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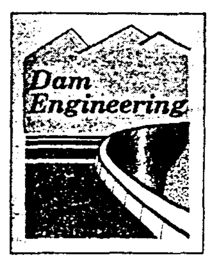

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A critical appraisal of 'no-tension' dam design: a fracture mechanics viewpoint

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SUMMARY

Concrete dams have been designed under the assumption that the material cannot resist tension. This "no-tension" design is used in two versions: (1) simple analysis based on a linear stress distribution throughout the uncracked portion of a horizontal cross section, and (2) finite element elastic/perfectly plastic limit analysis based on a yield condition with a zero value of tensile strength. This paper examines version (1) in detail, using linear elastic fracture mechanics, with its applicability justified by the very large sizes of dams. It is shown that, for certain lengths of a horizontal crack in the dam, tensile stresses, characterized by a large value of the crack-tip stress intensity factor, do occur for no-tension designs. Regarding version (2), it is deduced that, if a certain critical size of a dam is exceeded, the exact no-tension solution of any cracked dam gives a larger maximum load than linear elastic fracture mechanics for any given value of fracture toughness of the material. Although detailed finite element studies of version (2) are needed, it can already be concluded that it is imprudent to suppose that the no-tension design is safe.

Introduction

Because of the uncertainty of the value of the tensile strength of concrete as well as the perception that fracture mechanics might be too difficult to use, concrete dams have been designed under the assumption that the tensile strength of concrete is zero. This "no-tension" design has generally been believed to be on the safe side, that is, the failure load obtained by no-tension analysis has been believed to be lower than that obtained by an analysis in which the tensile strength is taken into account. This approach, however, has never been justified theoretically. This brief paper, stimulated by the discussions at a recent international research workshop (Dungar, Saouma and Wittmann, 1990), will demonstrate by a simple example that this approach is not guaranteed to be safe, and that situations in which it is unsafe do exist.

1 No-tension design

There are two versions for the no-tension design of concrete structures:

1. A simplified elastic analysis in which the distribution of the compressive normal stresses throughout the cross section is specified in advance, in the case of dams the analysis tends to be linear (Figure 1a), and no tensile stresses are allowed; and

2. Finite element analysis in which the tensile strength of the material is assumed to be zero.

Version (1) has for a long time been used in the design of reinforced concrete beams for bending and axial loads, and has been proven safe for those applications. However, this does not mean that the simplification gained by the no-tension hypothesis is on the side of safety in the case of unreinforced and massive structures such as dams.

Version (2) is properly implemented as a special case of a finite element program for plasticity in which the tensile yield limit is taken as zero (for example, Zienkiewicz, 1977). A suitable type of the yield condition is the Rankine yield surface or the limit case of the Mohr-Coulomb yield surface, in which the tensile yield strength is zero (Owen and Hinton, 1980). In the theory of plasticity it has been proven (for example, Hodge 1959) that if the yield surface A lies within the yield surface B, then the limit load for surface A is not larger that the limit load for surface B. This fact seems to suggest that the no-tension design should be safe. But this would be true only if concrete were actually a plastic material, that is, if the stress-strain diagram exhibited a horizontal yield plateau after the attainment of the yield limit. Concrete in tension in fact exhibits post-peak strain softening. The consequence is that the failure condition can no longer be characterized by a material failure surface expressed in terms of the stresses and strains, but must involve energy. It turns out that the failure of such materials depends on the energy release rate of the structure with respect to the length of extension of the failure zone. Thus one must consider the propagation of failure throughout the stucture. In plasticity, by contrast, the failure progress at the limit load is simultaneous (non-propagating), proceeding in proportion at all points of the failure surface.

The following observation suggests that the no-tension hypothesis might be unsafe: if a structure has a finite tensile strength, it can store more energy than the same structure with zero tensile strength. Therefore, at the moment of failure, more energy can be released into the fracture front, which helps to drive the fracture propagation.

The present brief study deals mainly with the first version of no-tension analysis, normally used in the design of dams. Nevertheless, certain properties of the exact no-tension solutions that are relevant to the second (finite-element) version will also be pointed out.

2 Example

We consider a dam that is loaded by a vertical force P representing its own weight, together with any uplift forces, and by a horizontal force H representing the resultant of water pressure from the reservoir (Figure 1a). Both forces may be modified to approximate in a static manner seismic actions, and are amplified by appropriate safety factors. The dam is rigidly fixed at the base. We suppose there may be a horizontal crack of length a at the base, and assume the dam to be designed in such a manner that the resultant of the applied forces P and H at the base cross section, represented by the vertical force C in Figure 1b, will be located *exactly* at the 2/3-point of the uncracked portion of the cross section ($d - a$, called the ligament, as shown in the figure). In this case, the linear stress distribution that balances the force C has a zero stress value at the tip of the crack (Figure 1a), which represents a no-tension design (according to version 1).

