creep

There have been repeated attempts to directly modify various empirical forms of the compliance function for variable temperature and variable humidity. However, such attempts are unsatisfactory since violations of the basic laws of thermodynamics are likely to occur.

To ensure that the laws of thermodynamics are properly satisfied, it is necessary to base the creep law on a rheologic model such as the Kelvin or Maxwell chain and introduce the effects of variable temperature and humidity into the equations for the dashpot viscosities and spring moduli in these models. At constant temperature, the effect of aging is properly introduced by writing the equation for a spring in a rheologic model as $\dot{\sigma} = E(t)\dot{\epsilon}$, and the equation for a dashpot as $\sigma = \eta(t)\dot{\epsilon}$, in which σ and ϵ are the stress and strain in the spring of the dashpot, respectively, E is the spring modulus, η is the dashpot viscosity, which depends on age t , and the superior dots denote derivatives with respect to time t .

Some form of Dirichlet series expressions for the compliance function introduced in the past imply for the aging spring or dashpot the relation $\sigma = E(t)\epsilon$ or $\dot{\sigma} = \eta(t)\dot{\epsilon}$, which, however, has been shown to violate the second law of thermodynamics, given that the material is solidifying (in this regard, the important fact is that the layers of new solidified material must be stress-free at the moment they solidify).

The effect of temperature and humidity variation is twofold: to alter the rate of the chemical reactions of hydration, and to alter the viscosities of the processes involved in creep deformation, which are characterized by the equations for the dashpots. According to this physically justified approach, the equations for the springs and dashpots in a rheologic model should be written as:

$$\dot{\epsilon} = \frac{\dot{\sigma}_s}{E(t_e)}, \quad \dot{\epsilon} = \frac{\sigma_d}{\eta(t_e)} \psi(t) \quad \text{or} \quad \frac{d\epsilon}{dt_r} = \frac{\sigma_d}{\eta(t_e)} \quad (21)$$

in which σ_s and σ_d are stresses in the spring or dashpot and

$$t_r = \int \psi(t) dt, \quad \psi(t) = \psi_T(t) \psi_h(t) \quad (22)$$

$$\psi_T(t) = \exp \left[\frac{Q_v}{RT_0} - \frac{Q_v}{RT(t)} \right], \quad \psi_h(t) = 0.1 + 0.9h^2 \quad (23)$$

$$t_e = \int \beta(t) dt, \quad \beta(t) = \beta_T(t) \beta_h(t) \quad (24)$$

$$\beta_T(t) = \exp \left[\frac{Q_h}{RT_0} - \frac{Q_h}{RT(t)} \right], \quad \beta_h(t) = [1 + (5 - 5h)^4]^{-1} \quad (25)$$

Here t_r is reduced time (characterizing the rate of viscous processes), t_e is equivalent age (characterizing the process of solidification), T is absolute temperature, T_0 is reference temperature, Q_v and Q_h are activation energies for the viscous processes (creep rates) and for the solidification (or hydration), approximately $Q_h/R \approx 2700$ K and $Q_v/R \approx 5000$ K, respectively, R is universal gas constant, and h is relative humidity (vapor pressure) in the capillary pores of concrete. Whereas the expressions for coefficients ψ_T and β_T (Fig. 2(b)) are based on the rate-process theory of thermally activated processes, the expressions for coefficient ψ_h and β_h are empirical (Fig. 2(b)), describing the fact that the creep rate as well as the hydration rate is lower for a lower water content of concrete (Fig. 2(a)). It must be emphasized that the effect of pore humidity in

the foregoing equations describes only the effect of pore humidity on the basic creep. It does not describe the additional drying creep, which we will discuss later. The basic creep is defined as the creep at no moisture movements, that is, in sealed specimens. Thus, to test the basic creep at a lowered pore humidity h , the specimen must first be predried to moisture equilibrium at environmental humidity h , and only then loaded. This is of course feasible only for extremely thin specimens.¹⁹

In the solidification theory, the effects of viscosity and solidification rate are sufficiently separated to make it possible to express the compliance rate explicitly, in a closed form. If we adopt for the nonaging constituent the Kelvin chain model, we can generalize the foregoing rate equations for the Kelvin chain units at variable temperature and humidity as follows:

$$E_{\mu} \dot{\gamma}_{\mu} + \frac{\eta_{\mu}^0}{\psi(t)} \dot{\gamma}_{\mu} = \sigma \quad (\mu = 1 \dots N) \quad (26)$$

in which η_{μ}^0 are the viscosities of the dashpots in Kelvin chain units at reference temperature T_0 . Integration yields the creep compliance function for the nonaging solidifying constituent in the form

$$\Phi(t - t') = \sum_{\mu=1}^N \frac{1}{E_{\mu}} \left(1 - e^{-(t_r - t'_r)/\tau_{\mu}} \right) \quad (27)$$

in which the reduced times are $t_r = t_r(t)$, $t'_r = t'_r(t')$. Similarly to Eq. (17), we may consider Eq. (27) to be the expansion of $\Phi(t - t') = q_2 \ln\{1 + [(t_r - t'_r)/\lambda_0]^n\}$, and so

$$\dot{\Phi}(t - t') = \frac{q_2 n \xi^{n-1}}{1 + \xi^n}, \quad \xi = \frac{t_r - t'_r}{\lambda_0} \quad (28)$$

The effect of temperature and humidity on aging is introduced through the volume change function as follows

$$\frac{1}{v_e(t)} = \frac{1}{v[t_e(t)]} = \left[\frac{\lambda_0}{t_e(t)} \right]^{-m} + \alpha \quad (29)$$

In the viscous flow term, the viscosity must be modified by the creep rate coefficients ψ_t and ψ_h , and at the same time it must depend on the equivalent age expressing the effects of temperature and humidity on the hydration rate

$$\dot{\epsilon}^f(t) = \frac{\sigma(t)}{\eta(t)} = \psi(t) \frac{\sigma(t)}{\eta[t_e(t)]} \quad (30)$$

Combining the viscoelastic and flow terms, we obtain the macroscopic compliance rate for basic creep at variable temperature and humidity in the form:

$$j(t, t') = \frac{\dot{\Phi}(t-t')}{v_e(t)} + \frac{1}{\eta(t)} = \frac{n(q_2 t_e^{-m} + \alpha)}{(t_r - t'_r) + (t_r - t'_r)^{1-n}} + \frac{q_4}{t_e} \psi_T(t) \phi_h(t) \quad (31)$$

This represents the proper generalization of Eq. (11).

It is important to note the difficulties in generalizing the Dirichlet series expansion of the total compliance function of concrete. This expression reads:

$$J(t, t') = \sum_{\mu=1}^N \frac{1}{C_{\mu}(t')} \left(1 - e^{-(t_r - t'_r)/\tau_{\mu}} \right) \quad (32)$$

in which we have introduced the reduced time $t_r(t)$ to take into account the dependence of viscosities on temperature and humidity. However, it would be incorrect to replace functions $C_{\mu}(t')$ with functions $C_{\mu}(t'_e)$ of the equivalent time defined by Eq. (24).

