Constitutive equation of wood at variable humidity and temperature

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Summary. Theoretical analysis of the effects of variations in moisture content and temperature on the creep of wood is presented. Thermodynamics of the processes of diffusion of water in wood microstructure is discussed and distinction is drawn between macrodiffusion and microdiffusion. The constitutive relation for steady states of moisture content and temperature is formulated on the basis of Maxwell chain model whose viscosity coefficients depend on moisture content and temperature. It is shown that the apparent acceleration of creep due to simultaneous drying (or wetting) as well as heating (or cooling) may be modeled as additional, stress-induced shrinkage (or swelling) and stress-induced thermal expansion (or contraction), described by shrinkage and thermal expansion coefficients that depend on the absolute values of the rates of pore humidity and temperature. Certain other sources of irreversibility of creep are also discussed.

Introduction

At variable humidity, the deformations of stressed wood are extremely complex and have, so far, eluded realistic mathematical description. The intricacy of various phenomena exhibited by wood is most comprehensively, yet succinctly, described by Grossman (1976). An excellent overview of the state-of-the-art is available in recent NSF workshop proceedings (1978). Various significant advances to describe diverse aspects of the behavior have been made, e.g., by Grossman (1971, 1976, 1978), Armstrong (1972), Leicester (1971), Schaffer (1972), Cave (1978), Zahn (1978), Schniewind et al. (1972, 1968, 1967), Ugolev (1976), Sadoh (1981), Dinwoodie et al. (1981), Johnson (1978), Ranta-Maunus (1973), Arima and Grossman (1978), Hunt (1982), Armstrong and Kingston (1962), Christensen (1962), Armstrong and Christensen (1962), and others.

It is remarkable that the deformations of stressed wood at variable humidity exhibit characteristics very similar to certain other materials of a completely different chemistry and microstructure, especially portland cement concretes (Bažant, 1975, 1972, 1970, 1975, 1982). If we accept that this resemblance cannot be by

chance, there must exist certain essential characteristics common to these materials:

1) These materials are porous with a broad range of pore sizes, including large capillary pores, as well as subcapillary pores of molecular dimensions.
2) The material forming the pore walls is strongly hydrophylic, with strong adsorption properties.

The present study, based on a 1982 report (Bažant 1982) attempts setting up a speculative model which is based on these two essential characteristics and seems capable of describing, at least in the qualitative sense, all the essential phenomena displayed by stressed and unstressed wood at variable humidity.

Physical mechanism of water diffusion and creep

Rapid drying as well as wetting causes an acceleration of creep of wood. When this acceleration was observed, it was at first proposed that the viscosity coefficient in the constitutive equation should be considered to depend on the magnitude of the moisture flux which, in turn, depends on the magnitude of the gradient of moisture concentration, or of pore pressure. This hypothesis, however, was soon disproved by observing experimentally that a steady-state diffusion of water through a stressed specimen of wood does not seem to appreciably accelerate creep (e.g., Armstrong 1972; Grossman 1976; Johnson 1978). If, however, we look more closely at the microstructure of the pore system, the idea that the moisture flux affects the creep rate seems feasible in a modified form.

Wood consists of very elongated, thread-like cells (Fig. 1 a), typically 2 mm in length and 0.02 mm in width. Each cell has a hollow core, or lumen, and a fibrous wall made up of microfibrils (Dinwoodie 1975; Kollmann, Côté 1968; Siau 1971;
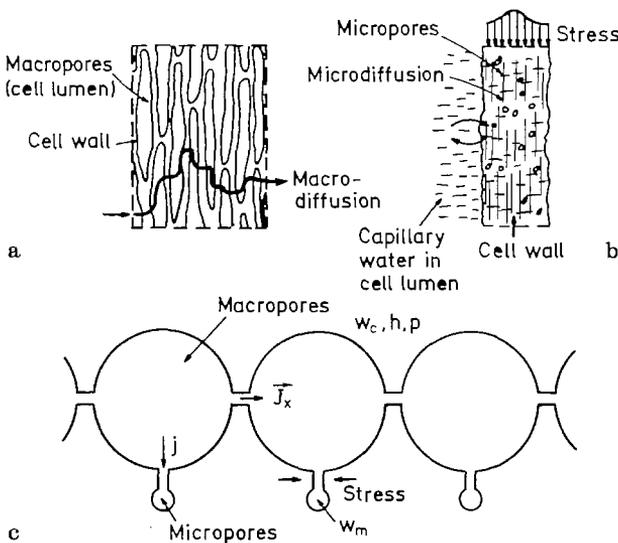
Fig. 1. a, b Macro- and micropores; **c** idealized pore system

Mark 1965). The cell lumens represent relatively large, capillary pores (of typical diameter 0.01 mm), in which the water is in a liquid or vapor state and does not participate in the transmission of permanent applied load. The fibrous cell walls (Fig. 1 b) contain numerous microscopic pores (micropores) which are of sub-capillary dimensions (gell-type pores). Water molecules in these micropores are strongly held (adsorbed) to the pore walls, and if the gap between the opposite walls is only a few molecules in width, the gap may be completely filled by adsorbed water molecules. Water molecules in these micropores are no doubt capable of transmitting significant permanent loads, as is known from the theory of disjoining pressure in hindered adsorbed water layers (cf. Bažant 1972). Thus when the molecules in the micropores are displaced in the process of drying or wetting, it must affect the deformation due to applied load. By contrast, no such effect should occur in the macropores, since water in them does not resist the applied load.

The sizes of the micropores are not the same. Some are larger and some are smaller (Banks 1981). The macroscopic transport of water tends to pass principally through the passages of least resistance, which is through the widest and shortest pores. Assuming viscous flow, the flux of water (either liquid or vapor) would then vary directly as the cube of the pore width (and so, e.g., if some micropores are 5-times wider than others, 125-times more water flows through them). The water passages include the capillary pores (cell lumen) and, since these pores are not continuous, also the largest and shortest micropore passages through the cell wall, many of which may be provided by the pit structure. Thus, it appears that in the process of macroscopic water transport through wood, most of the migrating water molecules should pass only through the macropores and only through a small portion of the micropores whose volume is probably negligible. Most of the water in the micropores would not participate in the macroscopic water transport. The micropores get filled or emptied in a local diffusion process which transports water between the micropores and the adjacent macropores locally while having a zero resultant flux vector macroscopically.

In wood microfibrils, there exist so-called amorphous regions where weak cross links (H-bonds) are known to break and reform due to thermal motion, at a rate depending on temperature according to the activation energy theory. Creep is viewed as an accumulation of dislocations due to the breakages of cross links. Unsatisfied cross link sites are hydrophylic, i.e. attract water molecules, and are responsible for the hygroscopic deformations (shrinkage and swelling). Thus, the migration of water molecules through the micropores may be expected to increase the rate of cross-link breakages, thereby accelerating creep. This happens to a significant degree only in the microscopic (local) diffusion. At steady state, the macroscopic diffusion may be expected to bypass most of the micropores (Fig. 1), passing only through those few micropores which lie on the passages of least resis-tance. Accordingly, a steady-state transport of water through wood, in which the cell walls do not change their water content but merely permit water molecules to pass through, should not cause any significant acceleration of creep, as observed in experiments.

