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Size Effect in Diagonal Shear Failure: Influence of Aggregate Size and Stirrups

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Fracture mechanics of brittle failures of concrete structures due to the propagation of a cracking zone indicates that the nominal stress at failure should not be constant but should decrease as the structure size increases. This size effect may be described by a simple size-effect law recently derived by dimensional analysis and similitude arguments. A formula for diagonal shear failure based on this size-effect law was published, verified, and calibrated by a large set of data from the literature. The present paper improves this previous formula in two respects: (1) the effect of maximum aggregate size, distinct from the effect of the relative beam size, is incorporated and (2) the formula is extended to cover the effect of stirrups on the shear capacity of concrete. The new formula is verified and calibrated according to a larger data set than before, consisting of several hundred test results compiled from the literature. It is shown that the new formula achieves an appreciable reduction in the coefficient of variation of the deviations of the measured data from the prediction formula.

Keywords: aggregate size; beams (supports); cracking (fracturing); crack propagation; **failure;** loads (forces); optimization; reinforced concrete; **shear properties;** statistical analysis; **stirrups;** tests.

The diagonal shear failure of reinforced concrete beams is a classical yet formidable problem that has not been resolved to complete satisfaction despite several decades of study.[1-37] Although the design formulas for diagonal shear have been gradually improved, they still exhibit large errors that are in fact much greater than the random errors in the measurements of strength or of fracture energy. So there is clearly room for further improvement.

Until recently, the formulas proposed for diagonal shear failure were either purely empirical or based on plastic limit analysis. A new viewpoint was introduced in 1981 by Reinhardt,[38,39] who suggested that the design formula be based on linear elastic fracture mechanics and substantiated his suggestion by certain test results. Subsequently, it was established[40,41] that the size effect implied by linear elastic fracture mechanics is too strong in the case of concrete, and that brittle failures of concrete structures are better described by nonlinear fracture mechanics, which is based on the existence of a large cracking zone at the fracture front and yields a considerably weaker size effect.

At the same time, a simple approximate size-effect law for brittle failures due to fractures that are blunted by a zone of distributed cracking was derived[41-44] and shown to be approximately applicable to various brittle failures of concrete structures, including the present problem of diagonal shear failure in both nonprestressed[45] and prestressed beams,[46] the punching shear failure of slabs,[47] the ring and beam failures of unreinforced pipes,[48] and torsional failures of longitudinally reinforced beams.[49,50] A formula for diagonal shear failure of beams without stirrups, incorporating the size-effect law, was developed[45] and shown to offer significantly reduced errors in comparison to the existing formulas of ACI[51] or CEB-FIP,[52] which were developed on the basis of crack initiation rather than failure.

The purpose of the present study is to: (1) improve the previously proposed formula based on the size-effect law by incorporating, in addition to the effect of relative beam size, the effect of aggregate size; (2) recalibrating the formula with a larger data base, encompassing essentially all the adequately documented test results that can be found in the literature; and (3) extending the formula to beams with stirrups.

PREVIOUS FORMULA INCORPORATING THE SIZE-EFFECT LAW

In a previous paper,[45] Bažant and Kim derived and justified by existing test data the following formulas for the diagonal shear failure of beams with longitudinal reinforcement but without stirrups [Fig. 1(a)]

$$v_c = v_c^0 \left(1 + \frac{d}{\lambda_0 d_a}\right)^{-1/2} \quad (1)$$

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$$v_c^0 = k_1 \rho^{1/3} \left(\sqrt{f_c'} + 3000 \sqrt{\frac{\rho}{(a/d)^5}} \right) \qquad (2)$$

where

v_c = V_c/bd = nominal shear strength at failure
V_c = shear force at failure
b = beam width
d = beam depth measured to the axis of longitudinal reinforcement
d_a = maximum aggregate size
a = shear span (equal to the distance of the concentrated load from the support when a simply supported beam is loaded symmetrically by two concentrated loads)
ρ = steel ratio for longitudinal reinforcement
f_c' = standard cylindrical compressive strength in psi (6895Pa)
k_1, λ_0 = empirical constants (k_1 = 10)

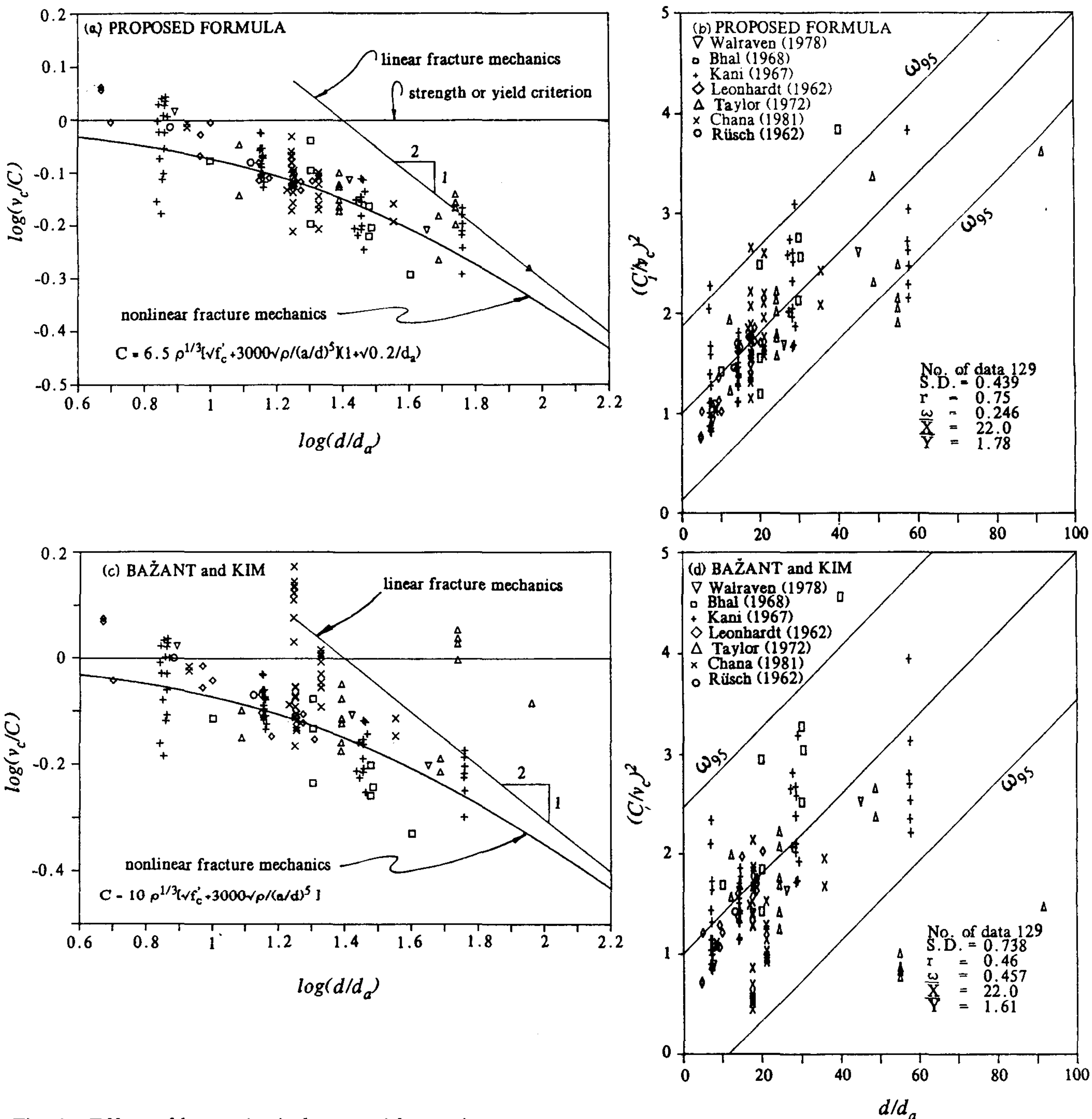

Fig. 1—Effect of beam size in beams without stirrups

Eq. (2), which is quite similar to the formula currently used by ACI, has been derived by analyzing the arch action and the composite beam action, and summing their contributions.[4]

The size effect is due to the release of strain energy from the beam into the cracking zone as the cracking zone extends; the larger the structure, the greater is the energy release. Eq. (1) expresses Bažant's size-effect law,[42] which was derived by dimensional analysis and similitude arguments from two simplifying assumptions, namely that the energy loss due to cracking is a function of both the fracture length and of the area of the cracking zone assumed to have a constant width at its front, proportional to the maximum aggregate size. Although this size-effect law is approximate, it was shown to capture the basic trend correctly when geometrically similar specimens of different sizes, failing in a brittle manner, are considered.

For very small structure sizes d, such that $d/\lambda_0 d_a \ll 1$, Eq. (1) approaches the limiting case of plastic limit analysis, which is characterized by absence of the size effect. For very large structure sizes d, such that $d/\lambda_0 d_a \gg 1$, Eq. (1) approaches the limiting case of linear elastic fracture mechanics. For normal beam sizes, Eq. (1) describes a smooth transition between these two limit cases.

Applicability of the size-effect law in Eq. (1) to structures that contain no notches is further contingent upon the assumption that the failure surfaces (cracks) in beams of various sizes are geometrically similar, and that the failure occurs when the front of the final crack reaches approximately the same relative position within the beam regardless of the size. These assumptions appear to agree with experimentally observed behavior.

It should be mentioned that the most general size-effect law for geometrically similar structures or specimens may be described by an asymptotic series expansion presented in Reference 43. The approximate size-effect law in Eq. (1) is then found to represent the first two terms of this asymptotic series expansion. It has been also shown[45,53] that an improved size-effect law for blunt fracture can be obtained as $v_c = v_c^0 (1 + \xi_v^r)^{-1/2r}$ with $\xi = d/\lambda_0 d_a$, in which r is an empirical constant. In the course of the present study, however, it appeared that the existing test data for concrete, due to their scatter as well as limited range, are insufficient for determining r. The optimum values of r for the existing diagonal shear data for beams ranged from 0.75 to 1; the fits were about equally good for this entire range. Therefore, the value $r = 1$ has been adopted.

Eq. (1) and (2) involve no size effect of beam width. This means that the cracking is assumed to occur roughly simultaneously through the thickness rather than propagate across the thickness.

Fracture mechanics is not the only theory that yields a size effect. A competing theory is the statistical Weibull theory. Although both theories can describe the same size-effect data if their size range is rather limited, they have very different extrapolations to very large sizes. While Weibull theory is certainly applicable to brittle failures of structures that can be modeled as a chain of brittle suddenly failing elements, it is questionable to apply it to diagonal shear failure.[44]

There is no aspect in this problem that could be regarded as a failure of a one-dimensional chain of elements. The release of strain energy into a localized extending fracture front, which is the basis of fracture mechanics, has nothing to do with Weibull's assumptions. It has also been pointed out[44] that if the material parameters of the extreme value strength distribution underlying Weibull theory are calibrated by typical test data for homogeneously stressed tensile specimens, and the same material parameters are then used for the cracking front zone, the statistical Weibull-type contribution to the size effect is found to be negligibly small, due to the fact that the volume of the frontal cracking zone is rather small compared to the beam size.

In very large beams, significant size effect may also be caused by diffusion phenomena, such as conduction of hydration heat and diffusion of pore water. Larger beams heat up more, and they also lose moisture more slowly than small beams. For lack of information, the diffusion size effects must be ignored in the analysis of existing test data.

Eq. (1) and (2) were calibrated[45] by a set of 296 data points extracted from the literature. Unfortunately, most of these data were not obtained with the size effect in mind, and therefore, the conclusions for the size effect cannot be definite. Nevertheless, plotting the existing data in the proper variables, the existence of the size effect was clearly demonstrated and it was shown that the size effect did not disagree with the theoretical size-effect law [Eq. (1)]. The scatter of the data was large, since it was necessary to include in the comparison data obtained in different laboratories, with different concretes and test beams that were not geometrically similar. This produces a scatter of the data that is large and obfuscates the precise trend with regard to the beam size.

As for comparisons with the existing formulas of ACI[51] and CEB-FIP,[52] the improvement achieved by Eq. (1) and (2) was significant; the standard deviation of the errors in v_u compared to the formula was found to be 128 psi for the ACI formula, 148 psi for the CEB-FIP formula, and 44.6 psi for Eq. (1) and (2). On the other hand, the improvement over another previous excellent formula due to Zsutty[18,21] was negligible when the entire data set was considered. However, Zsutty's formula gives no size effect and the presence of the size effect is clearly verified by those scant data in which beams of significantly diffent sizes were included in the test series (see Fig. 5 of Reference 45).

PROPOSED GENERALIZED FORMULA

To cover the effect of maximum aggregate size d_a, Bažant[53] derived by fracture mechanics considerations the following generalization of Eq. (1)

$$v_c = v_c^0 \; 1 + \sqrt{c_0/d_a} \Big/ \sqrt{1 + \frac{d}{\lambda_0 d_a}} \qquad (3)$$

where c_0 = empirical material constant having the dimension of length. By optimizing the fits of the test data described in the sequel, the optimum values of the empirical constants were found to be $c_0 = 0.2$ in., $\lambda_0 = 25$ and $k_1 = 6.5$. Note that for the aggregate size $d_a = 0.69$ in. (17.5 mm), Eq. (2) and (3) are equivalent to the original Eq. (1) and (2). In Reference 53 it was also shown that the factor $1 + (c_0/d_a)^{1/2}$, which gives the effect of aggregate size, agrees reasonably well with the test results of Taylor[25] and of Chana,[30] which include a wide range of maximum aggregate sizes.

With regard to the identification of material parameter values, Eq. (3) has the advantage that it can be algebraically transformed to a linear regression plot. This plot is described by the equation $Y = AX + C$ in which

$$X = \frac{d}{d_a} \qquad Y = \left[\frac{1}{v_c}\left(1 + \sqrt{\frac{0.2}{d_a}}\right)\right]^2 \tag{4}$$

with $C = 1/v_0^2$ and $A = C/\lambda_0$. By linear regression of the test data plotted as Y versus X, one finds the slope A and the Y-intercept C of the regression line, and the material parameters are then obtained as $v_0 = 1/C$ and $\lambda_0 = C/A$. This regression plot reveals the effect of the relative size d/d_a.