To show it, we write the total strain according to the principle of superposition, $\epsilon(t) = \int_0^t J(t, t') d\sigma(t') = \sum_{\mu} \epsilon_{\mu}(t)$. It is now easy to check that the partial strains ϵ_{μ} , corresponding to the individual Kelvin units, satisfy the differential equations

$$\left[\frac{\tau_{\mu}}{t_r} \dot{\epsilon}_{\mu} \right]' + \dot{\epsilon}_{\mu} = \frac{\dot{\sigma}}{C_{\mu}} \quad \text{or} \quad \left[\frac{C_{\mu} \tau_{\mu}}{t_r} \dot{\epsilon}_{\mu} \right]' + \left[C_{\mu} - \frac{\tau_{\mu} \dot{C}_{\mu}}{t_r} \right] \dot{\epsilon}_{\mu} = \dot{\sigma} \quad (33)$$

Now the stress in a dashpot of a Kelvin unit is $\eta_{\mu} \dot{\epsilon}_{\mu}$ and the stress rate in the spring of a Kelvin unit is $E_{\mu} \dot{\epsilon}_{\mu}$, and so the equation of equilibrium in the Kelvin unit is $(\eta_{\mu} \dot{\epsilon}_{\mu})' + E_{\mu} \dot{\epsilon}_{\mu} = \dot{\sigma}$. Comparing this to Eq. (33), we conclude that the viscosities and spring moduli of the Kelvin units are expressed as:^{3,12}

$$\eta_{\mu}(t) = \frac{\tau_{\mu}}{t_r(t)} C_{\mu}(t), \quad E_{\mu}(t) = C_{\mu}(t) - \frac{\tau_{\mu} \dot{C}_{\mu}(t)}{t_r(t)} \quad (34)$$

To introduce the effect of aging, that is, the acceleration of hydration due to temperature or humidity, one must replace t with t_e in the functions η_{μ} and E_{μ} . Equation (34) can be reduced to a single differential equation for C_{μ} , namely $\eta_{\mu}(t) \dot{C}_{\mu} - C_{\mu} [E_{\mu}(t) - C_{\mu}] = 0$. This is a nonlinear differential equation from which C_{μ} as a function of time cannot be solved in a closed form. If the dependence of η_{μ} on T and h is introduced into the last differential equation, it becomes clear that the solution C_{μ} depends not only on t_e but also on the histories of T and h .

Therefore, generalization of Eq. (32) to aging at various temperatures and humidities cannot be accomplished in a closed form. On the other hand, such a generalization can be made in a closed form when creep is characterized by the relaxation function $R(t, t')$ expressed in the form of a Dirichlet series.^{3,12} However, the use of the relaxation function is less convenient for concrete because the material data are obtained predominantly from creep rather than relaxation tests.

Crack Opening Rate

The interaction of creep with damage or fracture is an important subject that has only recently come to the forefront of attention. Although classical fracture mechanics is a time- and rate-independent theory, fracture is in reality a time-dependent process. There is no instantaneous fracture.

Fracture results from the breakage of bonds between atoms or molecules, and breakage of bonds is a thermally activated process promoted by stress. The thermal vibration energies of molecules or atoms have random values following the Maxwell distribution. Applying stress, the probability that an atom or molecule would jump over its activation energy barrier Q is altered in proportion to the stress. This leads to the following dependence of the bond breakage rate (and thus the rate of displacement v due to fracture) on the temperature:

$$\dot{w} = C_T \sinh \left[\frac{T_0}{T} k_0 \sigma_b \right] \quad (35)$$

with

$$\sigma_b = \sigma - \phi(w), \quad C_T = \exp \left[\frac{Q}{RT_0} - \frac{Q}{RT} \right] \quad (36)$$

Here w is the crack opening displacement, used in the cohesive (fictitious) crack model, σ is the crack macroscopic bridging stress (cohesive stress, Fig. 3(d)), Q is the activation energy of bond breakage, k_0 is a constant (depending on the activation volume), and σ_b is assumed to be proportional to the typical bond stress. The function $\phi(w)$ is similar to the dependence of the crack bridging stress on the crack opening displacement used in the time-independent cohesive (fictitious) crack model, but is scaled down by a ratio of approximately 0.8–0.9 and is shifted up to approximately $0.1 f'_c$ (see Fig. 3(e)).

Let us now outline a simplified derivation of Eqs. (35) and (36). The atoms or molecules in solids are in permanent random vibratory motion about their equilibrium positions representing the minimum potential of the binding forces. The potential energy surface of the bond force, sketched in Fig. 3(a,b), exhibits maxima representing energy barriers called the activation energy, Q . If the energy of the atom or molecule exceeds Q , the atom or molecule can jump over the activation energy barrier. This represents rupture of the bond (Fig. 3(c)).

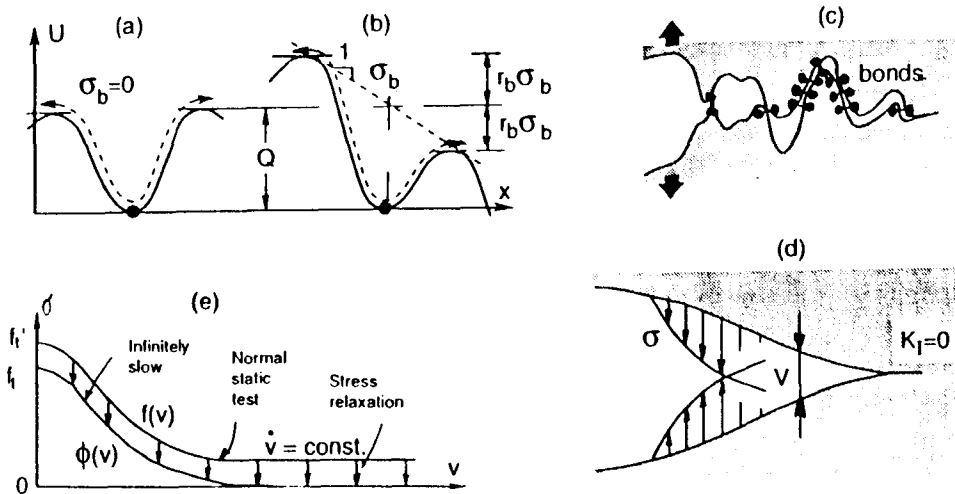


Fig. 3. (a) Potential of bond forces, (b) change of potential due to stress, (c),(d) bond ruptures and bridging stresses in a cohesive crack, (e) stress-displacement relation (adapted from Ref. 43).