Let us now try to make a rough quantitative estimate of the effect of pore width, w. First we should recognize that the larger the pore, the smaller the stress s_w that is transmitted across a pore of width w by the cross links and the adsorbed

water molecules, since the deformation of both the smaller and the larger pores must conform to the same macroscopic deformation. Similar to the probabilistic analysis in Çinlar et al. (1977), imagine for simplicity flat pores between parallel planar walls subdivided into layers (e.g., monomolecular adsorption layers) with regularly distributed sites which can be either empty or occupied by a bonded solid particle. Let the area fraction of solids within each layer be a (a ≪ 1), which represents the probability of a solid particle being at a given site. Stress can be transmitted from one layer to the next only at those sites where there is a solid particle in both adjacent layers, and the probability of this to happen is a^2 (joint probability). For three layers, the probability is a^3, etc. Generally, for a pore of width w, the probability that stress can be transmitted across the pore at a given site of the pore wall is $p_1 = a^{w/h_1} = e^{-k_1 w}$, where h_1 = thickness of one layer and $k_1 = -(\ln a)/h_1$ = positive constant. At the same time, the stiffness of the stress-transmitting cross-link bridge is proportional to w. Thus, $s_w = c_0 s\, e^{-k_1 w}/w$ in which c_0 = constant, and s = mean stress in the porous material of the wood cell wall.

It may now be imagined that the rate of cross-link breakages that cause creep is proportional to the flux of water through the cell wall pores. Assuming this flow to be like a viscous flow, the flux J_1 of water through a given pore should then be $c_1 w^3$ where c_1 = constant. At the same time, the contribution to the creep rate from one pore should be proportional to s_w. Thus, the contribution to the creep rate per unit value of the flux of water through all pores is $C_1 = c_0 s\, e^{-k_1 w} c_1 w^3/w$, i.e.,

$$C_1 = k_0 s\, w^2 e^{-k_1 w} \tag{1}$$

which is a function that decays rapidly with increasing w if k_1 is small ($k_0 = c_0/c_1$).

For illustration, assume that in the thinnest pores ($w = h_1$) the area fraction that transmits stress is $a = 0.01$, in which case $k_1 = -(\ln 0.01)/h_1 = 4.6/h_1$. Consider three pores of widths $w = 2h_1$, $4h_1$, and $6h_1$. Their contributions to creep are, according to Eq. (1), in the ratio $11 \times 10^6 : 4.400 : 1$. Even though this estimation is crude, we see that, if some cell wall pores are at least 3-times wider than others, the flow of water through them can have virtually no effect on creep. We of course exclude from consideration the pores which may be so narrow that no water can pass through.

As a result of these considerations, we may introduce the following three basic hypotheses:

1. The pores may be subdivided into macropores and micropores (Fig. 1 c).

2. A steady-state macroscopic diffusion of water through wood leaves most of the micropores undisturbed (Fig. 1 c) and has no direct effect on creep.

3. Microscopic diffusion of water through the micropores accelerates creep, regardless of whether the diffusion is out of the micropores or into them.

As we will mathematically confirm, the microdiffusion takes place only if the water content of the large, capillary pores varies. In the case of a steady-state moisture flux through a test specimen, the moisture content of the capillary pores remains constant with time at any point of the specimen. Therefore, the steady-state moisture flux does not produce any microdiffusion, and should not, therefore, have any effect on deformation, as observed in experiments.

There is a question whether Hypothesis 2 could not be abandoned, considering a modified model of diffusion in the micropores. One can, e.g., abandon Hypo-

thesis 2 and assume that the steady-state flow of water through a micropore might have no effect because, as one water molecule leaves a bonding site (cross link site) another one is substituted for it, while the total number of water molecules in the pores remains the same. The creep acceleration would then have to be attributed to a change in the number of the water molecules in the pore. However, it would then be difficult to explain that both a decrease and an increase in the number of water molecules in the pores would accelerate creep.

Thermodynamic formulation of microdiffusion

To formulate the idea mathematically, we need to introduce the driving force of the microdiffusion in the cell walls. Based on the experience with modeling concrete at variable humidity and temperature, it seems that the diffusion is best described by considering it as pressure-driven. We introduce, therefore, the relative humidity (relative vapor pressure) in the macropores (capillary pores, or cell lumen), h, as the basic variable. (The value of h is permitted to exceed 1.0, in which case h represents the ratio of hydraulic pressure of water to the saturation vapor pressure.) As for the micropores, the state of water in them cannot be characterized by relative humidity since these pores do not contain vapor. Rather, it is more appropriate to introduce as the basic variable the spreading pressure π_m in the layer of water in the micropores. The chemical potential (Gibbs' free energy per unit mass) of the micropore water, μ_m, is then a function of π_m and temperature T. At the same time, there must exist for the micropores a certain sorption relation giving π_m as a function of the water content w_m of the micropores per unit volume of the material, and of T. Thus, we may consider μ_m to be a function of w_m and T. For the macropores, the chemical potential of water vapor, μ_v, is a function of h and T, and so we may consider that

$$h = f_1 (\mu_v, T) \tag{2}$$

where f_1 is a certain function. At thermodynamic equilibrium between the micropores and the macropores, the chemical potentials must be equal, $\mu_v = \mu_m$. It is convenient to characterize the state of water in the micropores by relative humidity χ for which the water in the macropores and the micropores is in equilibrium based on Eq. (2) in which μ_v is replaced by μ_m, i.e., $\chi = f_1 (\mu_m, T)$. Substituting for μ_m, one may thus in theory determine the function (Fig. 2 b):

$$\chi = f (w_m, T) . \tag{3}$$

The condition of thermodynamic equilibrium ($\mu_v = \mu_m$) may now be simply written as $h = \chi$. Consequently, the flux of water (microflow) from capillary pores into adjacent micropores (at equal temperatures) may be expressed as

$$j = b_1 (h - \chi) . \tag{4}$$

Flux j is a scalar since the microscopic fluxes involve all possible directions, and b_1 is a certain permeability coefficient which characterizes the permeability of the cell wall material. Coefficient b_1 may be expected to depend on w_m as well as T.

Conservation of mass requires that $j = \dot{w}_m$. Noting that the micropore water content w_m must be related to χ and T by a certain sorption relation, $w_m = w_m(\chi, T)$, and further that

$$\dot{w}_m = \frac{\partial w_m}{\partial \chi} \dot{\chi} + \frac{\partial w_m}{\partial T} \dot{T} = b_1(h - \chi)$$

we obtain the following equation governing the microdiffusion process:

$$\dot{\chi} = b_m(h - \chi) - c_m \dot{T} \tag{5}$$

in which

$$b_m = \frac{b_1}{(\partial w_m/\partial \chi)_T}, \qquad c_m = \frac{(\partial w_m/\partial T)_\chi}{(\partial w_m/\partial \chi)_T}. \tag{6}$$

Consider now the solution of this equation. It is no doubt physically reasonable to assume h to be a smooth function of time. Then, in a sufficiently short time interval Δt (Fig. 1 a), h may be assumed to vary linearly with time, i.e.,

$$h \simeq h_1 + \dot{h} t \tag{7}$$

where h_1 and $\dot{h}$ are constants and $t = $ time. Coefficients b_m and c_m may also be considered constant within a sufficiently short time interval. Then, adding the general solution of the homogeneous part of Eq. (5) to a particular solution of this equation, we obtain the following general solution of Eq. (5) for the short time interval

$$\chi \simeq \chi_1 + \dot{\chi}_1 t + C_1 e^{-b_m t} \tag{8}$$

in which χ_1, $\dot{\chi}_1$ and C_1 are certain constants, and superimposed dots denote time rates.