Another algebraic rearrangement of Eq. (3) yields a linear regression plot $Y' = A'X' + C'$ in which

$$X' = \frac{1}{\sqrt{d_a}} \qquad Y' = v_c\left(1 + \frac{d}{\lambda_0 d_a}\right)^{-1/2} \tag{5}$$

with $C' = v_c^0$ and $A' = v_{c0}\sqrt{c_0}$. After determining the slope and the Y'-intercept of this plot, the material parameters may be identified as $v_c^0 = C'$ and $c_0 = (A'/v_c^0)^2$. This type of regression reveals the effect of the maximum aggregate size.

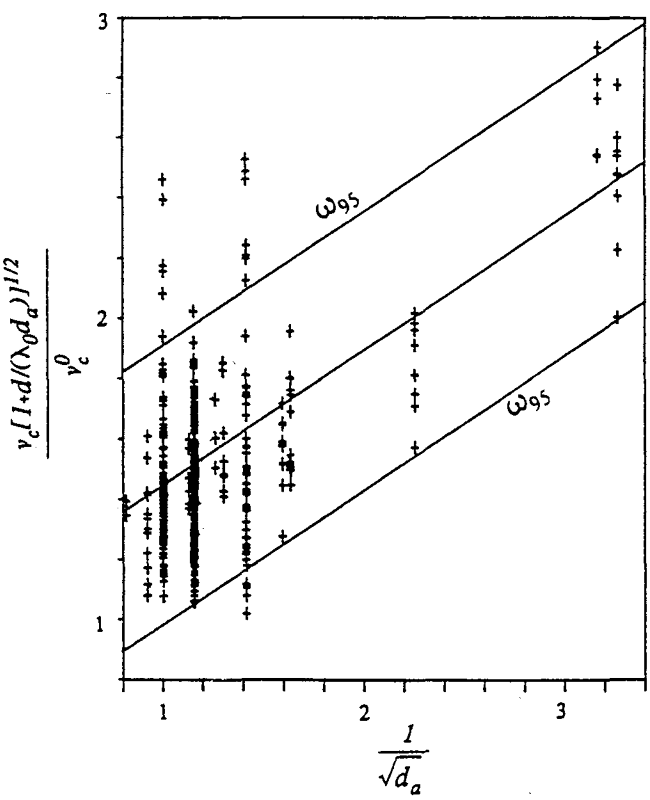

Fig. 2—Effect of aggregate size in beams without stirrups

To verify the present formulation, one would ideally need test data for geometrically similar beams of very different sizes. Unfortunately, no such test data exist in the literature. Most of the published test results pertain to beams of the smallest possible size that can be made with a given aggregate. Only a few test series, those indicated in Fig. 1(b), give at least some information on the size effect, although the size range of these data is relatively limited. These data, obtained by Walraven,[28] Bhal,[19] Kani,[16] Leonhardt and Walther,[8] Rüsch et al.,[10] Taylor,[25] and Chana,[30] are plotted in Fig. 1(a) through (d).

The large scatter of these data is due principally to the fact that the tests were made in different laboratories, on different concretes, and on geometrically dissimilar beams. Consequently, the error involves not only the error of Eq. (3) but also the error of Eq. (2), which enters Eq. (3). Nevertheless, the cost of producing a sufficiently large data set in one laboratory, on the same concrete, and for geometrically similar beams, would be high. First, one should exploit the information already available, even though it was not obtained for the purpose of studying the size effect.

From Fig. 1(a) through (c), it is clear that a significant size effect exists, since otherwise the trend of the data in these figures would have to be horizontal. Furthermore, it is clear that the size effect according to linear fracture mechanics, represented by the straight line of slope $-1/2$ in Fig. 1(a) and (c), would be too strong. The size effect is intermediate between the strength or yield criterion (limit analysis) and linear elastic fracture mechanics. This corroborates the acceptability of the assumption of a blunted fracture front from which Eq. (1) is derived. The size-effect plot of the new formula in Fig. 1(a) [with the corresponding linear regression according to Eq. (4) and (5) and Fig. 1(b)] is compared to the size-effect plot of the previous Bažant-Kim formula [Fig. 1(c) and Eq. (1)], with corresponding linear regression [Fig. 1(d)].

It is seen that the new formula, which includes the effect of aggregate size, significantly reduces the scatter. This is confirmed by the values of the coefficient of variation $\omega = \text{S.D.}/\bar{Y}$ for the vertical deviations of the data from the regression line, listed in Fig. 1, or the regression correlation coefficient r. The notations are: S.D. = standard deviation of the vertical differences from the regression line; $\bar{X}, \bar{Y}$ = coordinates of the centroid of the data; the lines of ω_{95} represent the 95 percent confidence limits; and, in Fig. 1, $C_1 = v_0[1 + (0.2/d_a)^{1/2}]$. Compared to the previous Bažant-Kim formula, the present improved formula reduces the coefficient of variation of the errors from 0.457 to 0.246, and increases the correlation coefficient from 0.46 to 0.75 [Fig. 1(b) and (d)].

The linear regression plot of the same data, showing the effect of maximum aggregate size and based on Eq. (5), is presented in Fig. 2. Despite the very large scatter, it is evident that, in addition to the effect of the relative beam size d/d_a, there is also an effect of the maximum aggregate size. If there were none, the trend of the data in Fig. 2 would have to be horizontal.

Fig. 3(a) and (b) show comparison of the present formulas [Eq. (2) and (3)] with 461 test results that have been extracted from the literature. In addition to the data used in Fig. 1, this large data set also includes the test results of Ahmad et al.[37] Bhal,[19] Bresler and Scordelis,[11] Chana,[30] Diaz de Cossio and Siess,[6] Elzanaty et al.,[36] Gaston, Siess, and Newmark,[4] Kani,[15,16] Krefeld and Thurston,[14] Leonhardt and Walther,[8] Mathey and Watstein,[12] Mattock,[20] Mattock and Wang,[33] Moody et al.,[5] Mphonde and Franz,[34] Placas and Regan,[24] Rajagopalan and Ferguson,[17] Swamy and Bahia,[35] Swamy,[31] H. Taylor,[25] Taylor and Brewer,[7,13] van den Berg,[9] and Walraven.[28]

Since the compilation of these data has been a rather tedious process (requiring correspondence with some of the authors to obtain various missing information), the complete set of 461 data points is listed numerically in Table 1. This makes it possible to verify the present rules as well as to use these data for possible future improvements of the diagonal shear formula.

Since most of the data in the complete set of 461 data points in Fig. 3 do not cover the size effect, the effect of the relative beam size d/d_a is not as conspicuous as in Fig. 1, but from the size-effect plot in Fig. 3(a) it is still clear that a size effect exists. Fig. 3(c) shows a plot of the measured values versus the calculated values of the nominal shear stress at failure. If there were no scatter and the formula were perfect, this plot would be a straight line of Slope 1, passing through the origin. So the deviations from this straight line are a measure of the error.

For comparison, Fig. 3(b) and (d) show the same plots for the previous Bažant-Kim formula. We see that the present formula achieves a reduction of scatter, diminishing the coefficient of variation of the errors ω from 0.292 to 0.242, and increasing the correlation coefficient r from 0.923 to 0.952. Again, the lines marked ω_{95} represent the 95 percent confidence limits.

GENERALIZATION FOR THE EFFECT OF STIRRUPS

For a beam with both longitudinal reinforcement and stirrups, the nominal ultimate shear stress at failure may be expressed as

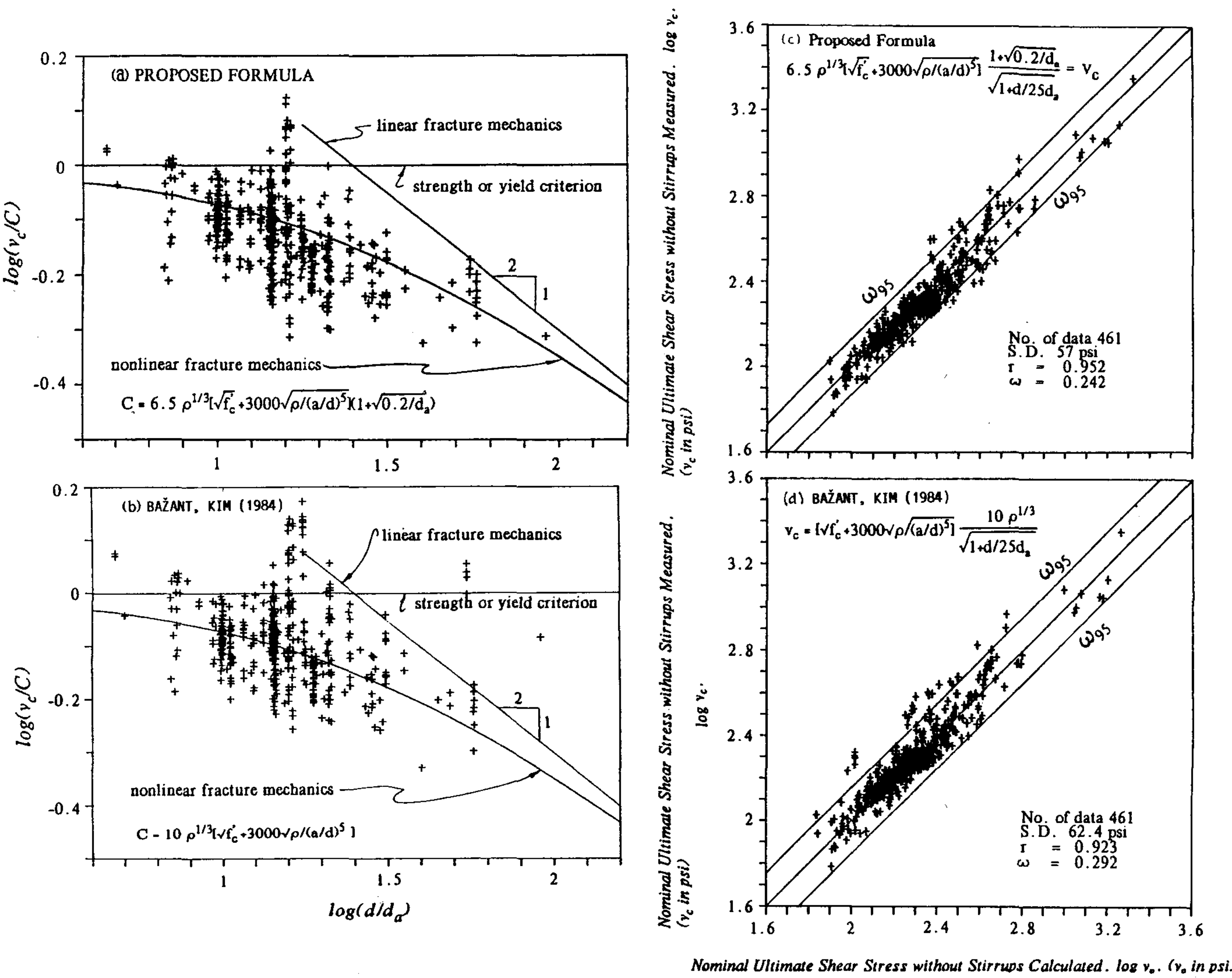

Fig. 3—Plots of measured versus calculated values of nominal shear strength for 461 available data for beams without stirrups (1 psi = 6895 Pa)