The internal force resultants acting in the base cross sections are the normal force P (positive for compression) passing through the centre of the cross section, the bending moment M about the centre, and the shear force H. According to version (1) of no-tension analysis, only the values of M and P matter, while the value of H has no effect on the result. Neither does the shape of the dam; only the values of d and a matter. This permits us to modify the shape of the dam and the magnitude and vertical coordinate y_H (Figure 1a) of the horizontal force, so as to be able to exploit the known fracture mechanics solutions given in the literature (Tada et al., 1985, and Murakami, ed., 1987). Accordingly, we choose our "dam" to be a rectangular two-dimensional body of width d, height h and thickness b, subjected (from the top) to a centric vertical load P and a horizontal load F (Figure 1b) such that $Fh = Hy_H = M =$ the bending moment of the actual applied forces. Solutions of this kind of "dam" for loading by either F or P are available in fracture mechanics literature, but only for $h/d = 1.25, 2, 4$ and ∞ and for P applied at infinity on an infinite strip.

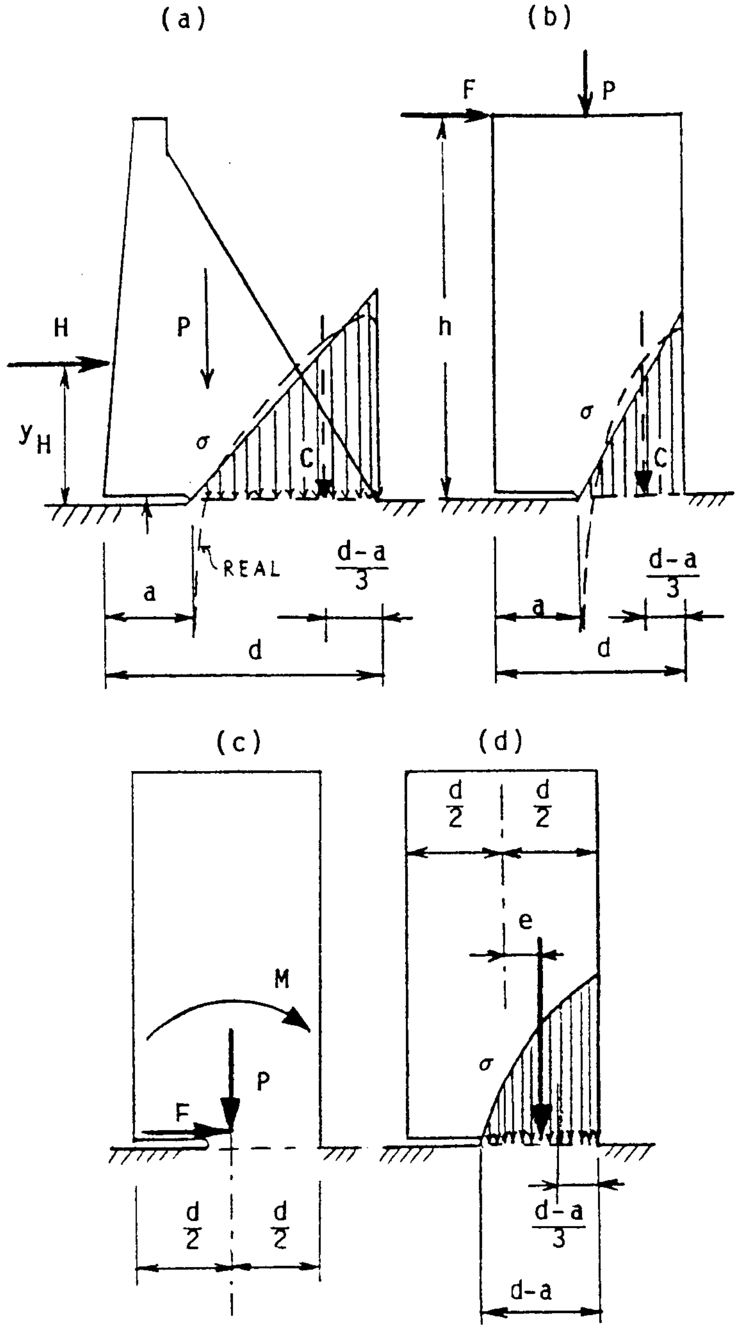

Figure 1. (a) Loading of a dam and stress distribution in no-tension design, (b) example problem, (c) internal force resultants in the base cross section, and (d) compression resultant C that yields $K_I = 0$