Noticing the exponentially decaying term in Eq. (8), we see that this term must be negligible for time durations $\gg 1/b_m$. The distance that any water molecule has to migrate in the microdiffusion process is obviously less than one-half of the thickness of the cell wall, and since this thickness is very small, the microscopic diffusion is likely to be relatively very fast. So we will assume the exponential term in Eq. (8) to be negligible for all practical purposes. Then, substituting Eq. (8) (with $C_1 = 0$) into Eq. (5), we obtain

$$\dot{\chi} = \dot{h}, \qquad \chi - h \simeq -\frac{1}{b} \dot{h} - \frac{c_m}{b_m} \dot{T} \tag{9}$$

and so we have

$$j \simeq \frac{b_1}{b_m} \dot{h} + \frac{b_1 c_m}{b_m} \dot{T} \tag{10}$$

where $b_1/b_m = (\partial w_m/\partial \chi)_T$. This leads to the following important conclusion:

If the water transport between the micropores and macropores, i.e., between the cell wall pores and the cell lumen, is rapid compared to the macroscopic water transport through the specimen, then the flux into (or out of) the micropores (cell wall pores) is proportional to $\dot{h}$ and $\dot{T}$.

Equation (10) also indicates one possible test of this conclusions. At a certain magnitude of temperature rate $\dot{T}$, the effect of $\dot{T}$ on the creep rate must be the same as the effect of a certain humidity rate $\dot{h}$. For concrete this test is affirmative; indeed, a rapid temperature change of either sign is known to accelerate creep. This phenomenon is called transitional thermal creep. For wood, this phenomenon does not appear to have been examined yet.

Macroscopic diffusion of water

We must now distinguish the water content, w_c, of the capillary pores (kg of water per m^3 of wood) from the water content, w_m, of the micropores, and from the total water content $w = w_c + w_m$ of the porous material. The condition of conservation of the mass of water requires that

$$\dot{w}_c = - \text{div } \mathbf{J}_c - j \tag{11}$$

in which $\dot{w}_c = \partial w_c / \partial t$, and $\mathbf{J}_c$ is the vector of the mass flux of capillary water (kg/s m^2). It is probably most reasonable to assume that the flow of capillary water is pressure driven, i.e., of Darcy type, and then we have

$$\mathbf{J}_c = - \frac{a}{g} \text{ grad } p \tag{12}$$

in which p represents the pressure of water vapor in capillary pores, if they are unsaturated (meniscus present), or the pressure of bulk (liquid) water, if they are saturated (no meniscus). Furthermore, a = permeability (dimension m/s), and $g = 9.806$ m/s^2 = gravity acceleration, which is included solely for reasons of dimensionality. Substituting Eq. (12) into Eq. (11), we obtain

$$\dot{w}_c = \text{div} \left(\frac{a}{g} \text{ grad } p \right) - j . \tag{13}$$

The water content of capillary pores is related to pressure and temperature through an experimental sorption relation (Fig. 2c; Kollmann, Côté 1968; Simpson 1979):

$$w_c = w_c (p, T) . \tag{14}$$

This relation holds locally also during macroscopic diffusion if the existence of local thermodynamic equilibrium is assumed. Then, by chain rule of differentiation,

$$\dot{w}_c = \frac{\dot{p}}{k_c} + \left(\frac{\partial w_c}{\partial T} \right)_p \dot{T}, \quad k_c = \frac{1}{(\partial w_c / \partial p)_T} . \tag{15}$$

Substituting now Eq. (15) and Eq. (10) into Eq. (13), we get

$$\dot{p} = k_c \text{ div} \left(\frac{a}{g} \text{ grad } p \right) - k_c \frac{\partial w_c}{\partial T} \dot{T} - \frac{b_l}{b_m} (\dot{h} + c_m \dot{T}) \tag{16}$$

in which $h = p/p_s(T)$, $p_s(T)$ being the saturation vapor pressure of water at temperature T. Thus $\dot{h} = (\dot{p}/p_s) - p \, p_s' \dot{T}/p_s^2$ where $p_s' = dp_s/dT$. This leads to the
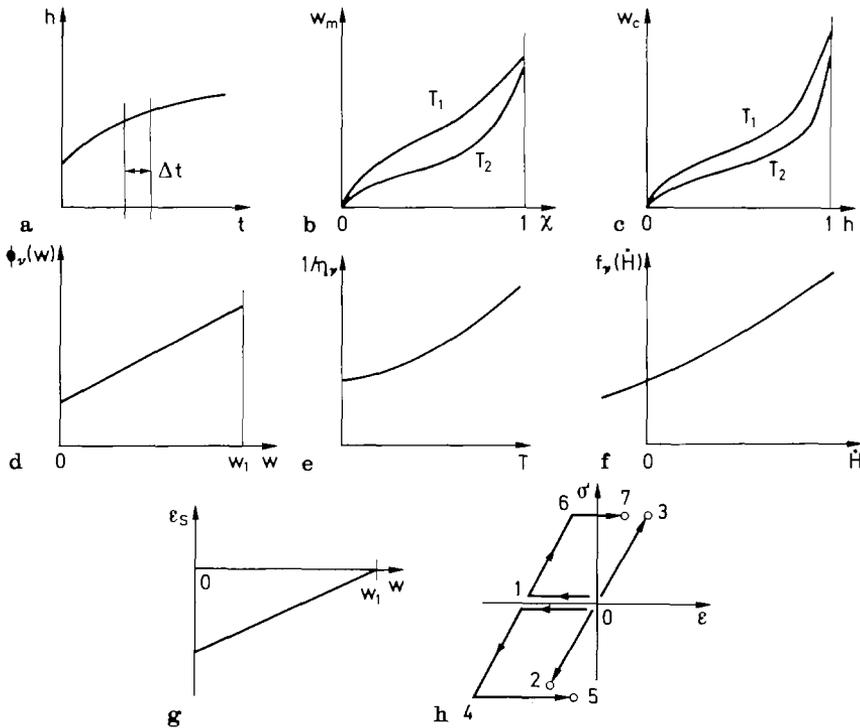
Fig. 2. Typical plots of various characteristic functions

following governing differential equation for the macroscopic flow of water through the material:

$$\dot{p} = k \, \text{div} \left(\frac{a}{g} \, \text{grad} \, p \right) - k_1 \, \dot{T} \tag{17}$$

in which we introduce the notation

$$k = k_c \left(1 + \frac{b_1}{b_m \, p_s} \right)^{-1}, \qquad k_1 = \left[k_c \left(\frac{\partial w_c}{\partial T} \right)_p + \left(\frac{p \, p_s'}{p_s^2} - c_m \right) \right] \left(1 + \frac{b_1}{b_m \, p_s} \right)^{-1}. \tag{18}$$

From the form of Eq. (17) we conclude that if the local water transport between macropores and micropores is much faster than the macroscopic water transport (which must always be true for a sufficiently large body of wood), then the differential equation governing the macroscopic diffusion is of the usual form, i.e., of the same form as that in absence of local diffusion. It is, therefore, not possible to observe the microdiffusion except on sufficiently small specimens, probably too small for practical measurements.

