Input of creep and shrinkage characteristics for a structural analysis program

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Presented is a computer program for the input of creep and shrinkage properties for a structural analysis program. The program either accepts numerical data on the compliance function and the shrinkage function at discrete load durations and ages at loading, or uses the recent BP Model for creep involving the double-power law. The most useful characteristic is the availability of thirteen different options for specifying the creep and shrinkage parameters; sealed conditions or drying conditions, creep values specified and interpolation to be used, parameter of a creep formula specified, or creep formula automatically fitted to given creep data, short time data specified and extrapolation done by a formula, etc. In addition to returning the compliance function, the program also calculates the relaxation function and the age-dependent elastic moduli of a Maxwell chain model that is equivalent to the given creep properties. The program is particularly suitable for the input of creep characteristics for a large-scale finite element analysis.

INTRODUCTION

Development of numerical structural analysis methods for creep and shrinkage of concrete has greatly enhanced our capability of realistic modeling and prediction of structural behavior. Implementation of such an analysis however requires the use of a realistic model for creep and shrinkage properties (constitutive equation) and optimum adaptation of this model to available data on the material.

Extending the work briefly outlined in a recent conference paper [1], we present here a complete listing of a computer program which serves the aforementioned purpose. This program allows for characterizing the creep and shrinkage properties by a set of measured or given values at discrete times or by formulas recently presented in this Journal [2]. Various mixed modes of input, combining partial experimental data and automatic fitting of a formula are also available in this program. The most useful feature of the program is the availability of numerous options for the specification of creep (and shrinkage) properties by the user and for the type of representation of the compliance function. In addition to generating the compliance function, the program also automatically calculates discrete values of the relaxation function as well as discrete values of the age-dependent elastic moduli of a Maxwell-chain rheologic model — a model which allows the most efficient analysis of large structural systems by finite elements.

The FORTRAN IV listing which we present here includes numerous comments defining the meaning of

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principal variables as well as the input. The output of the program is also made self-explanatory by the headings and comments. It is therefore possible to understand and use this program without a separate guide, which could not be anyhow condensed into a short paper. However, a detailed guide is available [3], as part of a manual for a complete finite element program for concrete creep at variable temperature.

CONSTITUTIVE RELATION

The present program characterizes creep in terms of a compliance function $J\ (t,\ t')$ (also called creep function), which represents the strain at age t caused by a unit constant stress acting since the age t'.

By using this fonction, we assume the constitutive equation to be linear, obeying the principal of superposition [4]. The linearity assumption is acceptable only at stresses less than about one-half of the strength of concrete. One must be aware, however, that even within this range significant deviations from linearity are observed upon sudden unloading and generally in regimes of decreasing strains. Further significant deviations from linearity are caused by a drying simultaneous with creep, chiefly due to microcracking produced by the drying. By considering the compliance function as a function of two variables $(t,\ t')$, the aging of concrete is taken into account.

To facilitate the analysis of large structural systems, the program also generates the material parameters for a rate-type constitutive law for the aging creep of concrete. The well-known Maxwell chain model with age-dependent elastic moduli is chosen for this purpose ([5], [6]). This formulation allows the storage of the history of stresses or strains to be dispensed with, and thus it greatly reduces the computational costs and increases the size of the structural system that can be handled on a given computer.

SIMPLIFIED FLOW CHART OF THE PROGRAM

1. Read the number of decades in log-time scale to be considered, the number of steps per decade, the number of elements in Maxwell chain, the time for the start of the first time step, and the first relaxation time.

2. Read the input option number and the corresponding material data from which the characterization of the compliance function is developed. When the case of drying is specified, read also the characteristics for shrinkage and drying creep.

3. Compute the discrete values of the relaxation function for various strain durations and various ages at the start of relaxation. (This consists in a direct numerical solution of a linear Volterra integral equation.) (RELAX).

4. Compute the discrete values of the Maxwell chain moduli at all discrete ages (MAXWL 1).

5. As a check, compute discrete values of the compliance function from the discrete values of Maxwell chain moduli for various ages (CRCURV). Print the resulting values of the compliance function and of their deviations from the initially specified values of the compliance function. Also calculate and print the coefficient of variation of these deviations.