When the horizontal load H acts alone (that is, $P = 0$), the stress state in the "dam" is obviously identical to that in one half of a simply supported beam of span $2h$, having a notch or crack of length a at the midspan and loaded at midspan by a concentrated load $2F$. According to Gettu and Bažant (1989) (for $h/d = 1.25$) and the handbooks of Tada et al.(1985) or Murakami (ed., 1987) (for other h/d values), the solutions of the crack-tip (mode I) stress intensity factor K_I^F are as follows:

$$K_I^{(r)} = \frac{6M}{bd^2}\sqrt{\pi a}\,F_r(\alpha), \qquad \alpha = \frac{a}{d}, \qquad M = Fh \tag{1}$$

in which $r = h/d$ and

$$F_{1.25}(\alpha) = \frac{1 - 2.5\alpha + 4.49\alpha^2 - 3.98\alpha^3 + 1.33\alpha^4}{(1-\alpha)^{3/2}} \tag{2}$$

$$F_2(\alpha) = \frac{1.99 - \alpha(1-\alpha)(2.15 - 3.93\alpha + 2.7\alpha^2)}{\sqrt{\pi}(1+2\alpha)(1-\alpha)^{3/2}} \tag{3}$$

$$F_4(\alpha) = 1.107 - 2.12\alpha + 7.71\alpha^2 - 13.55\alpha^3 + 14.25\alpha^4 \tag{4}$$

$$F_\infty(\alpha) = 1.122 - 1.40\alpha + 7.33\alpha^2 - 13.08\alpha^3 + 14.0\alpha^4 \tag{5}$$

The error of Eq.1 is under ± 1%, the errors of Eqs. 2 and 3 (for any α) are under ± 0.5%, and the error of Eq. 4 is under ± 0.2%. Eqs. 2 and 3 have the asymptotically correct form for $\alpha \to 1$.

When the vertical centric load P acts alone and the ratio h/d is sufficiently large (we assume that $h/d \geq 1.25$ is acceptable), then the stress intensity factor K_I^P at the crack tip is approximately the same as that in a single-edge-cracked infinite strip of width d, for which the following expression (after Gross and Srawley, 1964, and Brown and Srawley, 1967) is indicated in the handbooks of Tada et al. and Murakami, ed.:

$$K_I^P = -\frac{P}{bd}\sqrt{\pi a}\,F_P(\alpha) \tag{6}$$

in which, with an error within ± 0.5% for $\alpha \leq 0.6$,

$$F_P(\alpha) = 1.12 - 0.231\alpha + 10.55\alpha^2 - 21.72\alpha^3 + 30.38\alpha^4 \tag{7}$$

The sign at K_I^P is negative because load P is positive for compression.

Now we need to superpose these solutions in such a manner that M and P combined produce zero tensile stress at the crack tip (Fig. 1b or 1c) according to the no-tension design (version 1). This occurs when the eccentricity e of the compression resultant C (Fig. 1a or 1b) in the base cross section coincides with the 2/3 point of the ligament $d - a$, that is

$$e = \frac{M}{P} = \frac{d}{2} - \frac{d-a}{3} = (1+2\alpha)\frac{d}{6} \tag{8}$$

Expressing P from Eq. 5, we have

$$P = \frac{6M}{(1+2\alpha)d} \tag{9}$$

Now we substitute this into Eq. 6 and superpose Eq. 1. This furnishes the following stress intensity factor due to the combined action of M and P:

$$K_{I_{MP}}^{(r)} = \frac{6M}{bd^2}\sqrt{\pi a}\,[\,F_r(\alpha) - \frac{F_P(\alpha)}{1+2\alpha}\,] \tag{10}$$

If the design were really a no-tension design, the value of this stress intensity factor would have to be zero, but in general it is non-zero, as we will see. This suggest the question what should be the distance of the compression resultant C ($C = P$) from the downstream face of the dam in order to

Table 1. Crack-tip stress intensity factors $K_{I_{MP}}^{(r)}$, in $MNm^{-3/2}$, compared with the factor $K_I^{(\infty)}$ caused by moment alone