The diffusion process could alternatively be described using w, instead of p or h, as the basic variables. This would, however, be less convenient since grad T would have to be considered as a separate driving force. The advantage of using grad p is that a temperature gradient produces a gradient of p even at uniform w, thus describing both the mass transport due to concentration gradient (Fick's law) and the thermal mass flux (Soret flux).

Stress-strain relation at constant moisture content

Within the range of service stresses, wood at constant moisture content and constant temperature may be approximately considered as a linearly viscoelastic material. Rather than using memory integrals, it is more convenient to represent the viscoelastic behavior in terms of the Maxwell chain model (Fig. 3a), which is known to be capable of approximating any viscoelastic properties as closely as desired. The uniaxial stress-strain relation based on this model may be written as

$$\sigma = \sum_{v=1}^{N} \sigma_v, \quad \dot{\varepsilon} = \frac{\dot{\sigma}_v}{E_v} + \frac{\sigma_v}{\eta_v} \tag{19}$$

in which σ, ε = uniaxial stress and strain, σ_v = partial stresses (internal variables), visualized as the stresses in the individual Maxwell units of the chain, and E_v, η_v = the elastic moduli and viscosities of the individual units of chain, labeled by subscripts $v = 1, 2, \ldots, N$.

Creep tests have been conducted at various constant moisture contents w and at various constant temperatures T (Johnson 1978; Ugolev 1976; Dinwoodie, Paxton, Pierce 1981; Grossman 1978). From these it is known that for wood, similarly to many other materials, the viscosities depend on water content and temperature (Fig. 2e), which may be expressed as follows:

$$\frac{1}{\eta_v} = \frac{\varphi_v(w)}{E_v \tau_v} \exp\left(\frac{Q_c}{RT_0} - \frac{Q_c}{RT}\right) \tag{20}$$

in which Q_c = activation energy of creep, R = gas constant, T = absolute temperature, T_0 = reference temperature, φ_v = increasing functions of water content w (Fig. 3d), and τ_v = relaxation time of the v-th Maxwell unit at reference temperature T_0 and reference water content w_0. For the reference conditions we choose $\varphi_v = 1$. Furthermore, it is suitable to choose (Bažant, 1982) $\tau_v = \tau_1 \, 10^{v-1}$ for $v = 1, \ldots, N - 1$, and to consider the last relaxation time to be extremely long, e.g., $\tau_N = 10^9$ days, so that the last Maxwell unit would be essentially equivalent to a spring characterizing the long-term limiting elastic modulus.

Equation (20) may be also written as

$$\frac{1}{\eta_v} = \frac{c_Q}{E_v \tau_v} e^{(-Q_c + \psi_v)/RT} \quad \text{with} \quad \psi_v = RT \ln \varphi_v, \quad c_Q = e^{Q_0/RT_0} \tag{21}$$
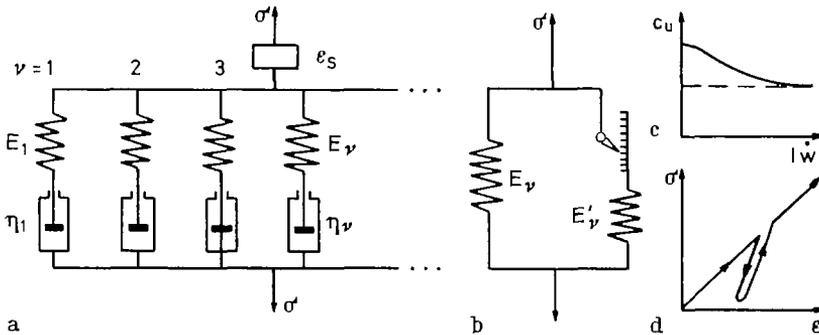
Fig. 3. a Maxwell chain model with shrinkage strain; b Ratchet model (b–d)

in which c_Q is a constant, and ψ_v may be interpreted as a moisture potential which represents the energy of straining the cross-link bonds whose ruptures and reformations are the cause of creep. The combined effect of temperature and moisture content as given in Eq. (21) can be also derived directly from the rate process theory (Schaffer 1972).

For practical purposes it is probably possible to assume that ψ_v is the same for all v, i.e., $\psi_v = \psi$ and $\varphi_v = \varphi$. It seems that a reasonable dependence of creep rate on the moisture content w may be obtained simply by assuming $1/\eta_v$ to be proportional to w. Then $\psi_v = c_1 + c_2 \ln w$, where c_1, c_2 = constants (Fig. 2d).

Based on the assumptions of the rate-process theory, one can also describe the nonlinear dependence of creep rate on stress. This approach usually works for moderately high stress levels, but not near failure. Thus, it could be shown that for moderately high stress levels Eq. (19) can be replaced by

$$\dot{\varepsilon} = \frac{\dot{\sigma}_v}{E_v} + \frac{c_R \, \sigma_v}{E_v \, \tau_v} \, e^{(-Q_o + \psi_v)/RT} \frac{RT}{V_c} \frac{1}{\sigma} \sinh\left(\frac{V_c}{RT}\,\sigma\right) \tag{22}$$

in which c_R is a constant and V_c is another constant interpreted as the activation volume.

Stress-strain relation at variable moisture content and temperature

Changes in moisture content of wood cause shrinkage or swelling. When the wood is simultaneously under stress, two phenomena take place:
1) The creep rate is increased, regardless of the sign of the change in moisture content; and
2) the shrinkage or swelling strain, ε_S, is superimposed.

As we argued at the outset, it is not reasonable to assume that the creep rate increases due to the macroscopic flux or the associated gradient since this flux should bypass the smallest micropores that participate in the transmission of load. It is much more reasonable to assume that the creep rate is affected only by the local diffusion flux j (microdiffusion) between the micropores and the macropores (cell wall pores and cell lumen). According to our result in Eq. (10) j is proportional to the quantity

$$\dot{H} = \dot{h} + c_m \, \dot{T} \tag{23}$$

where c_m is given by Eq. (6) H may be called the equivalent pore humidity rate; $\dot{H}$ coincides with $\dot{h}$ when temperature is constant. Considering the molecules of water in the micropores to participate in the transmission of stress, their displacement due to diffusion must facilitate creep, i.e., increase the creep rate. Thus, it is reasonable to extend Eq. (20) as follows:

$$\frac{1}{\eta_v} = \frac{1}{\eta_v^0} \, f_v(\dot{H}), \quad \frac{1}{\eta_v^0} = \frac{\varphi_v(w)}{E_v \, \tau_v} \exp\left(\frac{Q_c}{RT_0} - \frac{Q_c}{RT}\right) \tag{24}$$

in which $\varphi_v(w) \simeq k(w_1 - w)$, k = constant, w_1 = saturation water content; $f_v(\dot{H})$ are functions of $\dot{H}$.