Table 1—Test data from the literature

Beam No.	a (in)	b (in)	d (in)	d_a (in)	f_c (psi)	A_s (in^2)	V^c (lb)	V_c (lb)
1) Without stirrups, 3-point-bent								
Moody, K. G. et al. (1954)								
A1	31.5	7.	10.30	1.	4400	1.56	13000	13500
A2	31.5	7.	10.50	1.	4500	1.58	15000	15000
A3	31.5	7.	10.55	1.	4500	1.64	14000	17000
A4	31.5	7.	10.63	1.	4570	1.76	15000	16000
B1	31.5	7.	10.50	1.	3070	1.19	11450	12650
B2	31.5	7.	10.55	1.	3130	1.20	13500	13500
B3	31.5	7.	10.63	1.	2790	1.19	12000	12500
B4	31.5	7.	10.69	1.	2430	1.24	11900	12500
C1	31.5	7.	10.55	1.	920	.60	4500	4500
C2	31.5	7.	10.70	1.	880	.62	5500	5500
C3	31.5	7.	10.75	1.	1000	.60	5700	5700
C4	31.5	7.	10.80	1.	980	.62	5650	5650
Taylor, R. and Brewer, R. S. (1963) (*)								
A1	33.	7.35	8.69	.75	4060	1.24	10864	10864
A2	33.	7.35	8.69	.75	4180	1.24	10304	10304
A3	33.	7.40	8.69	.75	4060	1.24	10416	10416
A4	33.	7.35	8.69	.75	4260	1.24	8960	8960
A5	33.	7.38	8.69	.75	5430	1.24	11984	11984
A6	33.	7.38	8.69	.75	4520	1.24	11088	11088
A7	33.	7.38	8.69	.75	4520	1.24	11088	11088
A8	33.	7.38	8.69	.75	5490	1.24	9744	9744
A9	33.	7.40	8.75	.75	4260	.8	10416	10416
A10	33.	7.40	8.75	.75	4180	.8	9072	9072
A11	33.	7.40	8.75	.75	5430	.8	11200	11200
A12	33.	7.40	8.75	.75	5490	.8	10304	10304
Bresler, Boris and Scordelis, A. C. (1963)								
OA-1	72.0	12.2	18.15	.75	3270	4.	30000	37500
OA-2	90.0	12.0	18.35	.75	3440	5.	32500	40000
OA-3	126.0	12.1	18.17	.75	5450	6.	35000	42500
Mattock, Alan H. (1969)								
1	27.4	6.	10.	.75	2480	.62	8000	8200
3	27.4	6.	10.	.75	6800	.62	11500	12300
10	27.4	6.	10.	.75	2690	1.86	11500	12600
15	51.4	6.	10.	.75	3750	.62	7000	7000
18	51.4	6.	10.	.75	2620	1.24	8000	8000
22	51.4	6.	10.	.75	2340	1.86	9000	9000
24	51.4	6.	10.	.75	4230	1.86	11800	11800
Krefeld, William J. and Thurston, Charles W. (1966)								
4A3	36.	8.	15.36	1.	4440	2.54	22500	24700
5A3	36.	8.	15.36	1.	4330	3.81	22500	38300
11A2	36.	6.	12.36	1.	4380	2.54	15000	16500
12A2	36.	6.	9.36	1.	4360	2.54	12500	14400
18A2	36.	6.	12.44	1.	2800	2.00	13000	14200
18B2	36.	6.	12.44	1.	2880	2.00	13000	16200
18C2	36.	6.	12.44	1.	3280	2.00	12000	16500
18D2	36.	6.	12.44	1.	3200	2.00	12000	13500
13A2	36.	6.	12.56	1.	2890	.60	8500	10900
14A2	36.	6.	9.56	1.	3000	.60	6000	7900
15A2	36.	6.	12.44	1.	2920	1.00	9000	10300
15B2	36.	6.	12.44	1.	3000	1.00	11000	11700
16A2	36.	6.	9.44	1.	3220	1.00	8500	9400
17A2	36.	6.	9.56	1.	3190	1.20	9000	9900
18E2	36.	6.	12.44	1.	2870	2.00	12000	18400
19A2	36.	6.	9.44	1.	2980	2.00	9500	10400
20A2	36.	6.	9.36	1.	3050	2.54	10000	11400
21A2	36.	8.	9.36	1.	2890	3.81	14000	17200
2AC	48.	6.	10.00	1.	3340	.79	7000	8500
3AC	48.	6.	10.06	1.	3020	1.20	9000	9900
4AC	48.	6.	10.00	1.	2390	1.58	8500	8500
5AC	48.	6.	9.94	1.	2660	2.00	8500	9400
6AC	48.	6.	9.98	1.	3310	2.54	12000	12000
3CC	60.	6.	10.06	1.	2970	1.20	8000	8000
4CC	60.	6.	10.00	1.	2980	1.58	9000	9000
5CC	60.	6.	9.94	1.	2950	2.00	10000	10000
6CC	60.	6.	9.86	1.	2980	2.54	10000	10000
4EC	72.	6.	10.00	1.	3080	1.58	9400	9400
5EC	72.	6.	9.94	1.	2830	2.00	8900	8900
6EC	72.	6.	9.86	1.	2770	2.54	9500	9500
4GC	84.	6.	10.00	1.	3050	1.58	8000	8300
5GC	84.	6.	9.94	1.	3180	2.00	8500	9400
6GC	84.	6.	9.86	1.	3100	2.54	9100	9100
6C	36.	6.	9.94	1.	2920	2.00	11500	11500
3AAC	36.	6.	10.06	1.	5010	1.20	12000	12500
4AAC	36.	6.	10.00	1.	4235	1.58	12500	13000
5AAC	36.	6.	9.94	1.	4760	2.00	12000	12800
6AAC	36.	6.	9.86	1.	4990	2.54	13000	13500
3AC	48.	6.	10.06	1.	4620	1.20	11000	12000
4AC	48.	6.	10.00	1.	4420	1.58	11000	12100
5AC	48.	6.	9.94	1.	4760	2.00	11000	12200
6AC	48.	6.	9.86	1.	4950	2.54	12000	13300
4CC	60.	6.	10.00	1.	5570	1.58	11000	11800
5CC	60.	6	9.94	1.	5430	2.00	12000	12900
6CC	60.	6.	9.86	1.	5570	2.54	11500	14200
5EC	72.	6.	9.94	1.	5430	2.00	9000	12000
6EC	72.	6.	9.86	1.	4900	2.54	9000	11000
3AAC	36.	6.	10.06	1.	1820	1.20	9000	9100
4AAC	36.	6.	10.00	1.	1870	1.58	9000	9600
5AAC	36.	6.	9.94	1.	2230	2.00	10500	11300
6AAC	36.	6.	9.86	1.	1940	2.54	10000	14000
3AC	48.	6.	10.06	1.	1990	1.20	8000	8300
4AC	48.	6.	10.00	1.	1870	1.58	8000	9000
5AC	48.	6.	9.94	1.	2230	2.00	9500	9800
6AC	48.	6.	9.86	1.	1800	2.54	8000	9200
3CC	60.	6.	10.06	1.	1770	1.20	6000	7000
4CC	60.	6.	10.00	1.	2480	1.58	7900	7900
5CC	60.	6.	9.94	1.	2130	2.00	7500	7700
6CC	60.	6.	9.86	1.	1980	2.54	8500	8900
C	60.	8.	19.00	1.	2430	2.37	19000	19000
PCa	72.	6.	9.86	1.	5260	2.54	12000	12000
PCb	72.	6.	9.86	1.	5260	2.54	12000	12000
OCa	60.	6.	10.00	1.	5180	1.58	10000	10900
OCb	60.	6.	10.00	1.	5660	1.58	11500	11800
CCa	72.	10.	17.94	1.	5550	4.00	31000	33000
OCb	72.	10.	17.94	1.	5550	4.00	29000	30000

Beam No.	a (in)	b (in)	d (in)	d_a (in)	f_c (psi)	A_s (in^2)	V^c (lb)	V_c (lb)
1) Without stirrups, 3-point-bent								
Mphonde, Andrew G. and Frantz, Gregory C. (1984)								
A0-3-3b	42.	6.	11.75	.375	3273	2.37	12750	14500
A0-3-3c	42.	6.	11.75	.375	4277	1.64	15000	15000
A0-7-3a	42.	6.	11.75	.375	5938	2.37	18500	18500
A0-7-3b	42.	6.	11.75	.375	6562	2.37	18600	18600
A0-11-3a	42.	6.	11.75	.375	11812	2.37	20150	20150
A0-11-3b	42.	6.	11.75	.375	11766	2.37	20100	20100
A0-15-3a	42.	6.	11.75	.375	12823	2.37	21000	21000
A0-15-3b	42.	6.	11.75	.375	14768	2.37	-	22500
A0-15-3c	42.	6.	11.75	.375	14477	2.37	21500	22000
A0-11-2	29.375	6.	11.75	.375	12498	2.37	20000	25000
Bhal, N. S. (1968)								
B1	35.43	9.45	11.81	1.18	4054	1.43	-	15763
B2	70.87	9.45	23.62	1.18	5177	2.86	-	26345
B3	106.30	9.45	35.43	1.18	4807	4.29	-	36376
B4	141.70	9.45	47.24	1.18	4409	5.72	-	39683
B5	70.87	9.45	23.62	1.18	4651	1.43	-	23369
B6	70.87	9.45	23.62	1.18	4324	1.35	-	25132
B7	106.30	9.45	35.43	1.18	4765	2.15	-	30313
B8	106.30	9.45	36.02	1.18	4850	2.02	-	27557
Gaston, J. R., Siess, C. P. and Newmark, N. M. (1952)								
T2Ma	36.	6.	10.58	1.	4320	.88	-	8700
T2Mb	36.	6.	10.58	1.	4020	.88	-	9225
T2Mc	36.	6.	10.51	1.	4470	.88	-	11950
Placas and Regan (1971)								
R1	33.6	6.	10.	.75	3800	2.46	-	10100
R3	33.6	6.	10.	.75	3600	2.46	-	10100
D2	33.6	6.	10.	.75	4400	2.46	-	11800
2) Without stirrups, 4-point-bent								
Diaz de Cossio, Roger and Siess, Chester P. (1960)								
L-1	20.	6.	9.94	1.	3050	2.	15000	26100
L-2	30.	6.	9.94	1.	3120	2.	13000	17000
L-2a	30.	6.	9.94	1.	5320	2.	14300	18000
L-3	40.	6.	9.94	1.	4060	2.	12000	12000
L-4	50.	6.	9.94	1.	3740	2.	11500	11500
L-5	60.	6.	9.94	1.	4050	2.	11400	11450
Van den Berg, F. J. (1962) (*)								
D-1	49.5	9.	14.14	.75	8500	5.50	33000	34000
D-2	49.5	9.	14.14	.75	7330	5.50	29500	29500
D-3	49.5	9.	14.14	.75	6160	5.50	29000	29000
D-4	49.5	9.	14.14	.75	6060	5.50	30000	32500
D-5	49.5	9.	14.14	.75	7330	5.50	27500	29500
D-6	49.5	9.	14.14	.75	7050	5.50	28500	31500
D-7	49.5	9.	14.14	.75	5500	5.50	31500	31500
D-8	49.5	9.	14.14	.75	4350	5.50	25500	26500
D-9	49.5	9.	14.14	.75	2570	5.50	20000	20000
D-10	49.5	9.	14.14	.75	4550	5.50	28500	28500
D-11	49.5	9.	14.14	.75	3260	5.50	24500	24500
D-12	49.5	9.	14.14	.75	3970	5.50	22500	24000
D-13	49.5	9.	14.14	.75	3550	5.50	22300	22300
D-14	49.5	9.	14.14	.75	4080	5.50	24000	24000
D-15	49.5	9.	14.14	.75	3810	5.50	23000	23000
D-16	49.5	9.	14.14	.75	4420	5.50	25000	25000
D-17	49.5	9.	14.14	.75	3780	5.50	23500	23500
D-18	49.5	9.	14.14	.75	4160	5.50	23500	23500
D-19	49.5	9.	14.14	.75	4680	5.50	26000	26000
D-20	49.5	9.	14.14	.75	4130	5.50	24000	24000
E-1	60.0	9.	14.14	.75	11220	5.50	33500	33500
E-2	60.0	9.	14.14	.75	8060	5.50	32500	32500
E-3	60.0	9.	14.14	.75	7060	5.50	29000	29000
E-4	60.0	9.	14.14	.75	6190	5.50	29000	29000
E-5	60.0	9.	14.14	.75	3410	5.50	22000	22000
A5-1	49.5	12.	14.14	.75	7590	5.50	40000	41500
A5-2	49.5	9.	14.39	.75	3470	4.68	22500	22500
A5-3	49.5	9.	14.52	.75	4010	3.44	21500	21500
A5-4	49.5	9.	14.47	.75	3900	3.17	21500	21500
A5-5	39.0	9.	14.56	.75	4260	2.26	23000	23000
A5-6	39.0	9.	14.45	.75	4590	2.84	25000	27000
A5-7	49.5	9.	17.64	.75	4510	5.50	34500	34500
Moody, K. G., et al. (1954)								
B-1	36.	6.	10.56	1.	5320	1.2	11500	13000
B-2	36.	6.	10.56	1.	2420	1.2	7500	8000
B-3	36.	6.	10.56	1.	3740	1.2	11500	11750
B-4	36.	6.	10.56	1.	2230	1.2	8500	9100
B-5	36.	6.	10.56	1.	4450	1.2	10500	11700
B-6	36.	6.	10.56	1.	2290	1.2	7500	7750
B-7	36.	6.	10.56	1.	4480	1.2	10000	11500
B-8	36.	6.	10.56	1.	1770	1.2	7000	7000
B-9	36.	6.	10.56	1.	5970	1.2	11500	12000
B-10	36.	6.	10.56	1.	3470	1.2	9500	11000
B-11	36.	6.	10.56	1.	5530	1.2	11000	13500
B-12	36.	6.	10.56	1.	2930	1.2	10500	10600
B-13	36.	6.	10.56	1.	5480	1.2	10000	12500
B-14	36.	6.	10.56	1.	3270	1.2	9700	9700
B-15	36.	6.	10.56	1.	5420	1.2	11500	11500
B-16	36.	6.	10.56	1.	2370	1.2	8000	8500
Mathey, Robert G. and Watstein, David (1963)								
I-1	24.	8.	15.86	1.	3680	3.86	27500	70350
I-2	24.	8.	15.86	1.	3330	3.86	27500	69850
II-3	24.	8.	15.86	1.	3170	2.39	22500	58850
II-4	24.	8.	15.86	1.	3830	2.39	25000	70350
III-5	24.	8.	15.86	1.	3730	2.35	25000	64850
III-6	24.	8.	15.86	1.	3710	2.35	25000	65350
IV-7	24.	8.	15.86	1.	3500	2.36	25000	65375
IV-8	24.	8.	15.86	1.	3610	2.36	27500	68350
V-9	24.	8.	15.86	1.	3350	1.48	22500	50350
V-10	24.	8.	15.86	1.	3910	1.48	24000	60350
VI-11	24.	8.	15.86	1.	3680	1.48	20000	50350
VI-12	24.	8.	15.86	1.	3720	1.48	22500	60350
V 13	24.	8.	15.86	1.	3250	.954	17500	50000