The frequency of the jumps of atoms or molecules over their activation energy barrier Q controls the rate of the rupture process (it also controls many other processes, such as chemical reactions, diffusion, adsorption, creep, etc.). The statistical distribution of the thermal energies of atoms or molecules is well known. It follows a famous simple law called the Maxwell-Boltzmann distribution, which reads

$$f = c_1 e^{-\epsilon/RT} \quad (37)$$

where f is the frequency of attaining energy level ϵ at absolute temperature T , R is the universal gas constant, and c_1 is a constant.

Equation (37) may be derived in a manner similar to that used for a gas by Cottrell.²⁰ The key idea is to consider the laws governing energy interactions between adjacent atoms and molecules (which constitute the process by which heat is transmitted). Denote as v the maximum velocity that the particle (atom or molecule) has at the moment it crosses the equilibrium position. Let $F(v)dv$ be the number of particles with maximum velocities in the interval $(v, v + dv)$ in a certain specified direction. The rate of interactions of particles of velocity v_1 with adjacent particles of velocity v_2 is obviously proportional to both $F(v_1)$ and $F(v_2)$, and so it must be proportional to the product $F(v_1)F(v_2)$. This is in fact a joint probability. After the interaction (which represents a collision or a sufficiently close approach), the velocities of particles are denoted as $F(v_3)$ and $F(v_4)$. Again, the rate at which these particles interact must be proportional to the product $F(v_3)F(v_4)$. This product must be the same as before the collision. Taking logarithms, we may write this condition as

$$\ln F(v_1) + \ln F(v_2) = \ln F(v_3) + \ln F(v_4) \quad (38)$$

The maximum kinetic energies of the atoms or molecules are proportional to v_1^2 or v_2^2 . The kinetic energy must be preserved, and so

$$v_1^2 + v_2^2 = v_3^2 + v_4^2 \quad (39)$$

Equations (38) and (39) must be valid for any values of v_1, v_2, v_3 , and v_4 . This is possible if and only if $\ln F(v) \sim v^2$. Therefore, the distribution, called the Maxwell-Boltzmann distribution, must have the form:

$$F(v) = C_0 e^{-\kappa v^2} \quad (40)$$

in which C_0 and κ are certain constants.

Let us now determine the constant κ . Since it multiplies v^2 , it obviously should be related to the kinetic energy of the atoms or molecules, that is, to the absolute temperature. At this point we need to realize that velocity v is a vector, although we have so far neglected the vectorial aspect. We will now consider it, both for particle interactions in a plane or layer (two-dimensional) or in a space (three-dimensional). In the case of interactions in a plane or layer, the tips of all the velocity vectors lying between v and $v + dv$ occupy a circular annulus of area $2\pi v dv$, and so their number is $\kappa_1 v^m F(v) dv$ where m equals 1 and κ_1 is a constant. In the case of interactions in space, the tips of all the velocity vectors lying between v and $v + dv$ occupy a spherical shell of volume for $4\pi v^2 dv$, and so their number is $\kappa_1 v^m F(v) dv$ where m equals 2 and κ_1 is a constant. Thus, in general, the number of interacting particles with velocities between v and $v + dv$ in either a plane or a space may be written as $n(v) dv = \kappa_1 v^m F(v) dv$. Now, as known from thermodynamics, absolute temperature $T = (\text{total kinetic energy of all particles})/N$, where $N = \int_0^\infty n(v) dv =$ number of all interacting particles. It follows that:

$$T = \frac{\int_0^\infty \frac{m_p}{2} v^2 \kappa_1 v^m F(v) dv}{\int_0^\infty \kappa_1 v^m F(v) dv} = \frac{m_p}{2} \frac{\int_0^\infty v^{m+2} C_0 e^{-\kappa v^2} dv}{\int_0^\infty v^m C_0 e^{-\kappa v^2} dv} \quad (41)$$

$$= \frac{m_p}{2} \frac{\int_0^\infty \kappa^{-(m/2+1)} z^{m+2} C_0 e^{-z^2} \frac{dz}{\sqrt{\kappa}}}{\int_0^\infty \kappa^{-m/2} z^m C_0 e^{-z^2} \frac{dz}{\sqrt{\kappa}}}$$

where m_p is particle mass. After simplification, the result is

$$T = \frac{m_p}{2\kappa} C_1, \quad C_1 = \frac{\int_0^\infty z^{m+2} e^{-z^2} dz}{\int_0^\infty z^m e^{-z^2} dz} = \text{constant} \quad (42)$$

From this we obtain the meaning of constant κ ; $\kappa = m_p C_1 / 2T = 1/k_0 T$, $k_0 = 2/m_p C_1 = \text{constant}$. Thus we obtain the frequency of particle interactions as a function of absolute temperature:

$$F(v) = C_0 e^{-v^2/k_0 T} \quad (43)$$

Next we try to determine coefficient C_0 . The result would depend on whether we would consider interactions in space or plane, and it turns out it agrees with the conventional empirically established relations when we consider a plane (a layer). For the purpose of fracture, this is certainly not unreasonable since the ruptures of interparticle bonds that produce fracture happen on a plane (the future fracture surface), that is, are not dispersed through the volume of the material. The key consideration now is that the number of all particles ought to remain the same if temperature T is changed. This means that $\int_0^\infty 2\pi v F(v) dv = N = \text{constant}$. Then, substituting for function F from Eq. (43), we have $2\pi C_0 \int_0^\infty v e^{-v^2/k_0 T} dv = N$, in which the integral can be easily evaluated, providing $2\pi C_0 (k_0 T/2) = N$. Thus we conclude that $C_0 = C_1/T$ in which $C_1 = N/\pi C_0 k_0 = \text{constant}$. This leads to the following final equation for the frequency of encountering maximum particle velocity v at temperature T :

$$F(v) = \frac{C_1}{T} e^{-v^2/k_0 T} \quad (44)$$

From this frequency distribution of velocity, we can obtain the cumulative distribution giving the number of particles whose velocity v' exceeds a given velocity v in all directions of a plane (layer):

$$\begin{aligned} \nu(v) &= \int_0^\infty n(v') dv' = \int_v^\infty 2\pi v' f(v') dv' \\ &= \frac{2\pi C_1}{T} \int_v^\infty v' e^{-v'^2/k_0 T} dv' = \frac{2\pi C_1}{T} \left[-\frac{k_0 T}{2} e^{-v'^2/k_0 T} \right]_v^\infty \end{aligned} \quad (45)$$

Now we recognize that the maximum kinetic energy of the particle, that is, its energy as it passes its equilibrium position, is $\mathcal{E} = m_p v^2/2$, which means $v^2 = 2\mathcal{E}/m_p$. Substituting this into Eq. (45) and denoting $\bar{R} = m_p k_0/2$, we obtain Eq. (37) that we set forth to derive.