During the determination of the relaxation function, a subroutine for the compliance function is repeatedly called. This subroutine has three options to be specified by the user:

1. The compliance function is evaluated by interpolation from a given array of discrete values. The interpolation is linear in the logarithm of elapsed time and in the logarithm of the age at loading. For durations and ages falling outside the range, extrapolation is automatically used.

2. The compliance function is evaluated from a formula corresponding to the double power law [7].

3. If drying is specified, the compliance function is evaluated from a formula consisting of the double power law enhanced by a drying term [7].

The ultimate result obtained in the subroutine MATPAR is the array of discrete values of the moduli of the Maxwell chain. For any specified age, the values of the Maxwell chain moduli are computed (in subroutine function EMUF) from these discrete values by a linear interpolation in the logarithm of the age of concrete. For arguments falling outside the time range, a linear extrapolation from the two values at the end of the array is used.

INPUT OPTIONS FOR MATERIAL CHARACTERIZATION

In practical applications, many different situations can arise. Sometimes measured values of the creep function for many different times and ages at loading may be available, as is often the case in the design of nuclear structures. For other than special sensitive structures, the typical situation is that no measured creep data are available for the concrete to be used in the structure under design, and then some prediction formulas for creep and shrinkage need be employed, for which the BP model [2] is adopted here. Often, however, at least some experimental information may exist for the particular value of the elastic modulus, or even measured short-time creep deformations for one loading age or a few short load durations. Furthermore, even when full or partial experimental data may be available, they may be too scattered or uncertain, in which case a smoothing of the data by a realistic creep formula is appropriate. Similarly, the use of a creep formula is inevitable for the extrapolation of short-time creep data to long durations.

To treat the various possibilities just outlined, the following options for the material characterization are provided in the program MATPAR:

1. The compliance function is specified as an array of discrete values for various load durations and various ages at loading. No drying is considered.
2. Drying is considered, and the mean compliance function for the cross section is specified as in option 1. Also, the values of the mean shrinkage strain of the cross section are specified for various durations of drying and the given age at the start of drying.
3. The compliance function is given by the double power law, for which all of its five parameters are read. No drying is considered.
4. Same as option 3 but all double power law parameters except the 28-day elastic modulus are generated from the given strength and composition parameters of concrete (the mix ratio of water, cement, sand and gravel, the cement type, and the unit weight of concrete).
5. Same as option 4 except that the 28-day elastic modulus is also predicted from the strength and composition parameters.
6. Drying is specified and the compliance function is defined by the double power law enhanced by the term for the creep increase due to drying [2]. A formula for the shrinkage strain evolution in time (according to Ref. [2]) is also specified. And all parameters for these formulas are read.
7. Same as option 6 but all material parameters except the 28-day elastic modulus and the final shrinkage strain are predicted from the strength and the composition parameters of concrete (and also from the given ambient humidity).
8. Same as 6 but all parameters except the 28-day elastic modulus are predicted from the strength and composition of concrete.
9. Same as option 6 but all parameters are predicted from the strength and composition of concrete.
10. Two of the five double-power law parameters (namely, E_0 and φ_1, *see* Ref. [2]) are determined so as to obtain the best fit (in the least-square sense) of the given array of discrete values of the compliance function at various load durations and various ages. The remaining three double power law parameters are given. This option is used when the given array of discrete values for the compliance function is of limited range in time and/or age. No drying is considered in this option. As a check, the coefficient of variation for the deviations of the formula from the given limited array of values for the compliance function is automatically computed and printed.
11. Same as option 10 but the remaining three double power law parameters are not specified; they are predicted from given strength and composition of concrete.
12. Same as option 10 but drying is considered.
13. Same as option 11 but drying is considered.

The user selects his input option depending on the amount of information available to him before the analysis. If sufficient test data have been obtained, as is frequently done for nuclear concrete structures, then the measured discrete values of the compliance function should be used. However, if the range of these available data is too limited, it is preferable to approximate these values by the double power law.