Table 1 (cont.)—Test data from the literature

Beam No.	a (in)	b (in)	d (in)	d_a (in)	f_c (psi)	A_s (in^2)	V^c (lb)	V_c (lb)
2) Without stirrups, 4-point-bent								
V-14	24.	8.	15.86	1.	3870	.954	20000	50350
VI-15	24.	8.	15.86	1.	3700	.951	20000	40350
VI-16	24.	8.	15.86	1.	3310	.951	20000	42400
IIIa-17	60.	8.	15.86	1.	4240	3.23	17500	19800
IIIa-18	60.	8.	15.86	1.	3650	3.23	16500	18150
Va-19	60.	8.	15.86	1.	3410	1.18	12000	14225
Va-20	60.	8.	15.86	1.	3710	1.18	12500	14825
VIb-21	45.	8.	15.86	1.	3790	1.07	12500	16050
VIb-22	45.	8.	15.86	1.	3740	1.07	12000	14025
VIb-23	45.	8.	15.86	1.	4430	1.07	13500	16875
VIb-24	60.	8.	15.86	1.	3820	.594	10500	12250
VIb-25	60.	8.	15.86	1.	3740	.594	9000	11225
Taylor, R. (1960)								
1-18	18.	6.	11.	.75	2672	1.32	12320	26320
1-30	30.	6.	11.	.75	2728	1.32	10080	12320
1-42	42.	6.	11.	.75	2704	1.32	10528	10528
1-48	48.	6.	11.	.75	2944	1.32	9240	9352
1-54	54.	6.	11.	.75	2592	1.32	8960	8960
Ahmad, Shuaib H., Khaloo, A. R. and Proveda, A. (1986) (**)								
A1	32.	5.	8.	.5	9590	1.58	13000	13000
A2	24.	5.	8.	.5	9590	1.58	14000	15500
A3	21.6	5.	8.	.5	9590	1.58	14000	15500
A4	18.4	5.	8.	.5	9590	1.58	14300	21000
A5	16.	5.	8.	.5	9590	1.58	17000	37500
A6	8.	5.	8.	.5	9590	1.58	34500	90000
A7	32.76	5.	8.19	.5	9590	.71	8500	10500
A8	24.57	5.	8.19	.5	9590	.71	9500	11000
A9	22.11	5.	8.19	.5	9590	.71	11000	18000
A10	18.84	5.	8.19	.5	9590	.71	11000	18500
A11	16.38	5.	8.19	.5	9590	.71	12000	12500
A12	8.19	5.	8.19	.5	9590	.71	30000	50000
B1	31.76	5.	7.94	.5	10560	2.	11505	11510
B2	23.82	5.	7.94	.5	10560	2.	12750	15500
B3	21.44	5.	7.94	.5	10560	2.	14000	22500
B4	18.26	5.	7.94	.5	10560	2.	14000	32200
B5	15.88	5.	7.94	.5	10560	2.	17500	24000
B7	32.76	5.	8.19	.5	10560	.9	10000	10030
B8	24.57	5.	8.19	.5	10560	.9	10500	10500
B9	22.11	5.	8.19	.5	10560	.9	10507	18000
B10	18.84	5.	8.19	.5	10560	.9	12500	14400
B11	16.38	5.	8.19	.5	10560	.9	14000	27500
B12	8.19	5.	8.19	.5	10560	.9	25000	48000
C1	29.00	5.	7.25	.5	10140	2.4	12000	12200
C2	21.75	5.	7.25	.5	10140	2.4	11000	17000
C3	19.58	5.	7.25	.5	10140	2.4	9000	15500
C4	16.68	5.	7.25	.5	10140	2.4	12500	20000
C7	32.52	5.	8.13	.5	10140	1.32	8000	10200
C8	24.39	5.	8.13	.5	10140	1.32	10000	10000
C9	21.95	5.	8.13	.5	10140	1.32	10000	10200
C10	18.70	5.	8.13	.5	10140.	1.32	9250	12800
C11	16.26	5.	8.13	.5	10140.	1.32	14500	24000
C12	8.13	5.	8.13	.5	10140.	1.32	20000	55000
Kani, G. N. J. (1967)								
40	29.42	5.97	5.50	.75	3830	.85	-	7195
41	13.38	6.00	5.56	.75	3950	.87	-	11565
43	32.00	5.98	5.40	.75	4060	.88	-	6790
45	10.70	5.95	5.23	.75	3700	.88	-	14520
46	10.70	5.95	5.35	.75	3700	.88	-	15520
47	26.70	5.95	5.20	.75	3590	.88	-	6335
48	26.70	5.95	5.25	.75	3590	.88	-	6095
52	21.40	6.00	5.45	.75	3600	.88	-	6495
53	5.34	5.95	5.20	.75	3870	.88	-	34900
54	5.34	5.95	5.35	.75	3870	.88	-	35450
55	16.00	5.92	5.30	.75	3640	.88	-	7325
56	18.73	6.03	5.41	.75	3950	.87	-	6295
57	29.42	6.03	5.46	.75	3830	.86	-	7095
58	18.73	6.00	5.45	.75	3950	.87	-	6500
59	14.68	6.08	5.50	.75	3860	.88	-	11275
60	16.02	6.10	5.46	.75	3880	.88	-	8835
81	64.08	6.04	10.80	.75	3990	1.80	-	11500
83	32.04	6.14	10.68	.75	3980	1.80	-	14600
84	42.72	5.95	10.67	.75	3980	1.80	-	12450
91	64.08	6.08	10.58	.75	3980	1.74	-	11455
93	69.42	6.10	10.74	.75	4390	1.74	-	12095
94	21.36	6.03	10.76	.75	3670	1.80	-	24850
95	26.70	6.04	10.83	.75	3670	1.80	-	16350
96	42.72	6.03	10.83	.75	3670	1.80	-	12650
97	32.10	6.00	10.88	.75	3950	1.75	-	14050
98	26.75	6.03	10.81	.75	3800	1.75	-	17150
99	26.75	6.00	10.70	.75	3800	1.75	-	17350
100	21.40	6.03	10.62	.75	3950	1.76	-	25150
61	42.72	6.16	21.32	.75	3880	3.35	-	36700
63	85.44	6.08	21.37	.75	3800	3.60	-	20950
64	170.88	6.15	21.28	.75	3730	3.60	-	17750
65	53.50	5.89	21.75	.75	3910	3.61	-	25250
66	128.16	6.15	21.31	.75	3830	3.60	-	20400
67	21.36	6.19	20.80	.75	4400	3.54	-	123150
69	21.36	6.11	21.35	.75	3970	3.48	-	131600
71	64.08	6.10	21.42	.75	3970	3.48	-	22950
72	42.80	6.00	21.62	.75	3600	3.51	-	44250
74	64.20	6.00	20.60	.75	3950	3.51	-	24200
75	64.20	6.00	20.63	.75	3960	3.51	-	24250
76	53.50	6.00	20.38	.75	4460	3.51	-	25800
79	149.80	6.03	21.90	.75	3790	3.59	-	18800
3041	86.40	6.00	43.20	.75	3900	7.06	-	73300
3042	107.75	6.06	43.10	.75	3830	7.06	-	53250
3043	129.00	6.05	43.00	.75	3910	7.06	-	37100
3044	171.8	6.0	43.20	.75	4280	7.06	-	35750
3045	215.0	6.1	43.00	.75	4100	7.08	-	34250
3046	331.4	6.1	43.20	.75	3870	7.12	-	34650
3047	373.8	6.1	43.10	.75	3870	7.06	-	33050
271	64.2	24.06	10.58	.75	3910	6.99	-	48820
272	53.5	24.05	10.66	.75	3910	6.99	-	51200
273	42.8	24.1	10.68	.75	3940	6.99	-	46330
274	32.1	24.1	10.64	.75	3940	6.99	-	56225
Kani, G. N. J. (1966)								
265	16.02	6.04	10.60	.75	2630	.330	-	11900
266	26.50	6.03	10.69	.75	2620	.321	-	7300
2) without stirrups, 4-point-bent								
267	37.38	6.04	10.60	.75	3000	.330	-	5500
268	32.12	6.03	10.84	.75	2910	.321	-	6115
269	10.62	6.05	10.77	.75	2620	.321	-	20000
270	21.35	6.00	10.74	.75	2910	.321	-	9300
246	37.49	6.03	10.80	.75	4000	.330	-	5700
248	26.70	6.03	11.10	.75	4000	.330	-	8350
249	10.62	6.04	10.85	.75	4060	.321	-	18800
250	16.00	6.00	10.77	.75	4060	.321	-	12300
251	21.40	6.06	10.86	.75	3800	.318	-	9420
174	10.62	6.01	10.70	.75	5280	.330	-	19500
178	16.00	6.04	10.60	.75	5000	.330	-	13250
179	26.70	6.03	10.40	.75	4690	.330	-	7550
180	37.38	6.04	10.60	.75	5000	.330	-	5600
141	21.40	5.94	10.63	.75	2800	.511	-	10940
142	21.40	6.15	10.88	.75	2800	.513	-	13100
143	42.72	6.05	10.80	.75	2560	.486	-	6795
147	26.70	6.00	11.31	.75	2440	.475	-	9505
148	16.05	5.98	10.80	.75	2880	.508	-	17950
149	26.70	6.01	10.69	.75	2610	.500	-	9815
150	26.70	6.04	10.75	.75	2610	.500	-	10380
151	26.75	6.06	10.73	.75	2800	.505	-	8005
152	32.10	5.88	10.63	.75	2850	.494	-	7300
153	32.10	6.00	10.73	.75	2850	.491	-	7375
102	21.36	6.04	10.58	.75	3670	.487	-	10970
103	32.04	6.11	10.78	.75	4270	.486	-	8720
104	42.72	6.05	10.60	.75	3670	.486	-	7560
105	26.75	6.00	10.70	.75	3800	.496	-	9335
106	26.70	6.05	10.54	.75	4170	.486	-	10030
107	53.40	6.08	10.51	.75	3850	.486	-	5765
109	16.02	6.03	10.67	.75	3630	.486	-	16150
111	26.70	6.06	10.71	.75	3920	.491	-	9735
112	26.70	6.02	10.75	.75	3920	.491	-	8050
113	16.05	6.00	10.78	.73	3700	.500	-	19600
114	21.04	6.03	10.62	.75	3700	.512	-	13800
115	26.75	6.00	10.69	.75	3800	.494	-	10185
116	32.10	6.00	10.67	.75	3830	.497	-	8830
162	21.36	6.04	10.71	.75	4980	.498	-	13250
162	21.36	6.06	10.51	.75	4980	.487	-	13950
163	26.70	6.14	10.73	.75	5130	.498	-	9100
163	26.70	5.97	10.70	.75	5130	.498	-	8550
164	42.72	6.15	10.67	.75	4900	.476	-	8050
166	32.10	5.98	10.68	.75	5130	.499	-	9050
166	32.10	6.05	10.80	.75	5130	.496	-	8600
121	32.10	6.00	10.72	.75	2950	1.19	-	11000
122	42.80	5.90	10.85	.75	2880	1.18	-	8725
123	42.72	6.12	10.68	.75	2230	1.17	-	8500
124	53.40	6.08	10.68	.75	2230	1.17	-	7200
126	32.04	6.12	10.72	.75	2360	1.17	-	9600
129	16.02	6.09	10.81	.75	2550	1.17	-	32200
130	58.74	6.04	10.85	.75	2610	1.17	-	9000
131	26.75	5.95	10.79	.75	2630	1.19	-	11140
132	26.75	6.05	10.66	.75	2680	1.17	-	11680
133	53.50	6.06	10.76	.75	2880	1.18	-	8660
134	21.40	6.06	10.75	.75	2530	1.18	-	13450
135	21.40	5.88	10.79	.75	2530	1.18	-	17250
24	16.02	6.00	10.68	.75	4040	1.20	-	40900
25	21.36	6.00	10.68	.75	3560	1.20	-	23400
26	21.36	6.00	10.68	.75	3930	1.20	-	17550
27	26.70	6.00	10.68	.75	4320	1.20	-	11550
28	26.70	6.00	10.68	.75	4230	1.20	-	12200
29	48.06	6.00	10.68	.75	3560	1.20	-	9650
30	48.06	6.00	10.68	.75	3650	1.20	-	10400
35	37.50	6.11	10.61	.75	3780	1.18	-	10100
36	37.50	6.02	10.75	.75	3780	1.18	-	11600
181	21.36	6.06	10.69	.75	4920	1.16	-	14650
182	53.40	6.09	10.57	.75	4920	1.16	-	10960
184	16.02	6.05	10.67	.75	5090	1.16	-	36700
186	42.72	6.09	10.70	.75	5090	1.16	-	12450
188	21.36	6.04	10.90	.75	4800	1.16	-	20800
191	32.10	6.05	10.83	.75	4930	1.18	-	11940
193	26.70	6.04	10.93	.75	5020	1.19	-	12750
194	32.04	6.06	10.93	.75	5020	1.19	-	11500
195	42.72	6.03	10.84	.75	5020	1.19	-	10625
197	26.75	5.92	10.77	.75	5220	1.17	-	12000
199	21.40	6.00	10.75	.75	5220	1.18	-	17250
Leonhardt, et al. (1962) (*)								
D1/1	8.27	1.97	2.76	.59	6358	.093	-	1631
D1/2	8.27	1.97	2.76	.59	6358	.093	-	1609
D2/1	16.54	3.94	5.51	.59	6386	.360	-	4762
D2/2	16.54	3.94	5.51	.59	6386	.360	-	5225
D3/1	24.80	5.91	8.27	.59	6600	.791	-	10428
D3/2	24.80	5.91	8.27	.59	6600	.791	-	9634
D4/1	33.07	7.87	11.02	.59	6045	1.454	-	16645
D4/2	33.07	7.87	11.02	.59	6045	1.454	-	16027
C1	17.72	3.94	5.91	1.18	6699	.308	-	4850
C2	35.43	5.91	11.81	1.18	6699	.925	-	14550
C3	53.15	7.87	17.72	1.18	6699	1.851	-	22817
C4	70.87	8.86	23.87	1.18	6699	2.776	-	34171
Rajagopalan, K. S. and Ferguson, Phil M. (1968)								
S-1	40.	6.06	10.18	.5	5300	.88	-	8000
S-2	40.	6.06	10.44	.5	4800	.62	-	8400
S-3	44.	6.00	10.50	.5	4200	.51	-	6990
S-4	44.	6.00	10.56	.5	4800	.40	-	6300
S-5	44.	6.00	10.31	.5	4050	.33	-	7550
S-6	44.	5.94	10.53	.5	4500	.22	-	6150
S-7	44.	6.00	10.56	.5	4150	.16	-	6750
S-9	44.	5.98	10.30	.5	3640	.33	-	5500
S-12	44.	6.03	10.57	.5	4300	.16	-	5530
S-13	44.	5.99	10.43	.5	3440	1.08	-	9000
Taylor, Howard P. J. (1972) (*)								
A1	110.20	15.75	36.61	1.50	5280	7.78	-	80700
A2	110.20	15.75	36.61	.75	4620	7.78	-	73900
B1	55.12	7.87	18.31	1.50	4930	1.95	-	23400
B2	55.12	7.87	18.31	.75	4500	1.95	-	19600
B3	55.12	7.87	18.31	.375	5810	1.95	-	19200
C1	27.56	3.94	9.15	.75	4640	.487	-	5060
C2	27.56	3.94	9.15	.375	4640	.487	-	5390
C3	27.56	3.94	9.15	.375	4980	.487	-	6180
C4	27.56	3.94	9.15	.375	3770	.487	-	5060