The rate of fracture is proportional to the number of bond ruptures, which is equal to the number of particles whose kinetic energy exceeds a certain energy value ϵ representing a material property. The potential energy barrier that needs to be overcome by the kinetic energy of the particle at zero applied stress is called the activation energy, Q . Under applied stress σ , the bond potential is modified as shown in Fig. 3(b), where a gradient equal to σ is superimposed on

the potential energy diagram at zero stress (because the work of moving against the stress, per unit volume, is equal to σx). Therefore, the activation energy barriers against a particle making a jump opposite or along the stress direction (right or left in Fig. 3(b)) are $\epsilon = Q \pm \kappa \sigma$ ($\kappa = \text{constant}$). Substituting this into Eq. (37), we get the frequency f_1 of exceeding the activation energy barrier for jumps to the right and f_2 to the left. Subtracting these two, we get the net rate of movement of particles to the right:

$$\begin{aligned} \Delta f &= f_2 - f_1 = C_1 e^{-\frac{Q-\kappa\sigma}{RT}} - C_1 e^{-\frac{Q+\kappa\sigma}{RT}} \\ &= C_1 e^{-\frac{Q}{RT}} \left(e^{\frac{\kappa}{RT}\sigma} - e^{-\frac{\kappa}{RT}\sigma} \right) \end{aligned} \quad (46)$$

This result may be written as

$$\Delta f = C \sinh \left(\frac{\kappa}{RT} \sigma \right) e^{-Q/RT} \quad (47)$$

in which $C = C_1/2$ and κ and R are constants. From this, Eq. (35) follows.

The foregoing argument, strictly speaking, applies only when the kinetic energy of a particle is constant from collision to collision, which is true for gases but not solids. However, for the same reasons given by Cottrell²⁰ (p. 15), the same result should nevertheless apply also for solids, in which the kinetic energy varies during each vibration period between 0 and its maximum.

Interaction Creep with Fracture or Damage

Equation (35) needs to be complemented by the compliance relation between the crack stresses and the displacements due to the loads applied on the structure. The elastic form of this relation can be written in the form^{21,22}

$$w(x) = \int_{a_0}^a \bar{C}(x, \xi) E^{-1} \sigma(\xi) d\xi + \bar{C}_P(x) E^{-1} P \quad (48)$$

$$u = \int_{a_0}^a \bar{C}_P(\xi) E^{-1} \sigma(\xi) d\xi + \bar{C}_{PP} E^{-1} P \quad (49)$$

in which P is applied load, u is load-point displacement, a is length of the cohesive crack, a_0 is length of notch or initial traction-free crack, and \bar{C} , \bar{C}_P , and

\bar{C}_{PP} are the geometric compliances (that is, compliances for $E = 1$). These compliances can approximately be determined by the finite element method. In that case the integrals are replaced by discrete sums, which leads to a formulation essentially equivalent to that used by Petersson.²³

The foregoing compliance relations must be converted to creep. The most direct way is to implement the principle of superposition based on the compliance function of the material, $J(t, \tau)$. In that case Eqs. (48) and (49) are replaced by:

$$w(x, t) = \int_{a_0}^a \bar{C}(x, \xi) \int_{t_0}^t J(t, \tau) \sigma(\xi, d\tau) d\xi + \bar{C}_P(x) \int_{t_0}^t J(t, \tau) P(d\tau) \quad (50)$$

$$u(t) = \int_{a_0}^a \bar{C}_P(\xi) \int_{t_0}^t J(t, \tau) \sigma(\xi, d\tau) d\xi + \bar{C}_{PP} \int_{t_0}^t J(t, \tau) P(d\tau) \quad (51)$$

Here, for continuous variation of P and σ , $\sigma(\xi, d\tau) = [\partial\sigma(\xi)/\partial\tau]d\tau$, $P(d\tau) = dP(\tau) = [dP(\tau)/d\tau]d\tau$.

Computationally, however, Eqs. (50) and (51) are inefficient because long sums must be evaluated in each time step of a numerical integration procedure. It is much more efficient to use the rate-type form of the creep law, which leads to the following system of integrodifferential equations:

$$\dot{w}(x) = \int_{a_0}^a \bar{C}(x, \xi) \left[\frac{\dot{\sigma}(\xi)}{E_0} + \frac{\sigma(\xi)}{\eta(t)} \right] d\xi + \frac{1}{v(t)} \sum_{\mu} \dot{w}_{\mu}(x) \quad (52)$$

$$\dot{u} = \int_{a_0}^a \bar{C}_P(\xi) \left[\frac{\dot{\sigma}(\xi)}{E_0} + \frac{\sigma(\xi)}{\eta(t)} \right] d\xi + \frac{1}{v(t)} \sum_{\mu} \dot{u}_{\mu} \quad (53)$$

$$w_{\mu}(x) + \tau_{\mu} \dot{w}_{\mu}(x) = \frac{1}{E_{\mu}} \left[\int_{a_0}^a \bar{C}(x, \xi) \sigma(\xi) d\xi + \bar{C}_P(x) P \right] \quad (54)$$

$$u_{\mu} + \tau_{\mu} \dot{u}_{\mu} = \frac{1}{E_{\mu}} \left[\int_{a_0}^a \bar{C}_P(\xi) \sigma(\xi) d\xi + \bar{C}_{PP} P \right] \quad (55)$$

Here w_{μ} and u_{μ} are internal variables associated with w and u , and $v(t)$ is the volume growth function of the solidification theory.

From the foregoing relations, it is clear that fracture and creep interact closely. For concrete structures it is important to make a proper distinction between a short-term loading lasting a couple hours, dynamic loading lasting a few milliseconds, and long-term loading lasting many years. For example, in concrete dams, cracks may grow slowly over a period of 50 years and then propagate suddenly under seismic excitation. The fracture properties for these two regimes of fracture are very different, and the response is greatly affected by differences in creep.