Based on our preceding arguments, we expect that movements of water molecules of *any* sign, i.e., out of the micropores or into them, should promote creep. Therefore, f_v should actually be a function of $|j|$ rather than j, and according to Eq. (10), $f_v(\dot{H})$ should then be a function of $|\dot{H}|$. Assuming this function to be smooth, it may be expanded in Taylor series, and its linear terms should suffice if $|\dot{H}|$ is not very large. So we may reasonably assume $f_v(\dot{H}) \approx f_v^0(1 + a_1|\dot{H}|)$ where f_v^0 and a_1 are some positive constants; a_1 must be positive if movement of water molecules should increase rather than decrease creep. Thus we may write

$$f_v(\dot{H}) \simeq f_v^0(1 + a_1|\dot{h} + c_m\dot{T}|) . \tag{25}$$

At variable humidity and temperature, stess-independent strains representing shrinkage (or swelling) and thermal expansion, which are observed in stress-free specimens, must be added to the stress-produced strain (mechanical strain) given by Eq. (19) i.e.,

$$\dot{\varepsilon} = \frac{\dot{\sigma}_v}{E_v} + \frac{\sigma_v}{\eta_v} + \varkappa\dot{h} + \alpha\dot{T}, \quad \sigma = \sum_{v=1}^{N}\sigma_v \tag{26}$$

where $\varkappa$ is the shrinkage (or swelling) coefficient and α is the thermal expansion coefficient. If the stress-independent strain is experimentally determined as function F (h, T), then

$$\varkappa = \left[\frac{\partial F(h, T)}{\partial h}\right]_T, \quad \alpha = \left[\frac{\partial F(h, T)}{\partial T}\right]_h \tag{27}$$

$\varkappa$ and α are functions of h (or w) and T. Note, however, that direct measurement of F (h, T) is almost impossible since drying or wetting always produces residual stresses in a wood specimen, so that the specimen is not stress-free (except for extremely slow drying or wetting of very thin specimens).

Stress or strain-induced shrinkage or swelling

We will now show that the effect of humidity and temperature rate on the creep rate may be more conveniently described as stress-induced shrinkage (swelling) and thermal expansion. Substituting Eq. (25) into Eq. (24), Eq. (24) into Eq. (26), setting $|\dot{H}| = \dot{H}\text{ sign }\dot{H}$ where sign $\dot{H}$ is 1 if $\dot{H} > 0$ and -1 if $\dot{H} < 0$, and combining terms with $\dot{h}$ and $\dot{T}$, we find that

$$\dot{\varepsilon} = \frac{\dot{\sigma}_v}{E_v} + \frac{\sigma_v}{\eta_v^0} + \dot{\varepsilon}_S + \dot{\varepsilon}_T, \quad \sigma = \sum_{v=1}^{N}\sigma_v \tag{28}$$

with

$$\dot{\varepsilon}_S = \varkappa(1 + r_v\sigma_v\text{ sign }\dot{H})\dot{h}, \quad \dot{\varepsilon}_T = \alpha(1 + \varrho_v\sigma_v\text{ sign }\dot{H})\dot{T} \tag{29}$$

in which viscosities η_v^0 depend on h and T but not on $\dot{h}$ and $\dot{T}$ as specified by Eq. (20) or (21), and r_v and ϱ_v are certain positive constants or functions of h and T (or w and T); $\sigma_v = \alpha_1 f_v^0/\eta_v^0$ and $\varrho_v = \alpha_1\tau_m f_v^0/\eta_v^0$. $\dot{\varepsilon}_S$ and $\dot{\varepsilon}_T$ may be regarded as total shrinkage and thermal expansion rates, and $\varkappa r_v\sigma_v\dot{h}\text{ sign }\dot{H}$ and $\alpha\varrho_v\sigma_v\dot{T}\text{ sign }\dot{H}$

may be called the stress-induced shrinkage (or swelling) and the stress-induced thermal expansion (or contraction); note that their signs depend only on the sign of σ_v (tension or compression).

It should be noted that if stress-induced shrinkage (or drying creep) exists, stress-induced thermal expansion must exist, too. This is a consequence of the fact H depends on both $\dot{h}$ and $\dot{T}$, or that the specific Gibbs free energy of water per unit mass depends not only on pressure but also on temperature, and that the sorption relation between water content and pressure also involves temperature.

As a further simplification, coefficients r_v and σ_v may probably be assumed the same for all v, and σ_v in Eq. (29) may be replaced by σ; then

$$\dot{\varepsilon}_S = \varepsilon'_{sh}\,(1 + r\,\sigma\;\text{sign}\;\dot{H})\,\dot{h}, \quad \dot{\varepsilon}_T = \alpha\,(1 + \varrho\,\sigma\;\text{sign}\;\dot{H})\,\dot{T}. \tag{30}$$

Alternatively, $\dot{\varepsilon}_{sh}$ could be assumed to depend on $\dot{w}$ instead of $\dot{h}$. In that case

$$\dot{\varepsilon}_S = \tilde{\varepsilon}_{sh}\,(1 + r\,\sigma\;\text{sign}\;\dot{w})\,\dot{w} \tag{31}$$

in which $\tilde{\varepsilon}_{sh}$ is a function of w (or h) and T, different from ε'_{sh}.

Although quantitative fitting of test data is beyond the scope of the present project, it is apparent that various experimental results qualitatively support the idea of stress-induced shrinkage proposed here. The uniaxial compression tests (Bažant, Meiri 1982) indicate that the strain rate right after the start of drying becomes much larger than the sum of the initial creep rate and the shrinkage rate.

Particularly revealing are the bending measurements of Armstrong and Christensen (1971) and Johnson (1978). They indicate the bevahior sketched in Fig. 4b, determined on beams subjected to flexure under constant load and simultaneous humidity cycles. Beginning with the second cycle, the deflection decreases (recovers) during wetting (an increase of w). The recovery cannot be caused by stress, because negative work would be done, and the second law of thermodynamics rules this out. So, the decrease of deflection must be due to swelling. However, according to the customary assumption of stress-independent shrinkage or swelling, the swelling of the beam would be symmetrical and would result only
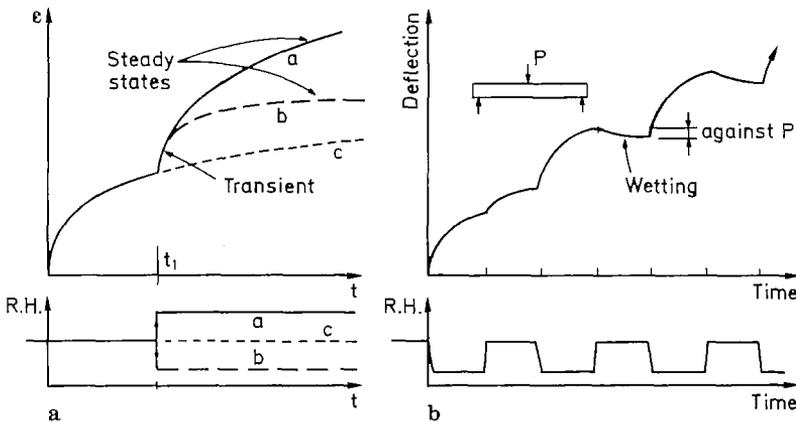
Fig. 4. a Creep after a decrease or increase of relative humidity; **b** typical deformations at humidity changes