Table 1 (cont.)—Test data from the literature

Beam No.	a (in)	b (in)	d (in)	d_a (in)	f'_c (psi)	A_s (in²)	v^c (lb)	V_c (lb)
2) Without stirrups, 4-point-bent								
C5	27.56	3.94	9.15	.375	4060	.487	-	6060
C6	27.56	3.94	9.15	.10	5220	.487	-	6180
D1	16.54	2.36	5.49	.10	5800	.122	-	2620
D2	16.54	2.36	5.49	.10	5800	.122	-	2720
D3	16.54	2.36	5.49	.10	5800	.122	-	2380
D4	16.54	2.36	5.49	.10	5800	.122	-	2560
Chana, P. S. (1981)								
2.1a	42.05	8.	14.02	.787	7150	1.89	-	21582
2.1b	42.05	8.	14.02	.787	7150	1.89	-	21829
2.2a	42.05	8.	14.02	.394	6034	1.89	-	19648
2.2b	42.05	8.	14.02	.394	6034	1.89	-	21222
2.3a	42.05	8.	14.02	.787	6556	1.89	-	22346
2.3b	42.05	8.	14.02	.787	6556	1.89	-	21672
3.1a	20.91	3.94	6.97	.394	5004	.479	-	5350
3.1b	20.91	3.94	6.97	.394	5004	.479	-	5373
3.2a	20.91	3.94	6.97	.394	5337	.479	-	5508
3.2b	20.91	3.94	6.97	.394	5337	.479	-	5733
3.3a	20.91	3.94	6.97	.394	5816	.479	-	5957
3.3b	20.91	3.94	6.97	.394	5816	.479	-	5216
D1	20.91	3.94	6.97	.394	4583	.479	-	4968
D2	20.91	3.94	6.97	.394	4699	.479	-	5261
D3	20.91	3.94	6.97	.394	6483	.479	-	4811
4.1a	12.52	2.36	4.17	.197	4482	.169	-	2208
4.1b	12.52	2.36	4.17	.197	4482	.169	-	1945
4.2a	12.52	2.36	4.17	.197	4482	.169	-	2017
4.2b	12.52	2.36	4.17	.197	4482	.169	-	2183
4.3a	12.52	2.36	4.17	.197	7571	.169	-	2630
4.3b	12.52	2.36	4.17	.197	7571	.169	-	2776
4.4a	12.52	2.36	4.17	.197	7571	.169	-	2163
4.4b	12.52	2.36	4.17	.197	7571	.169	-	2354
5.1a	20.08	7.87	6.69	.394	5845	.946	-	10746
5.1b	20.08	7.87	6.69	.394	5845	.946	-	10746
5.2a	20.08	7.87	6.69	.787	5758	.946	-	12364
5.2b	20.08	7.87	6.69	.787	5758	.946	-	12589
6.1	4.96	.925	1.65	.094	5192	.026	-	481
6.2	4.96	.925	1.65	.094	5511	.026	-	432
6.3	4.96	.925	1.65	.094	5671	.026	-	472
6.4	4.96	.925	1.65	.094	8702	.026	-	470
6.5	4.96	.925	1.65	.094	6802	.026	-	587
6.6	4.96	.925	1.65	.094	9282	.026	-	616
6.7	4.96	.925	1.65	.094	6251	.026	-	531
6.8	4.96	.925	1.65	.094	6179	.026	-	504
6.9	4.96	.925	1.65	.094	6527	.026	-	515
Walraven, J. C. (1978) (*)								
A1	14.76	7.87	4.92	.63	4978	.321	-	6704
A2	49.61	7.87	16.54	.63	4959	.964	-	15882
A3	85.04	7.87	28.35	.63	5041	1.77	-	22676
Elzanty, Ashraf H., Nilson, Arthur H. and Slate, F. O. (1986)								
F7	42.0	7.	10.7	.5	3000	.45	-	7600
F11	42.8	7.	10.7	.5	3000	.90	-	10000
F12	42.0	7.	10.5	.5	3000	1.80	-	12000
F8	42.8	7.	10.7	.5	5800	.75	-	10300
F13	42.8	7.	10.7	.5	5800	.90	-	10400
F14	42.0	7.	10.5	.5	5800	1.80	-	14300
F1	42.8	7.	10.7	.5	9500	.90	-	13100
F2	42.0	7.	10.5	.5	9500	1.80	-	14800
F10	42.0	7.	10.5	.5	9500	2.40	-	17300
F9	42.4	7.	10.6	.5	11500	1.20	-	14100
F15	42.0	7.	10.5	.5	11500	1.80	-	15000
F3	21.4	7.	10.7	.5	10000	.90	-	18100
F4	21.0	7.	10.5	.5	10000	1.80	-	25600
F5	64.2	7.	10.7	.5	9200	.90	-	9800
F6	63.0	7.	10.5	.5	9200	1.80	-	13600
Rusch, H. et al. (1962) (*) (**)								
X	15.75	3.54	4.37	.59	3421	.41	-	3285
Y	28.22	4.72	7.84	.59	3421	.982	-	6768
Z	37.29	7.09	10.31	.59	3602	1.93	-	12302
Kani, G. N. J. (1967) (**)								
44	32.	5.98	5.40	.75	4060	0.88	-	6790
88	10.68	6.01	10.47	.75	4560	1.77	-	80850
68	192.24	6.17	21.21	.75	3940	3.53	-	1700

v^c : strength at initial diagonal tension crack formed, for reference only.

(*) : concrete cubic strength

(**) : these data points not included in 461 data points

Beam No.	a (in)	b (in)	d (in)	d_a (in)	f'_c (psi)	A_s (in²)	v^c (lb)	V_c (lb)
3) Without stirrups, T-section								
Swamy (1971)								
M1N4	27.	6.	6.75	.75	5800	1.76	-	14870
M2N4	13.5	3.	3.37	.375	6000	.44	-	3980
M3N4	9.	2.	2.25	.25	6050	.20	-	1750
M1N3	20.25	6.	6.75	.75	6040	1.76	-	19490
M2N3	10.13	3.	3.37	.375	6200	.44	-	5040
M3N3	6.75	2.	2.25	.25	6550	.20	-	2560
M1N3/2	10.13	6.	6.75	.75	5800	1.76	-	48000
M2N3/2	5.06	3.	3.37	.375	5100	.44	-	10320
M3N3/2	3.37	2.	2.25	.25	6250	.20	-	6270
M1N3A	20.25	6.	6.75	.75	5750	1.76	-	17670
M1N3B	20.25	6.	6.75	.375	5700	1.76	-	18440
M1N3D	20.25	6.	6.75	1.5	4560	1.76	-	18400
M2N3A	10.13	3.	3.37	.75	6550	.44	-	5950
M2N3B	10.13	3.	3.37	.375	6220	.44	-	5060
M2N3C	10.13	3.	3.37	.25	5700	.44	-	4677
M3N3B	6.75	2.	2.25	.375	6700	.20	-	3020
M3N3C	6.75	2.	2.25	.25	5500	.20	-	2130

Beam No.	a (in)	b (in)	d (in)	d_a (in)	f_c (psi)	A_s (in²)	ρ_v (%)	V_u (lb)	f_{yv} (psi)
4) With stirrups, 3-point-bent									
Krefeld, William J. and Thurston, Charles W. (1966)									
26-1	72.	10.	17.94	1.	5820	4.	.166	46500	49500
29a-1	72.	10.	17.94	1.	5630	4.	.111	35900	49500
29a-2	72.	10.	17.94	1.	5460	4.	.111	36000	49500
213.5-1	72.	10.	17.94	1.	5640	4.	.074	33300	49500
29a-2	72.	10.	17.94	1.	5390	4.	.111	48700	54000
29b-2	72.	10.	17.94	1.	6000	4.	.111	45500	54000
29c-2	72.	10.	17.94	1.	3500	4.	.111	36300	54000
29d-2	72.	10.	17.94	1.	4410	4.	.111	37100	54000
29e-2	72.	10.	17.94	1.	7030	4.	.111	46400	54000
29g-2	72.	10.	17.94	1.	2280	4.	.111	33700	54000
213.5a-2	72.	10.	17.94	1.	5360	4.	.074	36300	54000
218a-2	72.	10.	17.94	1.	5450	4.	.055	36900	54000
29-3	72.	10.	17.94	1.	4970	4.	.111	40000	54000
318-1	72.	10.	17.94	1.	5880	4.	.122	49500	54000
321-1	72.	10.	17.94	1.	5620	4.	.105	36800	54000
318-2	72.	10.	17.94	1.	5640	4.	.122	39800	54000
321-2	72.	10.	17.94	1.	5510	4.	.105	37500	54000
313.5-3	72.	10.	17.94	1.	6190	4.	.163	48000	54000
318-3	72.	10.	17.94	1.	6240	4.	.122	39300	54000
321-3	72.	10.	17.94	1.	6240	4.	.105	31600	54000
Placas, Alexander and Regan, Paul E. (1971)									
R8	33.6	6.	10.	.75	3870	.876	.21	17900	39150
R9	33.6	6.	10.	.75	4290	.876	.41	23500	40600
R10	33.6	6.	10.	.75	4295	.585	.21	16900	39150
R11	33.6	6.	10.	.75	3800	1.17	.21	20100	39150
R12	36.0	6.	10.	.75	4920	2.5	.21	24600	39150
R13	36.0	6.	10.	.75	4680	2.5	.41	33600	40600
R14	33.6	6.	10.	.75	4210	.876	.14	20100	39150
R15	36.0	6.	10.	.75	4330	2.5	.41	31400	40600
R16	36.0	6.	10.	.75	4580	2.5	.41	31400	40600
R17	33.6	6.	10.	.75	1850	.876	.21	15700	39150
R18	33.6	6.	10.	.75	4540	.876	.21	19000	39150
R19	33.6	6.	10.	.75	4390	.876	.41	26900	40600
R20	33.6	6.	10.	.75	6230	.876	.21	20200	39150
R21	36.0	6.	10.	.75	6980	2.5	.41	33600	40600
R22	45.0	6.	10.	.75	4280	.876	.21	17900	39150
R24	50.5	6.	10.	.75	4480	2.5	.21	20700	39150
R25	36.0	6.	10.	.75	4470	2.5	.21	23500	39150
R27	36.0	6.	10.	.75	1980	2.5	.41	21300	40600
R28	36.0	6.	10.	.75	4580	2.5	.83	40300	39150
Mattock, Alan H. and Wang, Zuhua (1984) (*)									
C305-D0	37.2	5.91	12.4	.8	4710	1.91	.233	24260	51400
C310-D0	37.2	5.91	12.4	.8	4395	1.91	.467	29770	51400
C310-D20	37.2	5.91	12.4	.8	4440	1.91	.467	31420	51400
Bresler, Boris and Scordelis, A. C. (1963)									
A-1	72.	12.1	18.35	.75	3490	4.	.1	52500	47200
A-2	90.	12.	18.27	.75	3520	5.	.1	55000	47200
A-3	126.	12.1	18.35	.75	5080	6.	.1	52500	47200
B-1	72.	9.1	18.15	.75	3590	4.	.147	50000	47200
B-2	90.	9.	18.33	.75	3360	4.	.148	45000	47200
B-3	126.	9.	18.13	.75	5620	5.	.148	40000	47200
C-1	72.	6.1	18.25	.75	4290	2.	.199	35000	47200
C-2	90.	6.	18.28	.75	3450	4.	.202	36500	47200
C-3	126.	6.1	18.06	.75	5080	4.	.199	30500	47200
Clark, Authur P. (1951)									
A1-1	36.	8.	15.36		3575	3.81	.38	50016	48020
A1-2	36.	8.	15.36		3430	3.81	.38	47016	48020
A1-3	36.	8.	15.36		3395	3.81	.38	50016	48020
A1-4	36.	8.	15.36		3590	3.81	.38	55016	48020
5) With stirrups, 4-point-bent									
Johnston, Bruce and Cox, Kenneth C. (1939)									
B-1-I	36.	12.	12.	.75	3190	1.12	.104	22600	45000
B-2-II	36.	12.	12.	.75	3190	1.18	.104	22575	45000
B-3-I	36.	12.	12.	.75	3190	1.19	.104	26400	45000
B-3-II	36.	12.	12.	.75	3190	1.19	.104	26425	45000
I-1-I	36.	12.	12.	.75	3190	1.18	.104	23000	45000
I-1-II	36.	12.	12.	.75	3190	1.18	.104	22750	45000
T-1-I	36.	12.	12.	.75	3190	1.25	.104	27075	45000
T-2-II	36.	12.	12.	.75	3190	1.17	.104	22850	45000
T-3-I	36.	12.	12.	.75	3190	1.13	.104	21490	45000
T-3-II	36.	12.	12.	.75	3190	1.13	.104	22750	45000
Mattock, Alan H. and Wang, Zuhua (1984) (*)									
C205-D10	24.8	5.91	12.4	.8	4210	1.91	.233	30870	51400
C205-D20	24.8	5.91	12.4	.8	4410	1.52	.233	27560	51400
C210-D0A	24.8	5.91	12.4	.8	4950	1.91	.467	39690	51400
C210-80	24.8	5.91	12.4	.8	4170	1.91	.467	44090	51400
Elzanaty, Ashraf H., Nilson, Arthur H., and Slate, F.O. (1986)									
G4	42.	7.	10.5	.5	9100	2.40	.19	33500	55000
G5	42.	7.	10.5	.5	5800	1.80	.19	25000	55000
G6	42.	7.	10.5	.5	3000	1.80	.19	17500	55000
Leonhardt, Fritz und Walther, Rene' (1962) (*)									
E2	29.53	7.48	10.63	1.18	5405	1.96	.54	41006	37123
E3	29.53	7.48	10.63	1.18	5007	1.96	.54	41006	38830
Rajagopalan, K. S. and Ferguson, Phil M. (1968)									
S-8	44.	5.93	10.45	.5	3920	1.08	.211	13580	50000
S-10	44.	5.96	10.57	.5	4920	1.08	.211	13500	50000
S-11	44.	6.	10.40	.5	4340	1.08	.228	15000	45000
Moretto, Oreste (1945)									
1-V 1/4	32.	5.5	18.25	.75	3460	4.	.28	58300	55000
2-V 1/4	32.	5.5	18.25	.75	4805	4.	.28	67550	55000
1-I 1/4	32.	5.5	18.25	.75	3795	4.	.338	60475	55000
2-I 1/4	32.	5.5	18.25	.75	4700	4.	.338	66000	55000
1-D 1/4	32.	5.5	18.25	.75	3475	4.	.28	59275	55000
2-D 1/4	32.	5.5	18.25	.75	3455	4.	.28	70275	55000
1-V 3/8	32.	5.5	18.25	.75	3115	4.	.615	73425	47900
2-V 3/8	32.	5.5	18.25	.75	4120	4.	.615	71825	47900
1-I 3/8	32.	5.5	18.25	.75	3575	4.	.743	79500	47900