The interaction of creep and fracture, for example, is manifested by the effect of loading rate on the peak load, exemplified in Fig. 4(a), or by the depen-

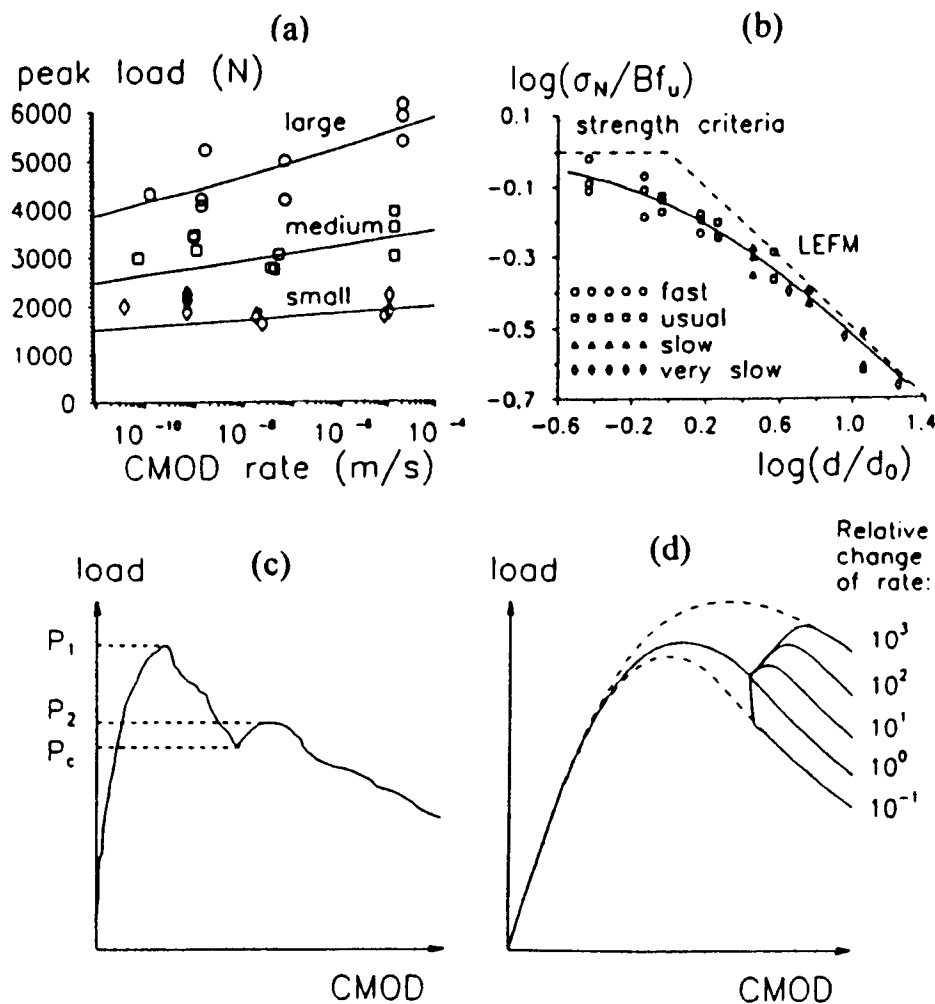


Fig. 4. Typical effects of loading rate and change of loading rate on fracture behavior (adapted from Refs. 25, 49, and 50).

dence of the nominal strength of a fracture specimen on the specimen size at various loading rates (Fig. 4(b)). A sudden change of the loading rate causes an immediate change of stiffness, and can even change a softening response to a hardening response,²⁴ as illustrated in Fig. 3(c,d) (this figure actually represents the results from a rate-dependent generalization of the *R*-curve model with creep).²⁵

Aside from the rate and creep effects on fracture, there is of course the effect of fracturing on creep. Creep at drying or creep at high temperatures is typically accompanied by cracking, and may cause distinct fractures. The distributed cracking is normally characterized by the stress-strain relations for creep, but a physically correct model would call for a material model that is based on the rate of growth of the distributed cracks.

Creep and Cracking Damage at Variable Humidity

This is a complex subject in which a gradual but significant progress has taken place over the last two decades (see the review in Ref. 8). The most important phenomenon is the increase of creep caused by drying, called drying creep or the Pickett effect. After rejection of several hypotheses, the physical explanation that is now considered viable involves two mechanisms.²⁶ The increase is caused by (1) the so-called stress-induced shrinkage, the mechanism of which takes place on the molecular level and consists in the fact that microdiffusion of water between gel pores and capillary pores increases the probability of bond breakages within the solid microstructure of gel, and (2) an apparent effect, consisting in the fact that microcracking caused by drying shrinkage reduces the strain observed in load-free shrinkage specimens. This means that the difference of deformation between the compressed and load-free specimens, commonly considered as creep, actually includes the strain due to suppression of cracking in the compressed specimens.

From the classical type of tests, these two mechanisms could not be separated except by an indirect analytical approach. Recently, a new type of test that separates these two mechanisms directly and clearly has been carried out.²⁷ The idea is to compare curvature creep rather than axial creep, and test two specimens subjected to the same bending moment but different axial forces. For specimens under small axial compression, there is cracking, and for specimens under high axial compression, the cracking is suppressed. From such tests, it transpired^{27,28} that more than half of the additional creep due to drying is caused by the stress-induced shrinkage, as measured on the specimens under high axial compression. The remaining part is due to microcracking (Fig. 5(c)).

For illustration, Fig. 5(a,b) shows the measured effective compliance as a function of time for sealed and drying specimens at small eccentricity of the axial load resultant (the case of high compression) and at large eccentricities (the case

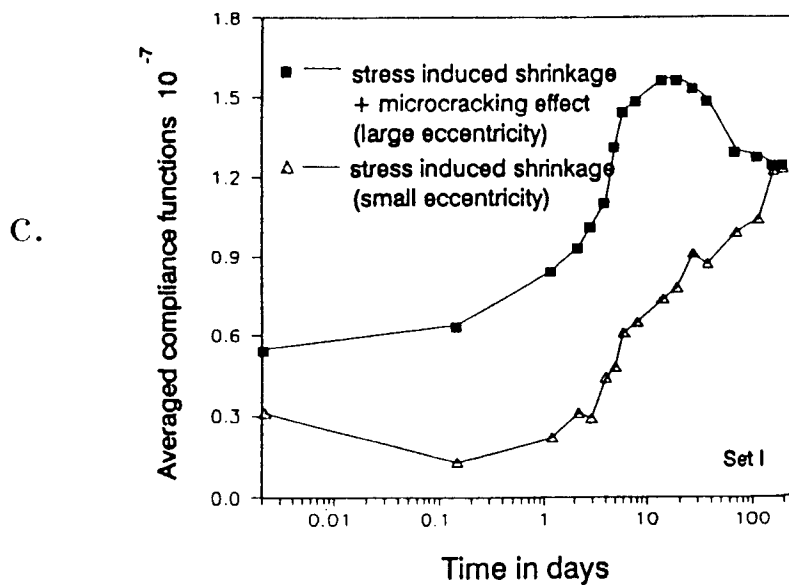
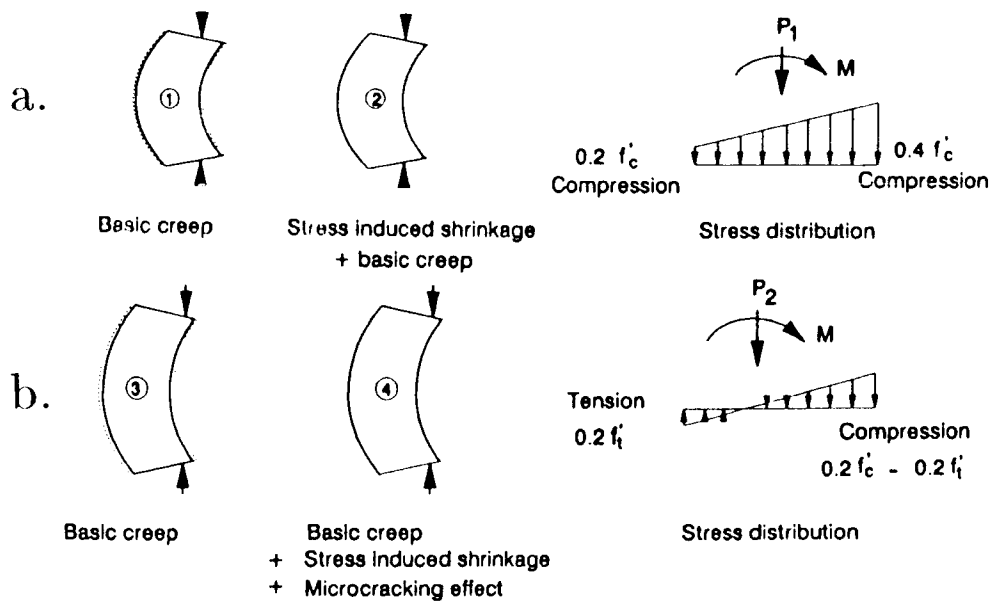