	$K_{I_{MP}}^{(r)}$				
α	$r = 1.25$	$r = 2$	$r = 4$	$r = \infty$	$K_I^{(\infty)}$
0.00	-0.01	0.00	0.00	0.00	-0.10
0.05	-1.46	0.43	-0.21	0.79	-21.48
0.10	-1.70	0.60	-0.75	1.63	-29.66
0.15	-2.19	0.51	-1.71	2.33	-36.20
0.20	-2.85	0.36	-2.95	2.97	-42.32
0.25	-3.55	0.30	-4.40	3.58	-48.55
0.30	-4.22	0.35	-6.06	4.15	-55.23
0.35	-4.87	0.47	-7.96	4.63	-62.70
0.40	-5.56	0.56	-10.15	4.96	-71.40
0.45	-6.42	0.47	-12.65	5.12	-81.91
0.50	-7.48	0.17	-15.39	5.15	-94.98
0.55	-8.64	-0.17	-18.23	5.19	-111.60
0.60	-9.36	0.06	-20.89	5.52	-132.97
0.65	-8.27	2.45	-22.92	6.57	-160.57
0.70	-2.21	10.55	-23.70	8.94	-196.17
0.75	15.97	32.28	-22.39	13.45	-241.82
0.80	63.34	86.47	-17.91	21.16	-299.91
0.85	186.64	224.43	-8.95	33.36	-373.16
0.90	552.52	629.04	6.10	51.62	-464.65

achieve a no-tension design according to linear elastic fracture mechanics. We express this distance as $\varrho(d-a)/3$ where coefficient ϱ is 1 for version 1 of no-tension design. Obviously,

$$\varrho = \frac{(d/2) - e}{(d-a)/3} \tag{11}$$

where e is the eccentricity of compression resultant C relative to the centre of the base cross section (Figure 1d). Superposing the stress intensity factors in Eq. 1 and 6, and substituting $P = M/e$, we obtain the following combined stress intensity factor which must vanish for the true no-tension design:

$$K_I^{(r)} = \frac{6M}{bd^2}\sqrt{\pi a}\,[F_r(\alpha) - \frac{d}{6e})F_P(\alpha)] = 0 \tag{12}$$

From this we solve:

$$e = \frac{F_P(\alpha)}{6F_r(\alpha)}\,d \tag{13}$$

It may be noted that the normal stresses in the base cross section near the crack tip are $\sigma_y = K_I(2\pi x)^{-1/2}$ where K_I is the stress intensity factor and x is the distance from the crack tip. The energy release rate, for the cases of positive K_I, is given as $\mathcal{G} = K_I^2/E'$ where $E' = E/(1-\nu^2), E =$ Young's elastic modulus, $\nu =$ Poisson's ratio (see, for example, Broek, 1986).

For a numerical example, consider a slice of the dam of thickness 10 m and width $d = 140$m. The bending moment at the base is taken as M = 140 000 MNm, which roughly corresponds to a 200 m-high dam. Tables 1 and 2 give the results obtained for various crack lengths a from Eq. 6, and Figure 2 shows the plot of factor ϱ.