Table 1 (cont.)—Test data from the literature

Beam No.	a (in)	b (in)	d (in)	d_a (in)	f_c (psi)	A_s (in²)	ρ_v (%)	V_u (lb)	f_{yv} (psi)
5) With stirrups, 4-point-bent									
2-I 3/8	32.	5.5	18.25	.75	4185	4.	.743	84000	47900
1-D 3/8	32.	5.5	18.25	.75	2855	4.	.615	64875	47900
2-D 3/8	32.	5.5	18.25	.75	3685	4.	.615	71825	47900
1a-V 1/4	32.	5.5	19.5	.75	3540	2.	.28	53050	46000
1a-V 3/8	32.	5.5	19.5	.75	3335	2.	.615	58675	52000
Clark, Authur P. (1951)									
B1-1	30.	8.	15.36		3388	3.81	.37	62685	48020
B1-2	30.	8.	15.36		3680	3.81	.37	57685	48020
B1-3	30.	8.	15.36		3435	3.81	.37	64030	48020
B1-4	30.	8.	15.36		3380	3.81	.37	60280	48020
B1-5	30.	8.	15.36		3570	3.81	.37	54280	48020
B2-1	30.	8.	15.36		3370	3.81	.73	67685	48020
B2-2	30.	8.	15.36		3820	3.81	.73	72435	48020
B2-3	30.	8.	15.36		3615	3.81	.73	75280	48020
B6-1	30.	8.	15.36		6110	3.81	.37	85280	48020
C1-1	24.	8.	15.36		3720	2.54	.34	62435	48020
C1-2	24.	8.	15.36		3820	2.54	.34	69935	48020
C1-3	24.	8.	15.36		3475	2.54	.34	55280	48020
C1-4	24.	8.	15.36		4210	2.54	.34	64280	48020
C2-1	24.	8.	15.36		3430	2.54	.69	65185	48020
C2-2	24.	8.	15.36		3625	2.54	.69	67685	48020
C2-4	24.	8.	15.36		3910	2.54	.69	64780	48020
C3-1	24.	8.	15.36		2040	2.54	.34	50280	48020
C3-2	24.	8.	15.36		2000	2.54	.34	45030	48020
C3-3	24.	8.	15.36		2020	2.54	.34	42280	48020
C4-1	24.	8.	15.36		3550	3.81	.34	69530	48020
C6-2	24.	8.	15.36		6560	3.81	.34	95280	48020
C6-3	24.	8.	15.36		6480	3.81	.34	97780	48020
C6-4	24.	8.	15.36		6900	3.81	.34	96359	48020
D1-1	18.	8.	15.37		3800	2.	.46	67685	48020
D1-3	18.	8.	15.37		3560	2.	.46	57685	48020
D2-1	18.	8.	15.37		3480	2.	.61	65185	48020
D2-2	18.	8.	15.37		3755	2.	.61	70185	48020
D3-1	18.	8.	15.37		4090	3.	.92	88781	48020
D4-1	18.	8.	15.37		3350	2.	1.22	70185	48020
D1-6	24.	6.	12.36		4010	2.54	.46	39281	48020
D1-7	24.	6.	12.36		4060	2.54	.46	40281	48020
D1-8	24.	6.	12.36		4030	2.54	.46	41781	48020
E1-2	25.	6.	12.36		4375	2.54	.73	49859	48020
D2-6	30.	6.	12.36		4280	2.54	.61	37859	48020
D2-7	30.	6.	12.36		4120	2.54	.61	35359	48020
D2-8	30.	6.	12.36		3790	2.54	.61	37859	48020
D4-1	30.	6.	12.36		3970	2.54	.49	37859	48020
D4-2	30.	6.	12.36		3720	2.54	.49	35359	48020
D4-3	30.	6.	12.36		3200	2.54	.49	37109	48020
D5-1	30.	6.	12.36		4020	2.54	.37	32859	48020
D5-2	30.	6.	12.36		4210	2.54	.37	35359	48020
D5-3	30.	6.	12.36		3930	2.54	.37	35359	48020
(*) concrete cubic strength									
6) With stirrups, T-section									
Palakas, Michael N., Darwin, Davis (1980) (3-point Bent)									
A00	61.	7.5	15.54	.75	4740	.765	0.	16244	-
A25	61.	7.5	15.38	.75	4720	.765	.052	19275	61000
A25a	61.	7.5	15.26	.75	4790	.765	.052	20772	61000
A50	61.	7.5	15.42	.75	3810	.765	.104	25954	71400
A50a	61.	7.5	15.49	.75	4060	.765	.104	24660	72500
A75	61.	7.5	15.56	.75	4670	.765	.157	31966	61900
B00	61.	7.5	15.70	.75	4640	.575	0.	16027	-
B25	61.	7.5	15.52	.75	4470	.575	.052	17670	62100
6) With stirrups, T-section									
B50	61.	7.5	15.39	.75	4390	.575	.104	24050	73600
C00	61.	7.5	15.41	.75	4270	1.09	0.	13270	-
C25	61.	7.5	15.33	.75	4100	1.09	.052	18650	62100
C50	61.	7.5	15.47	.75	4300	1.09	.104	30150	73600
C75	61.	7.5	15.57	.75	4260	1.09	.157	31020	65700
Placas, Alexander and Regan, Paul E. (1971) (3-point Bent)									
T1	33.6	6.	10.	.75	4050	.75	.21	24700	39150
T2	33.6	6.	10.	.75	4070	.876	0.	12300	-
T3	33.6	6.	10.	.75	3990	.876	.21	23500	39150
T4	33.6	6.	10.	.75	4710	1.17	.21	24600	39150
T5	33.6	6.	10.	.75	4890	.876	.41	31400	40600
T6	36.0	6.	10.	.75	3470	2.5	.83	46000	39150
T7	34.6	6.	10.4	.75	3970	1.8	.21	24600	39150
T8	36.0	6.	10.	.75	4530	2.5	.21	28000	39150
T9	36.0	6.	10.	.75	2930	2.5	.41	34700	40600
T10	33.6	6.	10.	.75	4090	.876	.14	19500	39150
T11	36.0	6.	10.	.75	5360	2.5	.41	36000	40600
T12	36.0	6.	10.	.75	4450	2.5	.21	32500	39150
T13	33.6	6.	10.	.75	1850	.876	.21	20200	39150
T14	36.0	6.	10.	.75	4840	2.5	.83	49300	39150
T15	72.0	6.	10.	.75	4810	2.5	.21	23500	39150
T16	72.0	6.	10.	.75	4740	2.5	.14	20800	39150
T17	72.0	6.	10.	.75	4790	2.5	.41	30100	40600
T18	33.6	6.	10.	.75	4980	.876	0.	16800	-
T19	54.0	6.	10.	.75	4340	2.5	.21	25500	39150
T20	54.0	6.	10.	.75	4655	2.5	.41	34600	40600
T22	33.6	6.	10.	.75	4980	.876	.21	24600	39150
T25	33.6	6.	10.	.75	7840	.876	.21	25800	39150
T26	36.0	6.	10.	.75	8260	2.5	.41	40300	40600
T27	36.0	6.	10.	.75	1740	2.5	.41	29700	40600
T31	33.6	6.	10.	.75	4495	.876	.21	21300	39150
T32	36.0	6.	10.	.75	4000	2.5	.83	48600	39150
T34	54.0	6.	10.	.75	4920	2.5	.21	25200	39150
T35	54.	6.	10.	.75	4880	2.5	.21	25800	39150
T36	36.	6.	10.	.75	3500	2.5	.41	40300	40600
T37	36.	6.	10.	.75	4615	2.5	.83	47100	39150
T38	36.	6.	10.	.75	4380	2.5	.83	53800	39150
Haddadin, Munther J., Hong, Sheu-Tien and Mattock, Alan H. (1971)									
A1	37.5	7.	15.		4275	4.	0.	26250	-
A2	37.5	7.	15.		4235	4.	.19	43680	52000
A3	37.5	7.	15.		4360	4.	.419	65520	50000
A4	37.5	7.	15.		4145	4.	.786	76965	50000
A5	37.5	7.	15.		3815	4.	1.26	87045	50000
B3	50.625	7.	15.		4015	4.	.419	61005	50000
C1	63.75	7.	15.		3760	4.	0.	19635	-
C2	63.75	7.	15.		4030	4.	.19	38955	52000
C3	63.75	7.	15.		3500	4.	.419	58485	50000
C4	63.75	7.	15.		3730	4.	.786	69825	50000
C5	63.75	7.	15.		4415	4.	1.26	80535	50000
D3	90.	7.	15.		4245	4.	.419	54180	50000
E1	37.5	7.	15.		2020	4.	0.	22365	-
E2	37.5	7.	15.		2200	4.	.19	38010	52000
E3	37.5	7.	15.		1980	4.	.419	42525	50000
E4	37.5	7.	15.		1945	4.	.786	56490	50000
E5	37.5	7.	15.		2475	4.	1.26	69090	50000
F3	37.5	7.	15.		6515	4.	.419	74130	50000
G3	37.5	7.	15.		3800	4.	.419	74760	66200
G4	37.5	7.	15.		3880	4.	.629	86205	66200
G5	37.5	7.	15.		3790	4.	1.05	96180	66200
H1	37.5	7.	15.		4240	2.	.419	63735	50000
H2	37.5	7.	15.		4090	6.	.419	71610	50000

1 in. = 25.4 mm; 1 in.² = 645.2 mm²; 1 lb = 4.448N; 1psi = 0.006895 MPa.
Note: The data of Clark (1951), Haddadin (1971), and T-section data of Rüsch (1972) are not used due to lack of information.

$$v_u = v_s + v_c \quad (6)$$

v_s represents the contribution of the yield forces in stirrups, and v_s represents the additional contribution due to concrete. For the contribution of stirrups, plastic limit analysis readily yields the expression

$$v_s = \frac{A_v f_{yv}}{bs}(\sin\alpha + \cos\alpha) \quad (7)$$

where

A_v = cross-sectional area of the stirrup
s = spacing of the stirrup
b = beam width
f_{yv} = yield strength of stirrups
α = angle between the stirrups and the longitudinal axis of the beam

Although it has been recognized that the presence of stirrups has some beneficial effect on the magnitude of v_c, this effect is neglected in the current code specifications, i.e., v_c is taken to be the same as in a beam with no stirrups. The assumption that the magnitude of v_c is unaffected by the presence of stirrups is more reasonable when the design formula is based on the initial cracking. However, design on the basis of initial cracking appears to be insufficient because the ratio of the ultimate load to the initial cracking load decreases as the beam size increases, thus causing the safety margin against failure to be nonuniform. The ideal approach would be to use design criteria for both the ultimate load and the first cracking load. However, if only one of these two criteria should be used, then the failure load criterion is probably more reasonable from the safety viewpoint.