in axial shortening, not in deflection. Thus, we conclude that swelling *must* depend on stress or strain. It must be larger on the compressed side of the beam, and smaller on the tension side. This phenomenon may be illustrated by the diagram in Fig. 2h where segments $\overline{45}$ and $\overline{67}$ represent the swelling of wood under compression or tension, and segment $\overline{01}$ represents shrinkage of stress-free wood, while segments $\overline{14}$ and $\overline{16}$ represent the deformations due to stress. This behavior is qualitatively in complete agreement with what we have deduced theoretically and formulated as the stress-induced shrinkage or swelling.

It is also possible that the additional strains due to humidity variations might be better described as strain-induced, rather than stress-induced, shrinkage; e.g.

$$\dot{\varepsilon}_S = \tilde{\varepsilon}_{sh}\, f_1(\varepsilon)\, \dot{w} \quad \text{or} \quad \dot{\varepsilon}_S = \varepsilon'_{sh}\, f_2(\varepsilon)\, \dot{h}. \tag{32}$$

A suitable form might be $f_1(\varepsilon) = e^{-a_1\varepsilon}$ and $\tilde{\varepsilon}_{sh} = c_1\, e^{a_1 c_* w}$. This has the advantage that for stress-free states, $\varepsilon = \varepsilon_S$, integration of Eq. (32) yields $\varepsilon = c_S\, w$ if $c_1 = c_S$, i.e., proportionality of shrinkage to water loss, which describes well many porous solids including wood.

The question of ratchet effect

The recovery of the creep deformation of wood upon unloading is incomplete; however, it can be made nearly complete when the moisture content is varied during recovery. It seems that some sort of internal blockage or friction prevents the recovery, and this blockage or friction is destroyed by simultaneous change in moisture content, e.g., by the flux between micropores and macropores, j. If we assume, however, a certain non-zero frictional limit on the deformation, we would have to deal with the theory of viscoplasticity rather than viscoelasticity. The deformations even at small stresses would be nonlinearly dependent on stress, and that does not seem to be the case.

Another approach is therefore desired, and one possibility is to postulate the so-called "ratchet effect", proposed by Bažant (1964, 1965) for fiber reinforced plastics and for concrete. This effect is represented by the model in Fig. 3b, showing a group of two springs which are intended to replace each spring of the Maxwell chain model. The second spring, E'_ν, is coupled in parallel with the spring E_ν but the stress is transferred into it by a ratchet which allows free displacement in only one direction and completely blocks the displacement in the opposite direction. Thus, for the first loading, the spring constants are E_ν, and for unloading (reversal of deformation, Fig. 3d) the spring constants are increased to

$$E_\nu = (1 + c_\nu)\, E^0_\nu \tag{32}$$

in which $E^0_\nu = $ constants and coefficients c_ν are defined as

$$c_\nu = \begin{cases} 1 & \text{if } |d\varepsilon| \geqq 0 \text{ and } |\varepsilon| = \max|\varepsilon| \\ c_u & \text{if } \begin{cases} \text{either } |d\varepsilon| < 0 \\ \text{or } |d\varepsilon| \geqq 0 \text{ and } |\varepsilon| < \max|\varepsilon|. \end{cases} \end{cases} \tag{33}$$

Here c_u is a coefficient for unloading, a constant larger than 1.0, and $\max|\varepsilon|$ represents the maximum strain magnitude achieved up to the current time. The

first case in Eq. (32) represents virgin loading, the second case represents unloading, and the third case represents reloading. Such a three-way distinction between various loading situations has recently been introduced with success for describing the nonlinear short-time deformations of concrete in triaxial loading (Bažant 1978; Bažant, Kim 1978, 1979). Instead of the absolute value of ε, we could use ε^2 in Eq. (32). In multiaxial generalizations, the absolute value of ε must be replaced by some suitable positive definite invariant of the strain tensor, such as J_2 or I_1^2.

The foregoing formulation has the advantage that all previous equations are preserved for the case of loading. For the case of unloading, the behavior is again governed (at constant w and T) by linear viscoelastic stress-strain relations which have however different constants, causing an incomplete recovery.

At a change of moisture content, the internal blockages which are visualized by the ratchet are disturbed, which may be interpreted as a drop in the value of c_u toward the value 1.0 that applies for loading. Thus, we may postulate the rule (Fig. 3c):

$$c_u = \frac{c_u^0 + b_u \dot{w}^2}{1 + b_u \dot{w}^2} \quad \text{or} \quad c_u = \frac{c_u^0 + b_u \dot{h}^2}{1 + b_u \dot{h}^2} \tag{34}$$

in which c_u^0 is a constant greater than 1.0, and b_u is a positive constant. At a sufficiently high rate of change of w or h, c_u would then approach 1.0, and the deformation would then fully recover because the last spring in the Maxwell chain is associated with an infinite relaxation time (i.e., no dashpot).

Effect of nonlinearity and bond ruptures produced by drying or wetting

In interpreting test results it is important to take into account possible tensile stresses due to nonuniformity of stress throughout the cross section of the specimen. The result may be nonlinear deformation and cracking. In other materials, such as concrete, drying or wetting may produce severe microcracking, undetectable to the unaided eye in case of small specimens. Microcracking may severely distort interpretations of test results if it is neglected, and greatly complicate interpretations if it is taken into account. Therefore, in testing wood specimens, it is important to produce moisture changes in such a way that no microcracking or nonlinear deformations are produced in the specimen. The safest way is to decrease the environmental humidity sufficiently slowly, so that the differences in humidity within the cross section do not exceed a certain small limit, e.g., 2%. The maximum admissible rate of change of environmental humidity can be calculated (Bažant, Raftshol 1982) and is found to be roughly proportional to the square of the thickness of the specimen. For materials of such a low diffusivity as concrete, this dictates extremely thin specimens (under 1 mm thickness), fragile to handle. For wood, the diffusivity is about 100-times larger than that for concrete, and this makes it possible to use specimens of greater thickness (several millimeters) and satisfy the foregoing condition by changing the environment from one value to another gradually, within a period not less than about one hour. Such testing has been carried out in the past for cement paste, but not for wood, as it seems.

Another possibility of alleviating the aforementioned deleterious effects is to put the entire specimen in hydrostatic compression, achieved by triaxial loading. This compression can assure that no tensile nonlinearities due to bond breakages can happen. However, one still faces the unpleasant task of evaluating test data for specimens in which the moisture content has been nonuniform. This task can be accomplished with the help of a finite element program.