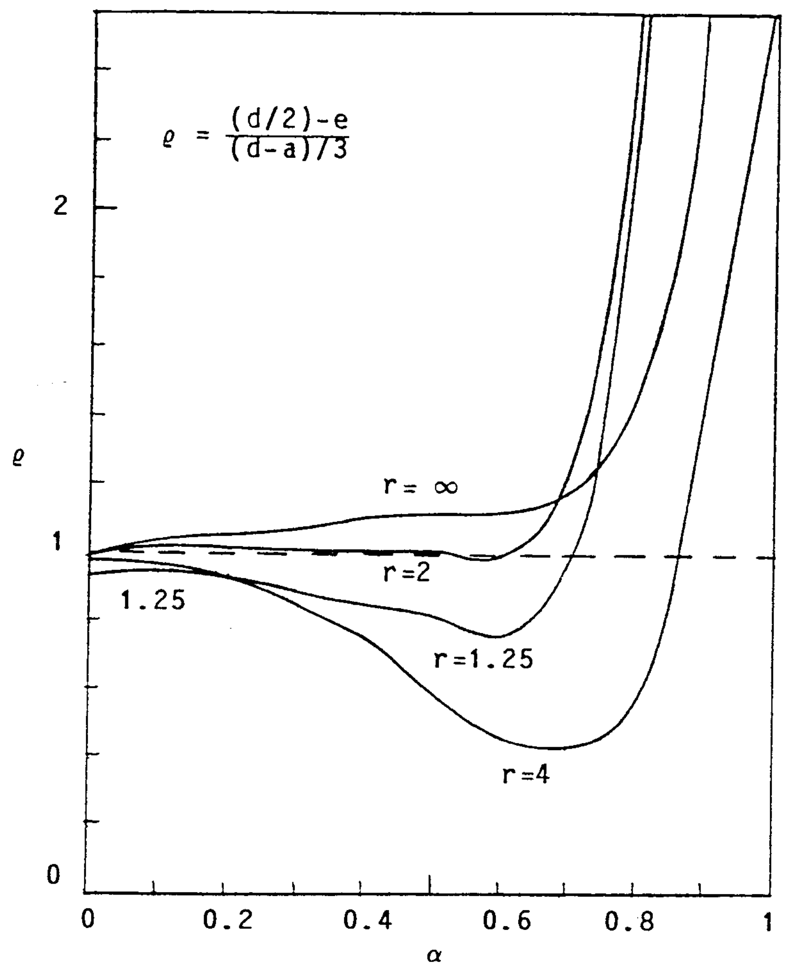

Figure 2. Factors ϱ characterizing the location of the compression resultant C for true no-tension design ($K_I = 0$) according to linear elastic fracture mechanics

Table 2. Calculated values of factor ϱ for adjusting the distance of compression resultant C from the downstream face to achieve true no-tension design ($K_I = 0$) according to fracture mechanics

α	$r = 1.25$	$r = 2$	$r = 4$	$r = \infty$
0.00	0.940	1.001	0.994	1.000
0.05	0.956	1.012	0.994	1.021
0.10	0.957	1.014	0.982	1.037
0.15	0.947	1.011	0.959	1.049
0.20	0.932	1.008	0.929	1.061
0.25	0.914	1.007	0.892	1.074
0.30	0.897	1.008	0.846	1.086
0.35	0.880	1.011	0.792	1.097
0.40	0.863	1.012	0.729	1.104
0.45	0.843	1.011	0.659	1.108
0.50	0.818	1.004	0.587	1.108
0.55	0.794	0.996	0.518	1.109
0.60	0.782	1.001	0.461	1.114
0.65	0.814	1.051	0.426	1.134
0.70	0.952	1.213	0.420	1.182
0.75	1.327	1.619	0.457	1.278
0.80	2.204	2.539	0.554	1.459
0.85	4.191	4.580	0.756	1.805
0.90	9.011	9.451	1.204	2.555

3 Discussion of numerical results

The positive values of $K_{I_{MP}}^{(r)}$ in Table 1 represent the cases where version (1) of the no-tension design does not avoid tensile stresses in the concrete and is therefore unsafe. In Table 2 and Figure 2, the unsafe situation is revealed by ϱ values that exceed 1. In many cases the positive stress intensity factor values are much less than the $K_I^{(\infty)}$ values caused by moment alone (last column of Table 1), and the ϱ values are not much larger than 1, which means that the design is nearly safe. There exist, however, cases where the stress intensity factor is relatively large and factor ϱ is much larger than 1 (for $\alpha > 0.7$ if $r = 1.25$, or $\alpha > 0.65$ if $r = 2$, or $\alpha > 0.25$ if $r = \infty$). In these cases, version (1) of no-tension design is significantly unsafe. It implies (according to linear elastic fracture mechanics) infinite tensile stress values at the crack tips (in which case the stresses solved by elastic finite element analysis increase beyond any bound as the mesh is refined).

It may be noted that version (1) of the no-tension design is exact for a vanishingly small crack in an infinite strip subjected to a uniform bending moment (case $\alpha = 1, r = \infty$ in Table 2). This of course must be so because the bending theory is exact for a very long beam with no crack. It may further be noted that, in the limit of a crack of vanishing length ($\alpha = 0$), version (1) of the no-tension design is generally quite close to a safe situation (ϱ not much larger than 1). This might have been expected because a very short crack causes a negligible disturbance of the stress field.

It should be kept in mind that none of the cases solved represents an actual dam. The real conditions for a dam would be approximated for $r \approx 0.5$, but this is beyond the range of fracture mechanics solutions in the literature. Calculations for this case should be carried out, considering also a realistic dam profile and load distribution.

The present results indicate that, according to linear elastic fracture mechanics, the exact stress distributions at the base of a dam with a horizontal crack can have positive (tensile) stress peak at the crack tip (see Table 1 and Figures 1d and 2). Negative stress peaks can also be obtained. The negative values of the stress intensity factor are of course fictitious, because they would imply overlap of the opposite crack faces, which is physically impossible. However, the case of a negative stress intensity factor is a safe situation and is not of concern here.