Fig. 5. New tests of curvature creep at equal bending moments and different axial forces, results of measurements indicating separation of stress-induced shrinkage and microcracking, and mesh used in finite element analysis (exaggerated deformation) (adapted from Refs. 27 and 28).

of small axial compression). These results have been used to calibrate the cracking model coupled with the solidification theory, based on finite element analysis of creep, shrinkage, and moisture diffusion in the specimen.

Another way to avoid cracking and measure the stress-induced shrinkage directly is to use extremely thin specimens (under 1 mm) and lower the environmental humidity gradually and sufficiently slowly, so that the differences in pore humidity within the specimen never exceed a few percent. Such tests have been done for cement paste,^{19,29} but for concrete they are impossible.

Exchange of moisture with the environment has a major effect on long-term deformations of concrete and often causes severe cracking. This effect is complicated and is described in the existing standard code-type recommendations for predicting concrete creep and shrinkage only in a crude way. If these recommendations are used, it normally makes no sense to engage in finite element analysis. Compared to simplistic hand calculations, the gain in accuracy achieved by finite element analysis is much less than the typical error of the creep and shrinkage material model in the code-type recommendations. The recommendations cannot be made more accurate as long as they deal with the average properties of the cross section at variable humidity. To get accurate results, it is inevitable to analyze the evolutions of moisture and stress distributions throughout the cross section.

A realistic and accurate material model, suited for finite element analysis, ought to have the following ingredients:

1. The profiles of the pore relative humidity and their evolution in time must be calculated by solving with finite elements the moisture diffusion problem, which is nonlinear.

2. The rate of local free shrinkage strains at various points must be determined from the space-time solution of pore humidity.

3. The stress-induced shrinkage—a large strain whose rate depends on the stress and rate of pore humidity—must be taken into account. If temperature varies, this term also depends on the rate of temperature (see, for example, the test data in Ref. 30). (These phenomena are the sources of the Pickett effect³¹ and the stress-induced thermal strains.^{30,32})

4. The decrease of the viscosities of the Kelvin (or Maxwell) chain units caused by a decrease of pore humidity (e.g., the test data in Ruetz³³) must be considered. If temperature varies, the temperature dependence of these viscosities, governed by activation energy, must be taken into consideration as well.

5. The effect of aging on the creep, shrinkage, and thermal properties must be calculated on the basis of the degree of hydration characterized for each point x and time t by its equivalent hydration period $t_e(x, t)$ (or maturity), whose rate of increase depends on the local pore humidity as well as (according to the activation energy of hydration) on the local temperature.

6. The strain-softening damage due to distributed cracking must be included in the analysis, and in the case of thicker cross sections it is also necessary to consider the localization of damage and use fracture mechanics to analyze propagation of distinct cracks.

Prediction of Creep of Concrete as a Composite

Aside from environmental effects, the main source of error in estimating the creep effects in concrete structures resides in predicting the creep properties of concrete. This task has so far been handled by empirical formulas. If an improvement should be achieved, it must be based on the analysis of concrete as a composite material.

In the simplest form, concrete can be regarded as a two-phase composite consisting of large aggregate pieces embedded in a matrix of portland cement mortar. Progress in this regard has recently been achieved by Granger and Bažant.³⁴ In the last part of the present review, we will briefly outline their approach.

Prediction of the mechanical properties of composites from the properties of their constituents is a subject that has been researched extensively.³⁵⁻³⁹ However, most works so far have dealt with elastic and plastic properties of composites. Ideally, one should apply to the problem of creep some of the powerful methods such as the Hashin-Shtrickman variational bounds, method of composite spheres,³⁵ self-consistent model,⁴⁰ (1965), Mori-Tanaka method,⁴¹ etc.

However, for creep with aging these methods appear far too difficult. Therefore, a simple model consisting of parallel-series coupling has been used by Granger and Bažant.³⁴ In this model, attention has been limited to uniaxial stress; however, generalization to volumetric and deviatoric deformations is possible because the Poisson ratio for creep is nearly constant and nearly equal to its elastic value ($\nu = 0.18$). The parallel-series model, depicted in Fig. 6, is a compromise between the parallel coupling and the series coupling, which are known to provide the upper and lower bounds on the stiffness of elastic composites, called the Reuss and Voigt bounds, respectively.³⁶

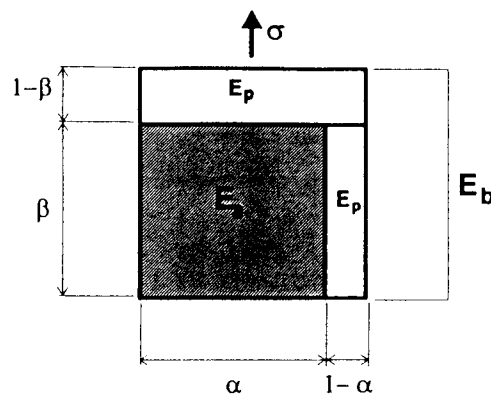


Fig. 6. Series-parallel coupling model for the interaction of aggregate and mortar in concrete (after Ref. 34).

According to the model in Fig. 6, $\alpha\beta = A_g =$ volume of aggregate grains per unit volume of concrete (thus, if A_g is given and β is chosen, $\alpha = A_g/\beta$). Obviously, according to the parallel-series model, the Young's elastic modulus of concrete, E_c , is given by

$$\frac{1}{E_c} = \frac{1 - \beta}{E_p} + \frac{\beta}{\alpha E_a + (1 - \alpha) E_p}, \quad \alpha = \frac{A_g}{\beta} \quad (56)$$

in which E_a and E_p are the elastic moduli of the aggregate and the paste.