Summary of constitutive relations

The stress-strain relation for wood is defined by Eq. (26) in which the equivalent pore humidity rate $\dot{H}$ is given by Eq. (23) (justified by Eq. (10)), the viscosities are given by Eqs. (24) and (23), the elastic moduli are given by Eq. (23), and the shrinkage strain is given by Eq. (29). The macroscopic water transport through wood is described by Eq. (17).

Conclusions

1. The acceleration of creep due to increase or decrease of moisture content may be explained by the transport of pore water through the cell wall material (microdiffusion), rather than by the macroscopic diffusion of water through the body of wood (macrodiffusion).

2. Considering that the micropores in the cell walls are of unequal sizes, the macroscopic transport of water through wood should pass mainly through the largest micropores, and should bypass the smaller micropores. Thus, in essence, the only transport of water through the smallest micropores is that which is caused by thermodynamic imbalance (i.e., by differences in Gibbs' free energy) between the micropores and the macropores, and is needed to restore thermodynamic equilibrium of the micropore water. No such thermodynamic imbalance is caused by a steady-state flow of water through wood.

3. Only the water molecules in the smallest micropores can be expected to effectively participate in transmitting applied loads, and only the movement of these can be considered to have an appreciable effect on the creep rate.

4. Considering that the microdiffusion, i.e., the water transfer between the cell wall pores (micropores) and the cell lumen (macropores), should be much faster than the macrodiffusion, which must always be true for a sufficiently large specimen, the effect of microdiffusion on the creep rate can be reduced to an effect of the rate of moisture content (or of pore relative humidity) on the creep rate. It also follows that the macroscopic gradient of moisture content should have no effect.

5. Since thermodynamic imbalance between the micropores and the macropores is produced not only by a change of water content but also by a change of temperature, the present model predicts creep acceleration to happen not only as a result of changes in the moisture content of wood, but also as a result of changes in temperature, particularly as an effect of the time rate of temperature on the viscosity coefficients. This offers one important possibility for checking the correctness of the hypothesis.

6. The acceleration of creep due to changes in moisture content may be introduced as a dependence of the viscosities of the Maxwell chain model on the magnitude of the rate of moisture content.

7. The aforementioned acceleration of creep is approximately equivalent to additional, stress-induced shrinkage (or swelling) and stress-induced thermal expansion (or contraction), which permits the creep viscosities to be considered independent of the rate of moisture content.

8. The irreversibility of deformation upon unloading may be reasonably described by a ratchet effect. The internal blockages which prevent deformation recovery and are modeled by the ratched effect are likely to be affected by the movement of water through the micropores of the cell wall.

9. To allow clear and unambiguous evaluation of test results it is important that the specimens be at an almost uniform moisture content during the humidity changes. This can be achieved by using sufficiently thin specimens and a continuous, sufficiently slow, variation of environmental humidity.

10. The essential characteristics of wood, which make its creep-rate sensitivity so similar to that of other porous materials (e.g., portland cement concretes), are the existance of pores of vastly different sizes (capillary macropores and subcapillary micropores) and the hydrophylic nature of the material forming the pore walls.

Finally, it must be emphasized that the present model is basically speculative and depends on a number of assumptions which appear reasonable but lack solid proofs. Obviously, much further work is required to reach complete understanding of this complex problem.

References

Arima, T.; Grossman, P. U. A. 1978: Recovery of wood after mechano-sorptive deformation. J. Inst. Wood Sci., 8 (2): 47–52

Armstrong, L. D. 1972: Deformation of wood in compression during moisture movement. Wood Sci. 5 (2): 81–86

Armstrong, L. D.; Christensen, G. N. 1961: Influence of moisture changes on deformation of wood under stress. Nature 191 (4791): 869–870

Armstrong, L. D.; Grossman, P. U. A. 1972: The behavior of particleboard and hardboard beams during moisture cycling. Wood Sci. Technol. 6: 128–137

Armstrong, L. D.; Kingston, R. S. T. 1962: The effect of moisture content changes on the deformation of wood under stress. Australian J. Appl. Sci. 13: 259–276

Banks, W. B. 1981: Addressing the problem of non-steady state liquid flow in wood. Wood Sci. Technol. 15: 171–177

Bažant, Z. P. 1965: Mathematical formulation of rheologic properties of fiber reinforced plastics (in German). Plaste und Kautschuk, Leipzig, 12: 592–600

Bažant, Z. P. 1970: Constitutive equation for concrete creep and shrinkage based on thermodynamics of multiphase systems. Materials and Structures (RILEM, Paris) 3: 3–36

Bažant, Z. P. 1978: Endochronic inelasticity and incremental plasticity. Intern. J. of Solids and Structures 14: 691–714

Bažant, Z. P. 1975: Theory of creep and shrinkage in concrete structures: A precis of recent developments. In: Mechanics Today, Vol. 2, pp. 1–93, New York: Pergamon Press

Bažant, Z. P. 1982: Mathematical models for creep and shrinkage of concrete. Chapter 7, pp. 163–256, in: Creep and Shrinkage in Concrete Structures, ed. by Bažant, Z. P. and Witmann, F. H. London: J. Wiley

Bažant, Z. P.; Wu, S. T. 1974: Thermoviscoelasticity of aging concrete. J. of the Eng. Mech. Div., Proc. of the Am. Soc. of Civil Engrs. 100: 275–297

Bažant, Z. P.; Wu, S. T. 1974: Creep and shrinkage law for concrete at variable humidity. J. of the Eng. Mech. Div., Proc. of the Am. Soc. of Civil Engrs. 100: 1183–1209

Bažant, Z. P.; Meiri, S. 1982: Compression creep tests of wood at humidity cycles, Report, Center for Concrete and Geomaterials, Northwestern University, Evanston, Illinois

Bažant, Z. P. 1982: Constitutive Equation of Wood at Variable Humidity and Temperature, Report to U.S. Forrest Products Laboratory, Center for Concrete and Geomaterials, Northwestern University, Evanston, Illinois

Bethe, E. 1969: Festigkeitseigenschaften von Bauholz bei Lagerung im Wechselklima unter gleichzeitiger mechanischer Belastung. Holz Roh- Werkstoff 27: 291–303

Byrd, V. L. 1972: Effect of relative humidity changes during creep on handsheets. Tappi 55: 247–252

Byrd, V. L. 1972: Effect of relative humidity changes on compressive creep response of paper. Tappi 55: 1612–1613

Cave, I. D. 1978: Modelling moisture-related mechanical properties of wood Part 1: Properties of the wood constituents. Wood Sci. Technol. 12: 75–86; Part II: Computation of properties of a model of wood and comparison with experimental data. Wood. Sci. Technol. 12: 127–139

Christensen, G. N. 1962: The use of small specimens for studying the effect of moisture content changes on the deformation of wood under load. Australian J. Appl. Sci. 13: 242–256

Çinlar, E.; Bažant, Z. P.; Osman, E. 1977: Stochastic process for extrapolating concrete creep. J. of the Eng. Mech. Div., Proc. of Am. Soc. of Civil Engrs. 103: 1069–1088

Dinwoodie, J. M.; Paxton, B. H.; Pierce, C. B. 1981: Creep in chipboard. Part III. Initial assessment of the influence of moisture content and level of stressing on rate of creep and time to failure. Wood Sci. Technol. 15: 125–144