The reason that the stirrups enhance the strength contribution of concrete is probably due to several ef-

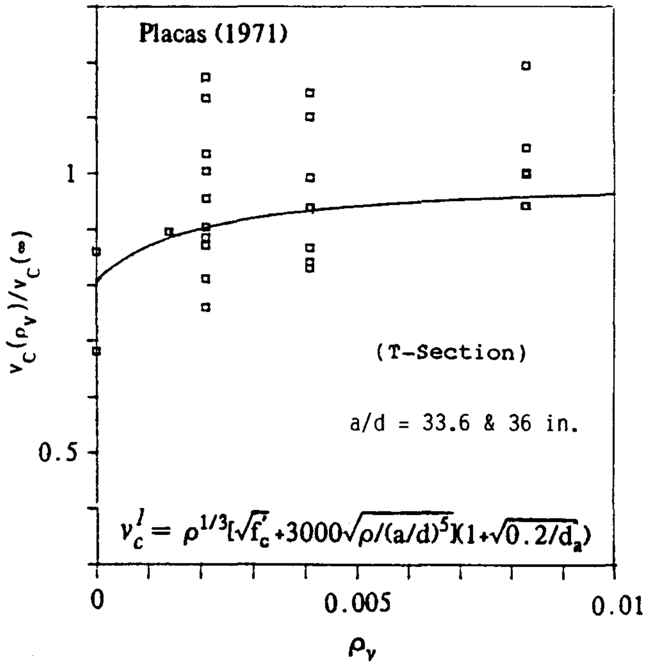

Fig. 4—Effect of stirrup reinforcement ratio on diagonal shear strength of beams without stirrups

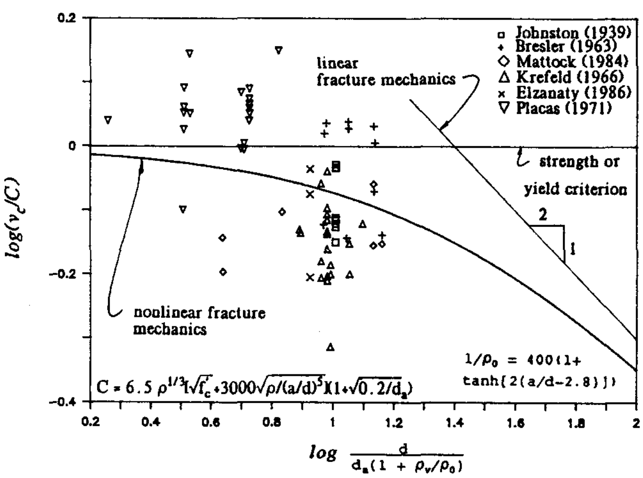

Fig. 5—Size effect in beams with stirrups and effect of stirrup ratio

fects. One of them is that cracks are forced to redistribute at a closer spacing and over a wider area. This results in a smaller crack width and therefore, in an increased shear and tension transfer capability. Another effect enhancing the beam strength arises from the fact that the stirrups support the longitudinal bar and thus prevent concrete splitting along the bar.

If one looks up the derivation of the size-effect law in Reference 42, it transpires that redistribution of cracking over a wider area should manifest itself in an increase of coefficient λ_0 in Eq. (1) or (3). Thus, from the viewpoint of the size effect, the proper way to model the influence of stirrups on the strength contribution of concrete appears to be an increase of coefficient λ_0 as a function of the stirrup-reinforcement ratio ρ_v, as proposed by Palakas and Darwin.[29] For the sake of simplicity, we assume a linear dependence, realizing that the scatter of the test data makes it impossible to calibrate any more complicated dependence. So we may write

$$\lambda_0 = 25\left(1 + \frac{\rho_v}{\rho_0}\right) \tag{8}$$

in which ρ_0 is an empirical coefficient indicating the stirrup reinforcement ratio ρ_v for which λ_0 is doubled compared to λ_0 for $\rho_v = 0$.

The increase of concrete shear capacity described by coefficient ρ can be realized only if the relative shear span is not too small; roughly $a/d \geqslant 1.5$. For smaller shear spans, the stirrups do not appear to develop their full yield capacity, as suggested by Zsutty.[21] Therefore, the value of coefficient ρ_0 cannot be constant, but must change from 0 for $a/d < 1.5$ to a finite value for larger shear spans. The transition should, of course, be expected to be smooth rather than abrupt, and a smooth formula is also preferable for certain numerical procedures or computer optimization of design. A simple empirical formula that gives a smooth transition from a zero value to a finite value is

$$\frac{1}{\rho_0} = a_0\left[1 + \tanh\left(2\frac{a}{d} - 5.6\right)\right] \tag{9}$$

in which a_0 is an empirical constant and coefficients 2 and 5.6 have been approximately determined by optimization of data fits.

The available experimental evidence on the effect of ρ_v on v_c is shown in Fig. 4, in which the ordinate is the ratio of the measured v_c to the value of v_c calculated for $\rho_v \rightarrow \infty$ (or $\lambda_o \rightarrow \infty$). The curve represents the trend according to Eq. (8). Although the scatter is very large, the increasing trend is nevertheless clearly apparent. The effect of ρ_v is also evident from the regression in Fig. 5.

Fig. 5 shows the effect of beam depth d on the contribution of concrete to the shear capacity of a beam with stirrups. The existing test data, plotted in the figure, are those of Johnston and Cox,[1] Bresler and Scordelis,[11] Mattock and Wang,[33] Krefeld and Thurston,[14] Elzanaty et al.,[36] and Placas and Regan.[24] The curve in Fig. 5 shows the present formula. Although the data scatter is very large, the presence of the size effect in beams with stirrups is nevertheless evident.

Fig. 6 shows, for beams with stirrups, a plot of the measured values of v_u versus the values calculated from the formulas. The plot in Fig. 6(c) is based on the presently proposed equations [Eq. (3), (2), (6), (8), (9), and (7)], and for comparison Fig. 6(d) shows the plot obtained when the effect of the stirrups is omitted, i.e., $1/\rho_0 = 0$. We see that by considering the effect of stirrups, the coefficient of variation of the deviations from the straight line is reduced from $\omega = 0.156$ to $\omega = 0.141$ for this set of 87 data points. The improvement due to considering the effect of stirrups on concrete ca-

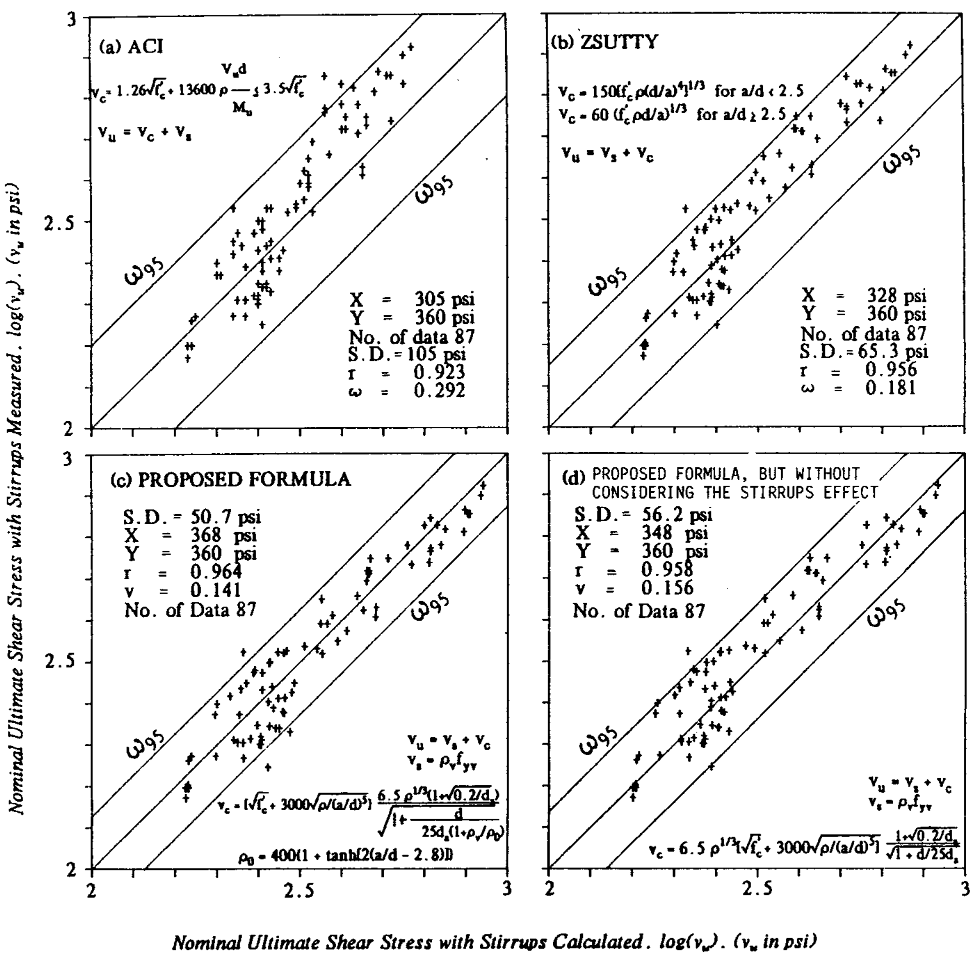

Fig. 6—Plots of measured versus calculated values of mean nominal shear strength for beams with stirrups (1 psi = 6895 Pa)

pacity is small but nevertheless appreciable. For comparison, Fig.6(a) and (b) show the same plots for the current ACI specifications[51] and for Zsutty's formula[18] [also see Eq. (15) and (18) of Reference 36]. We see that the coefficients of variation of the vertical deviations from the straight line are for these two formulas 0.292 and 0.181, which is distinctly larger than the value for the presently proposed formula. Note also the increased separation of the 95 percent confidence limits.

The same set of 87 data points is plotted in Fig. 7 to show the size effect, i.e., the dependence of v_c on the relative size d/d_a. Comparison with the proposed formulas is shown in Fig. 7(c) and (d), and despite large scatter the presence of a downward trend representing a size effect is clearly visible. By contrast, the scatter in Fig. 7(a) for the ACI specification is so large that no size effect is easily detected. This means that it would make no sense to introduce the size effect into the ACI formula without improving the form of the existing formula itself, which was previously shown possible for beams without stirrups.[45]

In all the preceding comparisons with tests, the formulas were made to represent the mean trend of the test data. The design formulas should, however, be introduced in such a manner that most of the test data are on the safe side of the predicted values. This is achieved by multiplying the formula for the mean trend by a factor. According to the experimental evidence, a suitable adjustment is achieved by changing the value of k_1 in Eq. (2) from 6.5 to 4.5. The plot of the measured versus calculated values of the nominal shear strength v_u for beams with stirrups is shown (for the design formula with k_1 = 4.5) in Fig. 8. In these plots, the majority of the test data should lie above the inclined straight line, and we see that this is well satisfied for Fig. 8(c) and (d) for the present formulas. However, for the existing ACI formula there are more data points falling significantly below this line, and also more data points high above the straight line — a case that represents overdesign.

CONCLUSIONS

1. The previously proposed formula of Bažant and Kim[45] for diagonal shear failure of longitudinally reinforced beams without stirrups is improved by introducing, in addition to the effect of the relative beam size, the effect of the maximum aggregate size. The new formula is calibrated according to a larger set of test data than before, consisting of 461 test results that were compiled from the literature.

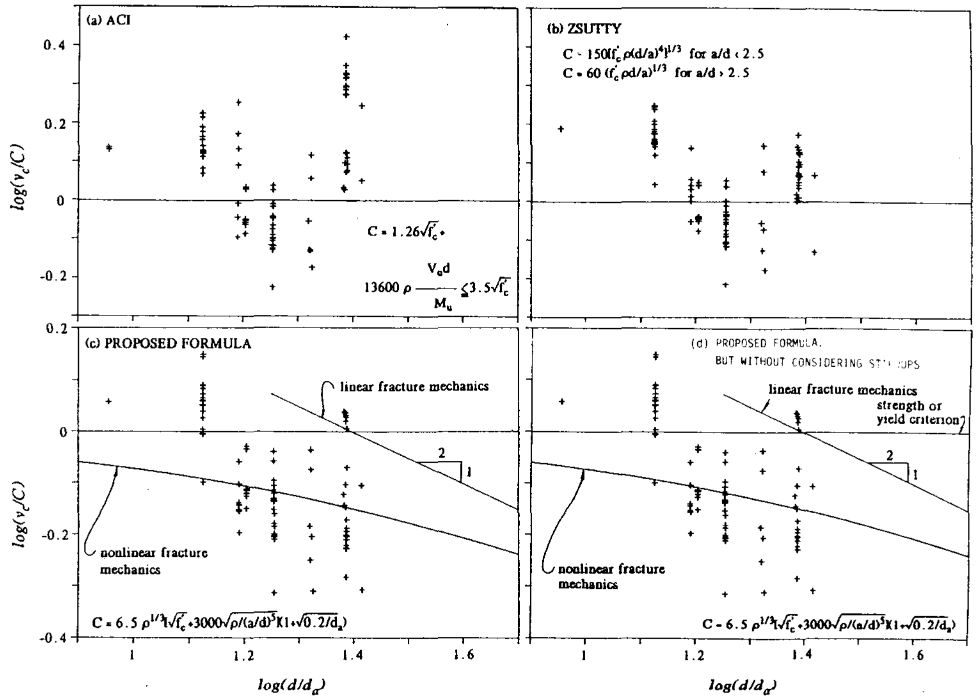

Fig. 7—Size-effect plots for various formulas for beams with stirrups in comparison with test data

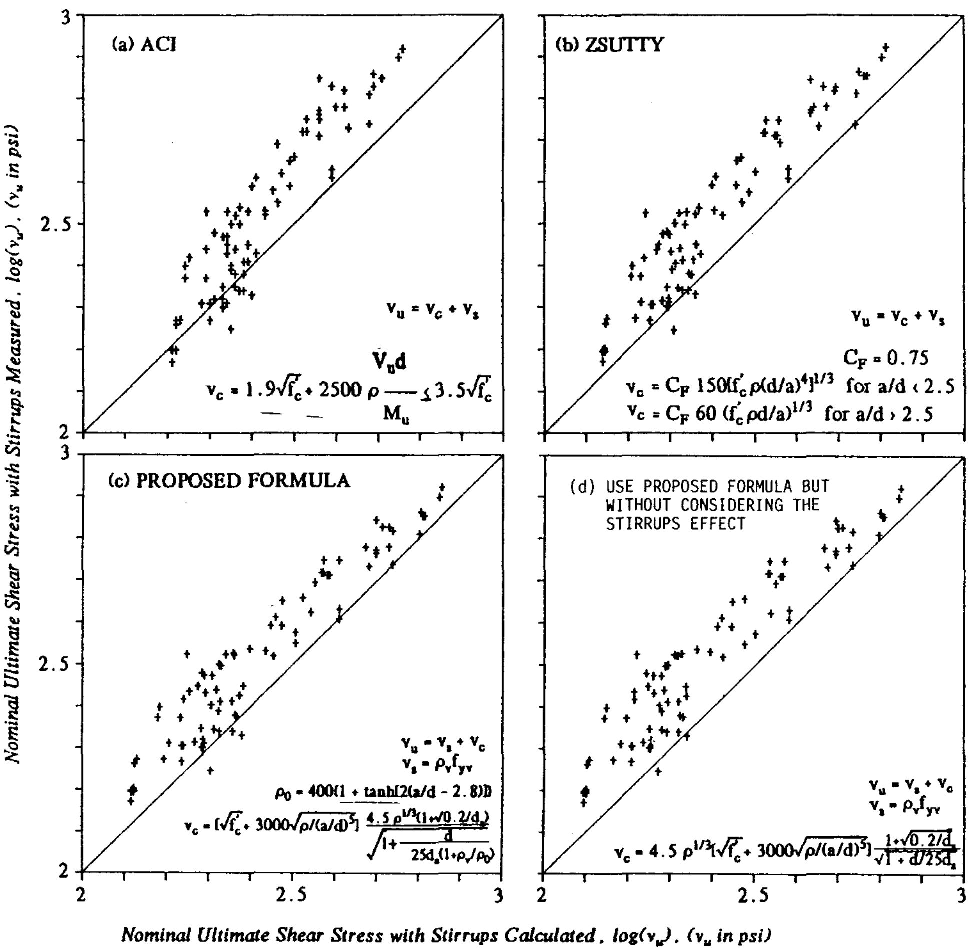

Fig. 8—Plots of measured versus calculated values of nominal shear strength for various design formulas

2. The formula is further extended to diagonal shear failure of reinforced concrete beams with stirrups. The generalization of the formula takes into account the fact that the presence of stirrups has a strengthening effect on the shear capacity of concrete. The degree of strengthening depends, however, on the shear span. The resulting formula is calibrated according to a set of 87 test results compiled from the literature. The results confirm that the size effect on the concrete shear strength still exists in the presence of stirrups, but it is milder than without stirrups.