The aging creep may be treated by the age-adjusted effective modulus method proposed by Bažant,¹ which reduces the problem to quasi-elastic analysis. The aging creep of concrete is characterized by the compliance function $J(t, t_0)$ representing the strain at age t caused by a unit uniaxial stress applied at age t_0 . Alternatively, the creep properties can be characterized by the relaxation function $R(t, t_0)$ representing the stress at age t caused by a unit constant strain applied at age t_0 . One of these functions can be obtained from the other by numerical solution of a simple Volterra integral equation. Approximately, but with very good accuracy, the relaxation function can also be obtained from the compliance function using the following one-line formula proposed by Bažant and Kim⁴² (in which t and t_0 must be given in days):

$$R(t, t_0) = \frac{0.992}{J(t, t_0)} - \frac{0.15}{J(t, t - 1)} \left[\frac{J(t - \Delta, t_0)}{J(t, t_0 + \Delta)} - 1 \right], \quad \text{with } \Delta = \frac{t - t_0}{2} \quad (57)$$

In the age-adjusted effective modulus method, one assumes that the strain history is a linear function of the compliance function, that is, $\epsilon(t) = \lambda + \mu J(t, t_0)$ where λ and μ are two arbitrary parameters. As proven in Ref. 1, the corresponding stress history is then $\sigma(t) = \lambda R(t, t_0) + \mu$. This property is easily understood if one notes that the strain history corresponding to a constant stress μ introduced at age t_0 is $\mu J(t, t_0)$, and the stress history corresponding to constant strain λ introduced at age t_0 is $\lambda R(t, t_0)$. Then the superposition of these two histories yields the aforementioned result.

This result is translated to a more convenient relation as follows: the initial strain at the time of first loading, t_0 , is $\epsilon(t_0) = \lambda + \mu E_0^{-1} = \sigma(t_0)/E_0$ where $\sigma(t_0)$ is the initial stress at the first loading and E_0 is the initial elastic modulus. Note that, by definition, $E(t_0) = 1/J(t_0, t_0)$. Using Δ to denote the changes from t_0 to the current time t , for example, $\Delta\epsilon(t) = \epsilon(t) - \epsilon(t_0)$, we have for these strain and stress histories $\Delta\epsilon = \mu(J - E_0^{-1})$ and $\Delta\sigma = \lambda(R - E_0)$. Then we solve λ and μ from the last two equations and substitute them into the foregoing expressions for $\epsilon(t)$ and $\sigma(t)$. Thus, defining the creep coefficient as $\phi(t, t_0) =$

$E(t_0)J(t, t_0) - 1$, we obtain the following familiar form of the basic relation of the age-adjusted effective modulus method:

$$\Delta \epsilon(t) = \frac{\Delta \sigma(t)}{E''} + \frac{\sigma(t_0)}{E(t_0)} \phi(t, t_0), \quad E'' = \frac{E(t_0) - R(t, t_0)}{\phi(t, t_0)} \quad (58)$$

In other words, the strain increment is the sum of the elastic strain increment based on the age-adjusted effective modulus E'' and ϕ -times the initial elastic strain. We now apply this rule to the stress and strain changes in the paste from t_0 to t , considering the model in Fig. 6.

Let us denote the stresses in the aggregate and in the portion of paste coupled in parallel as σ_a and σ_p , and their common strain as ϵ_a . To calculate the compliance function, we apply stress $\sigma(t_0)$ on the model at age t_0 and then hold it constant, that is, $\Delta \sigma = 0$. The stress-strain relations for the aggregate and the parallel portion of the paste (the latter based on the age-adjusted effective modulus method), and the equilibrium condition for the parallel coupling, are

$$\Delta \sigma_a = E_a \Delta \epsilon_a, \quad \Delta \sigma_p = E_p'' \left[\Delta \epsilon_p - \frac{\sigma_p(t_0)}{E_p(t_0)} \phi(t, t_0) \right] \quad (59)$$

$$\alpha \Delta \sigma_a + (1 - \alpha) \Delta \sigma_p = 0 \quad (60)$$

in which

$$\sigma_p(t_0) = \sigma(t_0) \frac{E_p(t_0)}{\alpha E_a + (1 - \alpha) E_p(t_0)} \quad (61)$$

where $\sigma(t_0)$ can be chosen as 1. From these relations,

$$\Delta \epsilon = (1 - \alpha) E_p'' \frac{\sigma(t_0)}{\alpha E_a + (1 - \alpha) E_p(t_0)} \frac{\phi_p(t, t_0)}{E_{ap}''}, \quad (62)$$

$$E_{ap}'' = \alpha E_a + (1 - \alpha) E_p''$$

where E_p'' is according to Eq. (58) expressed for the portion of paste coupled in parallel. Adding the compliance function for the portion of paste coupled in

series with the parallel coupling, we then finally obtain the compliance function of the composite that we have been seeking:

$$J(t, t_0) = \frac{\beta}{\alpha E_a + (1 - \alpha) E_p(t_0)} \left[1 + (1 - \alpha) \frac{E_p''}{E_{ap}''} \phi_p(t, t_0) \right] + (1 - \beta) J_p(t, t_0) \quad (63)$$

in which

$$E_p'' = \frac{E_p(t_0) - R_p(t, t_0)}{\phi_p(t, t_0)}, \quad \phi_p(t, t_0) = E_p(t_0) J_p(t, t_0) - 1 \quad (64)$$

This formula was presented in a recent conference paper by Bažant.⁴³

An intricate question is the determination of parameters α and β , which characterize the subdivision of the paste between the parallel and series couplings for concrete (Fig. 6). In this regard, the idea of the maximum compactness of aggregates, introduced by Granger and Bažant,³⁴ is useful. This idea was inspired by Larrard and Le Roy,⁴⁴ who introduced a similar idea for determining the elastic modulus of concrete.

The maximum compactness of the aggregate, $A_{g_{\max}}$, is defined as the maximum volume content (total volume of all grains per unit volume of granular material) that can be reached for aggregates with given maximum and minimum grain diameters d_{\min} and d_{\max} . The $A_{g_{\max}}$ value is attained when the aggregate size distribution (grading) follows a certain optimum grading curve (granulometric curve). Caquot⁴⁵ experimentally established the empirical formula

$$A_{g_{\max}} = 1 - 0.47 (d_{\min}/d_{\max})^{1/5} \quad (65)$$

The coefficient 0.47 pertains to essentially round aggregate shapes (for aggregates of other shapes, for example, with sharp corners or elongated, larger coefficients need to be used).