The factors ϱ exemplified in Table 2 and Figure 2 could serve as the basis of design. Using the condition that $K_I = 0$ at the crack tip, a table of these factors could be calculated for typical dam geometries. Having such a table or graph available, the designer could use the ϱ factors to determine the location of the compression resultant C for which the design is truly a no-tension design according to fracture mechanics. The design condition would simply be that the moment of all the applied forces about the location of this resultant at the base cross section must vanish. In this manner, the designer would be able to circumvent the use of fracture mechanics in the design procedure.

One might object that the linear elastic fracture mechanics, which underlies the present calculations, is not applicable to concrete and that non-linear fracture mechanics is the proper theory. This is true, but only for normal size structures. According to what has recently been learned about the size effect, particularly the size effect law for nominal strength (Bažant 1984, Bažant and Pfeiffer 1987, Bažant and Kazemi 1990), as well as the consensus of fracture researchers at the recent international research workshop (Wittmann, ed., 1990), it appears that linear elastic fracture mechanics is very applicable for the overall action of structures as large as concrete dams. Thus, the present simple approach is indeed relevant to the safety evaluations of cracked concrete dams.

4 General proof that the no-tension plasticity solution can be unsafe

4.1 Maximum load comparison

As is well known, the maximum load $P_u = P_u^{pl}$ (or ultimate load, limit load, collapse load) calculated by elastic/perfectly plastic limit analysis on the basis of a yield criterion with a zero tensile strength value can be either zero or non-zero, depending on the type of loading. In the case of dam design, this load is, of course, non-zero.

According to linear elastic fracture mechanics, the maximum load $P_u = P_u^{fr}$ of a structure with an existing crack is proportional to the value of the fracture toughness K_c, representing the critical value of the stress intensity factor K_I of the material ($K_c = \sqrt{G_f E}$, where G_f = fracture energy of the material; see, for example, Broek, 1986). It follows that for $K_c = 0$ the maximum load P_u^{fr} is zero, and thus less than P_u^{pl}. An exception is the case when the structural geometry and the type of loading are such that the stress intensity factor K_I calculated for the given loading is negative. In this case the fracture mechanics solution is invalid because the crack is closing (that is, the crack faces are in contact). However, in our numerical example we have shown that $K_I = K_{I_{MP}}^r$ is positive for certain crack lengths (Table 1, Figure 2).

4.2 Argument based on the size effect

The foregoing argument does not prove that P_u^{pl} can be larger than the fracture mechanics solution for a finite value of the fracture toughness K_c of the material. The possibility of this can be proven on the basis of the known size effects in plasticity and in fracture mechanics.

The size effect is defined by comparing the nominal strength σ_N (nominal stress at maximum load) of geometrically similar structures of different sizes d. The nominal strength is (for the case of two-dimensional similarity) defined as $\sigma_N = P_u/bd$ where b = thickness of the structure (that is, of the slice of the dam) and d = arbitrarily chosen characteristic dimension (size). Now the basic property of the elastic/perfectly plastic limit analysis is that, for geometrically similar structures made of the same material, the value of σ_N is independent of the structural size d. For linear elastic fracture mechanics, by contrast, the value of σ_N is inversely proportional $\sqrt{d}$. This indicates that for linear elastic fracture mechanics the plot of $\log \sigma_N$ versus $\log d$ is an inclined straight line of slope $-1/2$ (descending), while for elastic/perfectly plastic limit analysis this plot is a horizontal straight line, as shown in Figure 3.

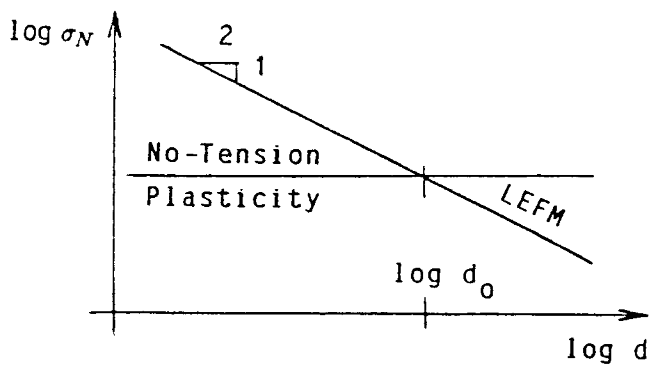

Figure 3. The size effects of stress-based failure criteria (including no-tension plasticity) and linear elastic fracture mechanics (LEFM)

Now the point is that these two lines must obviously intersect. Denoting by d_0 the size at the intersection point (Figure 3), we see that for structural sizes $d > d_0$ we must have

$$P_u^{fr} < P_u^{pl} \tag{14}$$

even if P_u^{pl} is based on zero tensile strength.