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REFERENCES

1. Johnston, Bruce, and Cox, Kenneth C., "High Yield-Point Steel as Tension Reinforcement in Beams," ACI JOURNAL, *Proceeding* V. 36, No. 1, Sept. 1939, pp. 65-80.

2. Moretto, Oreste, "An Investigation of the Strength of Welded Stirrups in Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 42, No. 2, Nov. 1945, pp. 141-164.

3. Clark, Arthur P., "Diagonal Tension in Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 48, No. 2, Oct. 1951, pp. 145-156.

4. Gaston, J. R.; Siess, C. P.; and Newmark, N. M., "An Investigation of the Load-Deformation Characteristics of Reinforced Concrete Beams up to the Point of Failure," Civil Engineering Studies, *Structural Research Series* No. 40, University of Illinois, Urbana, Dec. 1952.

5. Moody, K. G.; Viest, I. M.; Elstner, R. C.; and Hognestad, E., "Shear Strength of Reinforced Concrete Beams. Part 1—Tests of Simple Beams," ACI JOURNAL, *Proceedings* V. 51, No. 3, Nov. 1954, pp. 317-332.

6. Diaz de Cossio, Roger, and Siess, Chester P., "Behavior and Strength in Shear of Beams and Frames without Web Reinforcement," ACI JOURNAL, *Proceedings* V. 56, No. 8 Feb. 1960, pp. 695-735.

7. Taylor, R.,"Some Shear Tests on Reinforced Concrete Beams without Shear Reinforcement," *Magazine of Concrete Research* (London), V. 12, No. 36, Nov. 1960, pp. 145-154.

8. Leonhardt, Fritz, and Walther, René, "Beiträge zur Behandlung der Schubprobleme in Stahlbetonbau," *Beton- und Stahlbetonbau* (Berlin), Mar. 1962, pp. 54-64, and June 1962, pp. 141-149.

9. van den Berg, F. J., "Shear Strength of Reinforced Concrete Beams without Web Reinforcement. Part 2—Factors Affecting Load at Diagonal Cracking," ACI JOURNAL, *Proceedings* V. 59, No. 11, Nov. 1962, pp. 1587-1600.

10. Rüsch, Hubert; Haugli, Finn Robert; and Mayer, Horst, "Schubversuche an Stahlbeton-Rechteckbalken mit gleichmässing verteilter Belastung," *Bulletin* No. 145, Deutscher Ausschuss für Stahlbeton, Berlin, 1962, pp. 4-30.

11. Bresler, Boris, and Scordelis, A. C., "Shear Strength of Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 60, No. 1, Jan. 1963, pp. 51-74.

12. Mathey, Robert G., and Watstein, David, "Shear Strength of Beams without Web Reinforcement," ACI JOURNAL, *Proceedings* V. 60, No. 2, Feb. 1963, pp. 183-208.

13. Taylor, R., and Brewer, R. S., "The Effect of the Type of Aggregate on the Diagonal Cracking of Reinforced Concrete Beams," *Magazine of Concrete Research* (London), V. 15, No. 44, July 1963, pp. 87-92.

14. Krefeld, William J., and Thurston, Charles W., "Studies of the Shear and Diagonal Tension Strength of Simply Supported Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 63, No. 4, Apr. 1966, pp. 451-476.

15. Kani, G. N. J., "Basic Facts Concerning Shear Failure," ACI JOURNAL, *Proceedings* V. 63, No. 6, June 1966, pp. 675-692.

16. Kani, G. N. J., "How Safe Are Our Large Reinforced Concrete Beams?," ACI JOURNAL, *Proceedings* V. 64, No. 3, Mar. 1967, pp. 128-141.

17. Rajagopalan, K. S., and Ferguson, Phil M., "Exploratory Shear Tests Emphasizing Percentage of Longitudinal Steel," ACI JOURNAL, *Proceedings* V. 65, No. 8, Aug. 1968, pp. 634-638.

18. Zsutty, Theodore C., "Beam Shear Strength Prediction by Analysis of Existing Data," ACI JOURNAL, *Proceedings* V. 65, No. 11, Nov. 1968, pp. 943-951.

19. Bhal, N. S., "Uber den Einfluss der Balkenhöhe auf Schubtragfähigkeit von einfledrigen Stahlbetonbalken mit und ohne Schubberwehrung," dissertation, Universität Stuttgart, 1968, 124 pp.

20. Mattock, Alan H., "Diagonal Tension Cracking in Concrete Beams with Axial Forces," *Proceedings*, ASCE, V. 95, ST9, Sept. 1969, pp. 1887-1900.

21. Zsutty, Theodore C., "Shear Strength Prediction for Separate Categories of Simple Beam Tests," ACI JOURNAL, *Proceedings* V. 68, No. 2, Feb. 1971, pp. 138-143.

22. Swamy, R. Narayan, and Qureshi, Schamsuddin A., "Strength, Cracking, and Deformation Similitude in Reinforced T-Beams Under Bending and Shear," ACI JOURNAL, *Proceedings* V. 68, No. 3, Mar. 1971, pp. 187-195.

23. Haddadin, Munther J.; Hong, Sheu-Tien; and Mattock, Alan H., "Shear Effectiveness in Reinforced Concrete Beams with Axial Force," *Proceedings*, ASCE, V. 97, ST9, Sept. 1971, pp. 2277-2297.

24. Placas, Alexander, and Regan, Paul E., "Shear Failure of Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 68, No. 10, Oct. 1971, pp. 763-773.

25. Taylor, Howard P. J., "Shear Strength of Large Beams," *Proceedings*, ASCE, V. 98, STll, Nov. 1972, pp. 2473-2490.

26. Santiago, Samuel Diaz, and Hilsdorf, Hubert K., "Fracture Mechanisms of Concrete under Compressive Loads," *Cement and Concrete Research*, V. 3, No. 4, July 1973, pp. 363-388.

27. Park, Robert, and Paulay, Thomas, *Reinforced Concrete Structures*, John Wiley & Sons, New York, 1975, 769 pp.

28. Walraven, J. C., "The Influence of Depth on the Shear Strength of Lightweight Concrete Beams Without Shear Reinforcement," *Stevin Laboratory Report* No. 5-78-4, Delft University of Technology, 1978, 36 pp.

29. Palakas, Michael N., and Darwin, David, "Shear Strength of Lightly Reinforced Concrete Beams," *Structural Engineering and Engineering Materials Report* No. 3, University of Kansas Center for Research, Lawrence, Sept. 1980, 198 pp.

30. Chana, P. S., "Some Aspects of Modelling the Behaviour of Reinforced Concrete under Shear Loading," *Technical Report* No. 543, Cement and Concrete Association, Wexham Springs, July 1981, 22 pp.

31. Swamy, R. N., and Al-Ta 'an, Sa'ad A., "Deformation and Ultimate Strength in Flexure of Reinforced Concrete Beams Made with Steel Fiber Concrete," ACI JOURNAL, *Proceedings* V. 78, No. 5, Sept.-Oct. 1981, pp. 395-405.

32. Kotsovos, Michael D., "Mechanisms of 'Shear' Failure," *Magazine of Concrete Research* (London), V. 35, No. 123, June 1983, pp. 99-106.

33. Mattock, Alan H., and Wang, Zuhua, "Shear Strength of Reinforced Concrete Members Subject to High Axial Compressive Stress," ACI JOURNAL, *Proceedings* V. 81, No. 3, May-June 1984, pp. 287-298.

34. Mphonde, Andrew G., and Frantz, Gergory C., "Shear Tests of High-and Low-Strength Concrete Beams Without Stirrups," ACI JOURNAL, *Proceedings* V. 81, No. 4, July-Aug. 1984, pp. 350-357.

35. Swamy, R. N., and Bahia, H. M., "The Effectiveness of Steel Fibers as Shear Reinforcment," *Concrete International: Design & Construction*, V. 7, No. 3, Mar. 1985 pp. 35-40.

36. Elzanaty, Ashraf H.; Nilson, Arthur H.; and Slate, Floyd O., "Shear Capacity of Reinforced Concrete Beams Using High-Strength Concrete," ACI JOURNAL, *Proceedings* V. 83, No. 2, Mar.-Apr. 1986, pp. 290-296.

37. Ahmad, Shuaib H.; Khaloo, A. R.; and Poveda, R., "Shear Capacity of Reinforced High-Strength Concrete Beams," ACI JOURNAL, *Proceedings* V. 83, No. 2, Mar.-Apr. 1986, pp. 297-305.

38. Reinhardt, H. W., "Similitude of Brittle Fracture of Structural Concrete," *Advanced Mechanics of Reinforced Concrete*, IABSE Colloquim, Delft, 1981, pp. 201-210.

39. Reinhardt, H. W., "Scale Effect in Shear Tests in the Light of Fracture Mechanics," *Beton- und Stahlbetonbau* (Berlin), No. 1, 1981, pp. 19-21. (in German)

40. Bažant, Zdeněk P., and Oh, B. H., "Crack Band Theory for Fracture of Concrete," *Materials and Structures, Research and Testing* (RILEM, Paris), V. 16, No. 93, May-June 1983, pp. 155-177.

41. Bažant, Z. P., "Mechanics of Distributed Cracking," *Applied Mechanics Reviews*, V. 39, No. 5, May 1986, pp.675-705.

42. Bažant, Zdeněk P., "Size Effect in Blunt Fracture: Concrete, Rock, Metal," *Journal of Structural Engineering*, ASCE, V. 110, No. 4, Apr. 1984, pp. 518-535.

43. Bažant, Zdeněk P., "Fracture Mechanics and Strain-Softening of Concrete,"*Proceedings*, U.S.-Japan Seminar on Finite Element Analysis of Reinforced Concrete Structures (Tokyo, 1985), American Society of Civil Engineers, New York, 1986, pp. 121-150.

44. Bažant, Z. P., "Fracture Energy of Heterogeneous Materials and Similitude," *Proceedings*, RILEM-SEM International Conference on Fracture of Concrete and Rock (Houston, June 1987), Society for Experimental Mechanics, Bethel.

45. Bažant, Zdeněk P., and Kim, Jin-Keun, "Size Effect in Shear Failure of Longitudinally Reinforced Beams," ACI JOURNAL, *Proceedings* V. 81, No. 5, Sept.-Oct. 1984, pp. 456-468. Also, Discussion, *Proceedings* V. 82, No. 4, July-Aug. 1985, pp. 579-583.

46. Bažant, Zdeněk P., and Cao, Zhiping, "Size Effect of Shear Failure in Prestressed Concrete Beams," ACI JOURNAL, *Proceedings* V. 83, No. 2, Mar.-Apr. 1986, pp. 260-268.

47. Bažant, Zdeněk P., and Cao, Zhiping, "Size Effect in Punching Shear Failure of Slabs," *ACI Structural Journal*, V. 84, No. 1, Jan.-Feb. 1987, pp.44-53.

48. Bažant, Zdeněk P., and Cao, Zhiping, "Size Effect in Brittle Failure of Unreinforced Pipes," ACI JOURNAL, *Proceedings* V. 83, No. 3, May-June 1986, pp. 369-373.

49. Bažant, Z. P., and Sener, S., "Size Effect in Torsional Failure of Longitudinally Reinforced Concrete Beams," *Report* No. 86-9/428s, Center for Concrete and Geomaterials, Northwestern University, Evanston, Sept. 1986. Also *Journal of Structural Engineering*, ASCE, in press.

50. Bažant, Z. P.; Sener, S.; and Prat, P. C., "Size Effect Tests of Torsional Failure of Beams," *Report* No. 86-12/428s, Center for Concrete and Geomaterials, Northwestern University, Evanston, Dec. 1986, 18 pp.

51. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-83)," American Concrete Institute, Detroit, 1983, 111 pp.

52. *CEB-FIP Model Code for Concrete Strustures*, 3rd Edition, Comité Euro-International du Béton/Fédération Internatinale de la Precóntrainte, Paris (English Edition, Cement and Concrete Association, Wexham Springs), 1978, 348 pp.

53. Bažant, Zdeněk P., and Kim, Jin-Keun, Authors' Closure to Discussion of "Size Effect in Shear Failure of Longitudinally Reinforced Beams," ACI JOURNAL, *Proceedings* V. 81, No. 4, July-Aug. 1985, pp. 580-583.