Dinwoodie, J. M. 1975: Timber – A review of the structure-mechanical property relationship. J. of Microscopy 104: 3–32

Gerhards, C. C. 1982: Effect of moisture content and temperature on the mechanical properties of wood: An analysis of immediate effects. Wood Fiber 14: 4–37

Grossman, P. U. A. 1971: Use of Leicester's rheological model for mechano-sorptive deflections of beams. Wood Sci. Tech. 5: 232–235

Grossman, P. U. A. 1976: Requirements for a model that exhibits mechano-sorptive behavior. Wood Sci. Tech. 10: 163–168

Grossman, P. U. A. 1978: Mechano-sorptive behavior. Report of July 1978 NSF Workshop at Blue Mountain Lake, Minnowbrook, N.Y., ed. by Jayne, B. A., Johnson, J. A., and Perkins, R. W., Syracuse University, pp. 313–325

Grossman, P. U. A., and Kingston, R. S. T. 1954: Creep and stress relaxation in wood during bending. Aust. J. Appl. Sci. 5: 403–417

Halligan, A. F.; Schniewind, A. P. 1972: Effect of moisture on physical and creep properties of particleboard. Forest Prod. J. 22 (4): 41–48

Halligan, A. F.; Schniewind, A. P. 1974: Prediction of particleboard mechanical properties at various moisture contents. Wood Sci. Technol. 8: 68–79

Haygreen, J. G.; Hall, H.; Yang, K.-N.; Sawicki, R. 1975: Studies of flexural creep behavior in particleboard under changing humidity conditions. Wood Fiber 7 (2): 74–90

Hearmon, R. F. S.; Paton, J. M. 1964: Moisture content changes and creep of wood. For. Prod. J. 14: 357–359

Humphries, M.; Schniewind, A. P. 1982: Behavior of wood columns under cyclic relative humidity. Wood Sci. 15: 44–48

Hunt, D. G. 1982: Limited mechano-sorptive creep of beachwood. J. of Inst. Wood Sci. 9: 136–319

Johnson, J. A. 1978: Review of the interaction of mechanical behavior of moisture movement in wood. In General Constitutive Relations for Wood and Wood-Based Materials, Report of July 1978 NSF Workshop at Blue Mountain Lake, Minnowbrook, N.Y., (ed. by Jayne, B. A., Johnson, J. A., and Perkins, R. W., Syracuse University), pp. 282–303

Leicester, R. H. L. 1971: A rheological model for mechano-sorptive deflections of beams. Wood Sci. Tech. 5: 211–220

Leicester, P. H. L. 1971: Lateral deflections of timber beam-columns during drying. Wood Sci. Technol 5: 221–231

Libby, T. B.; Haygreen, J. G. 1967: Moisture content change induced by tensile stress in whole wood. J. Inst. Wood Sci. 18: 54–60

Lyon, D. E.; Schniewind, A. P. 1978: Prediction of creep in plywood – Part I. Prediction models for creep in plywood. Wood Fiber 10: 28–39

Mark, R. E. 1965: In: Côté, W. A. (ed.), Cellular ultrastructure of woody plants, Syracuse: Syracuse University Press

Narayanamarti, D.; Verma G. 1972: Swelling and shrinkage of wood under mechanical restraint. Holzforsch. Holzverwert. 24: 83–93

Palin, M. A.; Petty, J. A. 1981: Permeability to water of the cell wall material of spruce heartwood. Wood Sci. Technol. 15: 161–169

Pentoney, R. E. 1963: Time dependent mechanical properties of wood. In: the mechanical behavior of wood. Ed.: Schniewind, A. P., University of California, Berkeley, pp. 96–104

Pierce, C. B.; Dinwoodie, J. M. 1977: Creep in chipboard. Part I: Fitting 3- and 4-element response curves to creep data. J. of Materials Sci. 12: 1955–1960

Pierce, C. B.; Dinwoodie, J. M.; Paxton B. H. 1979: Creep in chipboard. Part II: The use of fitted response for comparative and predictive purposes. Wood Sci. Technol. 13: 265–282

Raczkowski, J. 1969: The effect of moisture content changes on the creep behavior of wood. Holz Roh-Werkst. 27: 232–237

Ranta-Maunus, A. 1973: Theory for creep of wood. Techn. Res. Centre, Finland, Publ. 4

Ranta-Maunus, A. 1975: The viscoelasticity of wood at varying moisture content. Wood Sci. Tech. 9: 189–205

Sadoh, T. 1981: Viscoelasticity properties of wood in swelling systems. Wood Sci. Technol. 15: 57–66

Schaffer, E. L. 1972: Modeling the creep of wood in a changing environment. Wood Fiber 3: 232–235

Schaffer, E. L. 1978: Temperature-time dependency of longitudinal mechanical behavior of dry douglas fir, NSF workshop on general constitutive relations for wood and wood-based materials, held at Blue Mountain Lake. Ed.: Perkins, R. W., Syracuse University, pp. 234–278

Schniewind, A. P. 1966: On the influence of moisture content changes on the creep of beach wood perpendicular to the grain, including the effects of temperature and temperature change. (CSIRO translation No. 8361) Holz Roh-Werkstoff 24: 87–98

Schniewind, A. P. 1968: Recent progress in the study of rheology of wood. Wood Sci. Technol. 2: 188–206

Siau, J. F. 1971: Flow in wood. Syracuse wood science series, No. 1. Syracuse: Syracuse University Press

Simpson, W. T. 1971: Moisture changes induced in red oak by transverse stress, Wood Fiber 3: 13–21

Simpson, W. T. 1980: Sorption theories applied to wood. Wood Fiber 12: 183–195

Simpson, W. T.; Howard, N. R. 1981: Equilibrium moisture content of wood at high temperatures. Wood Fiber 13: 150–159

Stamm, A. J.; Smith, W. E. 1969: Laminar sorption and swelling theory for wood and cellulose. Wood Sci. Technol. 3: 301–323

Szabo, T.; Ifju, G. 1970: Influence of stress on creep and moisture distribution in wooden beams under sorption conditions. Wood Science 2: 159–167

Ugolev, B. N. 1976: General laws of wood deformation and rheological properties of hardwood. Wood Sci. Technol. 10: 169–181

Ugolev, B. N.; Lapshin, Ju. G. 1971: On deforming of wood when loaded in the conditions of drying. Lesnoy Zhurnal (3): 63–65

White, R. H.; Schaffer E. L. 1980: Transient moisture gradient in fire-exposed wood slab. Wood Fiber 13: 17–38

Ylinen, A.; Jumppanen, P. 1967: Theory of the shrinkage of wood. Wood Sci. Technol. 1: 241−252

Yokota, T. 1959: Diffusion of sorption-water through the cell wall of wood. J. Japanese Wood Res. Society 5: 143−149

Zahn, J. J. 1978: Nonlinear constitutive relations for wood. Report on July 1978 NSF workshop at Blue Mountain Lake, Minnowbrook, N.Y. (Ed. Jayne, B. A., Johnson, J. A., and Perkins, R. W.), Syracuse: Syracuse University, pp 212−229

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