For the aggregate content $A_g = A_{g_{\max}}$, the concrete must be expected to exhibit the least creep possible since the number of contacts between the aggregate pieces is maximum and the paste is merely filling the remaining voids. This fact is suggested by the studies of shrinkage of the prepacked (or preplaced) aggregate concrete (also called "prepakt" by Neville and Dilger⁴⁶ and Neville et al.,⁴⁷ and also called by Naaman the "sifcon" or slurry infiltrated concrete). The

model for prepacked concretes should involve no series coupling because the portion of the paste coupled in series corresponds to the layers of paste separating the adjacent aggregate pieces. This is the key idea for estimating the portion coupled in parallel.

So we assume that the amount of paste that is needed to fill the voids at maximum aggregate compactness corresponds to the parallel coupling, and the excess amount of paste used in the concrete corresponds to the series coupling.

According to our preceding considerations, the minimum creep is reached if all the paste is coupled in parallel with the aggregates, in which case $\alpha = A_{g_{\max}}$ and $\beta = A_g/A_{g_{\max}} = 1$. For real concretes, for which the grading between d_{\min} and d_{\max} is not optimum, we can define A_g^* as the maximum relative compactness obtained when the maximum possible amount of paste is withdrawn from the concrete without creating further voids. During this withdrawal, the aggregates may be imagined to be brought into contact with each other to the maximum possible extent by means of nearly homogeneous contraction but the configurations of adjacent grains are not rearranged optimally as in vibration. So, this is not the same as compacting the aggregates by vibration, as done by Caquot. This means that, in order to fill the remaining voids, more paste is needed than for Caquot's optimum case, therefore $A_g^* < A_{g_{\max}}$.

Since Caquot's formula applies to the optimum size distribution (grading) between d_{\min} and d_{\max} , one must expect the difference $A_g^* - A_{g_{\max}}$ to depend on the grading, or at least on the sand/gravel ratio $r = s/g$ as the simplest characteristic of grading. Because A_g^* attains a minimum at $A_{g_{\max}}$, the dependence of A_g^* on s/g near this minimum must obviously be a concave function. Following Granger and Bažant,³⁴ we take it as a quadratic function of r in which λ , ρ , and r_0 are empirical constants to be found by data fitting:

$$A_g^* = 1 - \lambda(d_{\min}/d_{\max})^{1/5} [1 + \rho(r - r_0)^2], \quad r = s/g \quad (66)$$

Extensive test data on creep of concretes and mortars of various compositions have been reported by Ward, Neville, and Singh.⁴⁸ These data, comprising 14 different compositions, have been successfully matched by the present formulation.³⁴ Figure 7 gives an example of comparison of the predictions of the model (continuous curves) with the measured creep compliances for concretes of eight different compositions. The results have been quite satisfactory (see detail in Ref. 34).

Another series-parallel model, with a time-varying composition, has also been recently developed for the cement gel, as a generalization of the solidification theory.³⁴

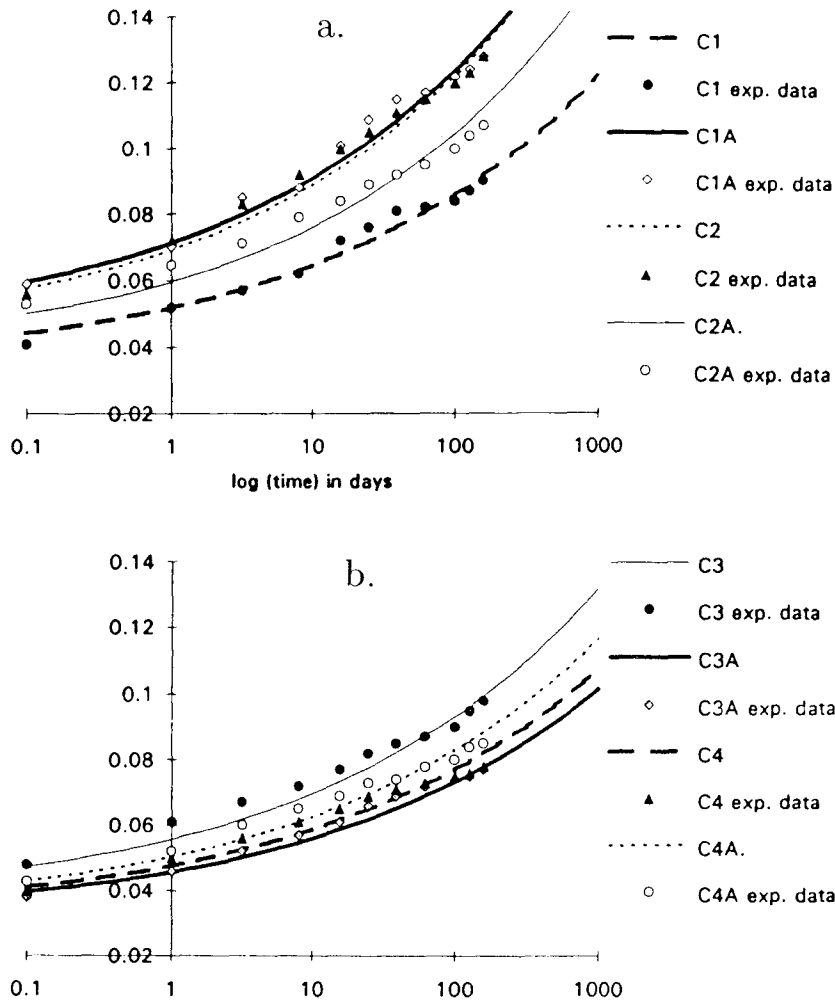


Fig. 7. Creep compliance data of Ward et al.⁴⁸ compared to predictions from series-parallel model of Granger and Bažant (after Ref. 34).

Concluding Remarks

Concrete creep is an intricate phenomenon whose understanding requires sophisticated mathematical modeling based on a physically justified although simplified description of the physical processes occurring in the microstructure. The complicating aspects of creep are (1) aging, caused by long-term chemical processes involved in the hydration of cement, (2) temperature and humidity effects, (3) effects of moisture diffusion through the pores of concrete, and (4) microcracking or fracture caused by drying, which represents one aspect of the interaction between creep and distinct fracture or distributed damage due to

cracking. Consideration of creep is also important in the modeling of rate- and time-dependent effects in fracture phenomena. The time dependence of the growth of fracture, which is due to the activation energy aspect of bond ruptures at the fracture front, can play a significant role in long-term deformations of concrete structures. Because concretes of many different compositions are used, the problem of predicting creep from the properties of constituents is of great importance for practice. It is also a problem of great difficulty, which has so far been handled by empirical equations that have large errors. Useful predictions can be obtained by the series-parallel model combined with the age-adjusted effective modulus method and Caquot's idea of maximum aggregate compactness.

The foregoing exposition has attempted to outline in a brief review fashion several advances made in these problems. Much further research, however, is needed to achieve a satisfactory knowledge of concrete creep as well as its interaction with damage and fracture.

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