More precisely,

$$\sigma_N = \frac{K_c}{\sqrt{g(\alpha)d}} \tag{15}$$

in which $g(\alpha)$ is the non-dimensionalized energy release rate for relative crack length α calculated by linear elastic fracture mechanics, which is positive in the situations we consider; see, for example, Bažant (1984) or Bažant and Pfeiffer (1987). Setting in the last equation $\sigma_N = \sigma_N^{pl} = P_u^{pl}/bd =$ nominal strength of the dam according to the elastic/perfectly plastic limit analysis with a zero value of tensile strength of the material, we find that the critical dam size is obtained as

$$d_0 = \frac{K_c^2}{g(\alpha)(\sigma_N^{pl})^2} \tag{16}$$

The value of $g(\alpha)$ can be easily calculated by linear elastic finite element analysis.

5 Conclusions

1. The no-tension design of a concrete dam with a horizontal crack, calculated under the assumption of a linear stress distribution through the uncracked portion of the cross section, is not guaranteed to yield a solution that would be on the safe side. Numerical examples that are similar, though not identical, to typical dam geometry reveal that in certain cases linear elastic fracture mechanics yields, for such a no-tension design, tensile stresses at the crack tip that are characterized by a relatively large positive value of the stress intensity factor.

2. In the case that the stress intensity factor for a no-tension design is positive, linear elastic fracture mechanics indicates that the maximum load of the dam is zero if the value of the fracture toughness K_c is considered to be zero.

3. For a finite value of fracture toughness K_c and for those situations in which the crack-tip stress intensity factor K_I is positive (see Table 1 and Figure 2 for $\varrho > 1$), there always exists a certain critical dam size d_0 such that for larger dam sizes d the maximum load of a cracked dam according to linear elastic fracture mechanics is less than the maximum load according to elastic/perfectly plastic limit analysis with a zero value of tensile strength.

4. The present brief analysis does not prove that: (a) positive K_I values can actually occur for no-tension designs based on finite element analysis, and (b) the critical size d_0 can be within the range of the actual dam sizes. But in the light of the present analysis it would be imprudent to assume that these situations cannot occur. (Finite element studies aimed to clarify these two points are in progress and will be reported soon.)

5. Fracture mechanics can be used to solve in advance a table of values of a correction factor ϱ that indicates the necessary shift of the compression resultant for which the design involves no tensile stresses according to linear elastic fracture mechanics. In the absence of a more realistic (non-linear) fracture analysis, this approach could be adopted for the practical design of dams. It would make it unnecessary for the designer to actually carry out fracture mechanics analysis.

Since large safety factors are used in the design of dams, the present results of course do not imply that a certain dam would actually collapse but only that its safety margin is less than assumed on the basis of conventional analysis. In theory, the safety factors should be correlated to the probability of non-failure of the dam, but the research has not sufficiently advanced for being able to do that.

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Bibliography

BAŽANT, Z. P., "Size Effect in Blunt Fracture: Rock, Concrete, Metal", *Journal of Engineering Mechanics ASCE*, 110(4); April, 1984.

BAŽANT, Z. P. AND PFEIFFER, P. A., "Determination of Fracture Energy from Size Effect and Brittleness Number", *ACI Materials Journal*, 84; November-December, 1987.

BAŽANT, Z. P. AND KAZEMI, M. T., " Size Effect in Fracture of Ceramics and Its Use to Determine Fracture Energy and Effective Process Zone Length", *J. of American Ceramic Society*, 73(7); 1990.

BROEK, D., " Elementary Engineering Fracture Mechanics", *Book*, fourth ed., Martinus Nijhoff, Dordrecht and Boston; 1986.

BROWN, W. F. AND SRAWLEY, J. E., " Fracture Toughness Testing Methods, in "Fracture Toughness Testing and Its Applications"", *Book*, ASTM, STP 381; 1967.

DUNGAR, R., SAOUMA, V. E. AND WITTMANN, F. H., ED., "Workshop Notes, Research Workshop on "Application of Fracture Mechanics to Dam Engineering"", *Proceedings*, Locarno, Switzerland, published by ETH (Swiss Federal Institute of Technology), Zürich; September, 1990.

GROSS, B. AND SRAWLEY, J. E., "Stress Intensity Factors for a Single-Edge-Notch Tension Specimen by Boundary Collocation of a Stress Function", *Report*, NASA TN D-2395; 1964.

GETTU, R. AND BAŽANT, Z.P., "Stress Intensity Factors for a Short Three-Point Bend Specimen", *Internal report*, Northwestern University; 1989.

HODGE, P. G., "Plastic Analysis of Structures", *Book*, McGraw Hill, New York; 1959.

MURAKAMI, Y., ED., "Stress Intensity Factors Handbook", *Book*, Pergamon Press, Oxford and New York, Vol. 1; 1987.

OWEN, D. R. J. AND HINTON, E., "Finite Elements in Plasticity", *Book*, Pineridge Press, Swansea, UK; 1980.

TADA, H., PARIS. P. C. AND IRWIN, J. K., "The Stress Analysis of Cracks Handbook", *Book*, second ed., Paris Productions, Inc., St. Louis, Missouri, USA; 1985.

ZIENKIEWICZ, O. C., "The Finite Element Method", *Book*, J. Wiley, Chichester and New York; 1977.

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ERRATA

The expression for the stress intensity factor in Eq. 4 of this paper was taken from the handbook of Murakami *et al*, which however contains a misprint. The correct expression, given in the original paper by Srawley, reads:

$$F_4(\alpha) = 1.106 - 1.522\alpha + 7.71\alpha^2 - 13.53\alpha^3 + 14.25\alpha^4 \tag{4}$$

This requires corrections in the column $r = 4$ of Table 1 and in the curve $r = 4$ in Figure 2, as shown below. These corrections, however, do not require any changes in the conclusions of the paper.

α	$r = 1.25$	$r = 2$	$r = 4$	$r = \infty$
0.00	0.940	1.001	—	1.000
0.05	0.956	1.012	1.010	1.021
0.10	0.957	1.014	1.021	1.037
0.15	0.947	1.011	1.030	1.049
0.20	0.932	1.008	1.039	1.061
0.25	0.914	1.007	1.049	1.074
0.30	0.897	1.008	1.059	1.086
0.35	0.880	1.011	1.068	1.097
0.40	0.863	1.012	1.075	1.104
0.45	0.843	1.011	1.079	1.108
0.50	0.818	1.004	1.080	1.108
0.55	0.794	0.996	1.083	1.109
0.60	0.782	1.001	1.092	1.114
0.65	0.814	1.051	1.117	1.134
0.70	0.952	1.213	1.170	1.182
0.75	1.327	1.619	1.274	1.278
0.80	2.204	2.539	1.465	1.459
0.85	4.191	4.580	1.825	1.805
0.90	9.011	9.451	2.603	2.555

Table 1. Calculated values of factor ϱ for adjusting the distance of compression resultant C from downstream face to achieve true no-tension design ($K_I = 0$) according to fracture mechanics.

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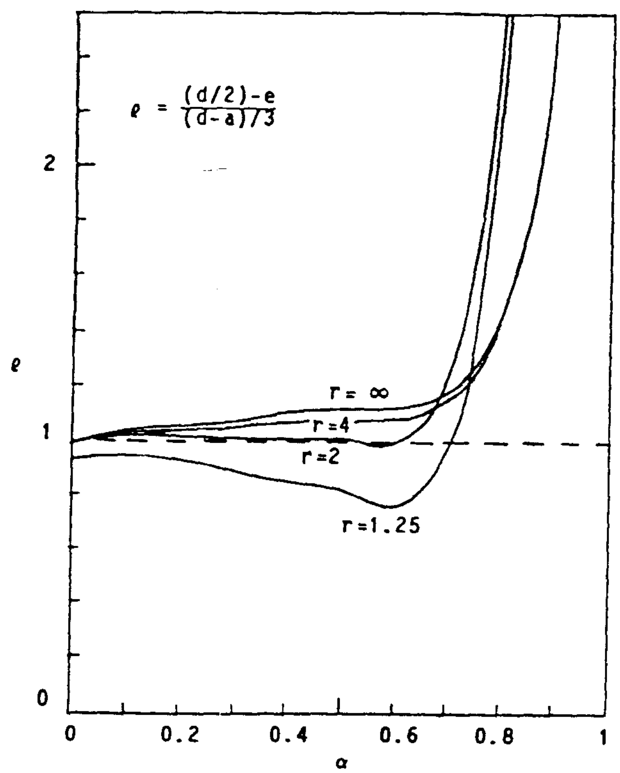

Figure 2. Factors ϱ characterizing the location of the compression resultant C for true no-tension design ($K_I = 0$) according to linear elastic fracture mechanics.