If no experimental information is available, the double power law parameters are predicted from the strength and composition of concrete. However, if there exists some information on the double law parameters for a similar concrete, an adjustment of some of its parameters is appropriate. Moreover, if the short-time deformation (elastic modulus) is measured, it should be also used to improve the parameters of the double power law.

The choice of the proper input option has a great effect on the accuracy in representing the concrete properties.

OUTPUT AND APPLICATION

The ultimate result of the program is the subroutine EMUF which generates the values of the Maxwell chain moduli, and in case that drying is specified, also the subroutine SHRINK which generates the values of the mean shrinkage strain of the cross section. Subroutine EMUF may then be called from a structural analysis program performing a step-by-step numerical solution.

As an alternative in case of a structural system that does not involve too many degrees of freedom, one may prefer characterizing the creep properties in terms of the compliance function $J(t, t')$ and performing the structural analysis on the basis of the principle of superposition, which leads to integral equations in time. In that case, subroutines RELAX, MAXWL1, CRCURV, and EMUF can be discarded and the calls for these subroutines eliminated from the main program MATPAR.

CONCLUDING REMARKS

The present program would hopefully help increasing the accuracy of material representation in computer creep analysis. Available finite element programs make an accurate analysis of complex structures possible, but this possibility can be realized only if the material is characterized with a commensurate accuracy. It makes no sense to carry out a sophisticated finite element analysis and at the same time use some crude model for representing creep, a model of the type intended for hand calculations in a design office. In case of the analysis of concrete structures for creep and shrinkage, it is currently the material model which represents by far the greatest source of error. Thus, devoting more effort to material modeling than to the structural calculations has the greatest potential pay-off.

The salient feature of the present program is its adaptiveness to the amount of information supplied. If the user supplies very little information about his concrete, the program automatically predicts reasonable values of material parameters but the resulting prediction of creep is of course crude. Various options allow the user to supply more information with the advantage of better prediction of creep.

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RÉSUMÉ

Introduction des caractéristiques de fluage et de retrait dans un programme d'analyse structurale. — *On présente l'informatisation des propriétés de fluage et de retrait qui entre dans un programme d'analyse structurale. Le programme soit accepte les données numériques de la fonction d'adaptation plastique et de la fonction de retrait pour des durées de chargement et des âges au chargement discrets, soit utilise le récent modèle de fluage BP à loi de double puissance. La caractéristique la plus utile est qu'on dispose de treize options différentes pour spécifier les paramètres de fluage et de retrait : conditions d'étanchéité ou conditions de séchage, valeurs spécifiées de fluage et interpolations, paramètres d'une formule spécifiée de fluage ou formules de fluage telles qu'elles donnent automatiquement les valeurs de fluage, valeurs spécifiées pour de courtes durées et extrapolations à partir d'une équation, etc. Outre qu'il restitue la fonction d'adaptation, le programme permet aussi de calculer la fonction de relaxation et les modules élastiques en relation avec l'âge d'un modèle HN de Maxwell, soit l'équivalent des propriétés de fluage données. Le programme se prête particulièrement à l'introduction des caractéristiques de fluage pour une analyse par éléments finis à grande échelle.*

APPENDIX-FORTRAN IV listing of program MATPAR *

```
      PROGRAM MATPAR(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C*********************************************************************
C
C     GENERATES CREEP AND SHRINKAGE PROPERTIES FROM GIVEN DATA AND
C     CALCULATES DISCRETE VALUES OF RELAXATION MODULI EMU
C
C     INPUT
C      T1,TP1=SMALLEST LOAD DURATION AND AGE AT LOADING IN THE
C        STRUCTURAL PROBLEM.
C      TAU1=SMALLEST RELAXATION TIME MAY BUT NEED NOT BE GIVEN.
C      ATL,ATPL=ARRAYS OF LOG(T-TPRIME) AND LOG(TPRIME) FOR DEFINING
C        DISCRETE VALUES OF J, AND N1=NO. OF DISCRETE (TPRIME), N2= NO.
C        OF DISCRETE (T-TPRIME).
C      NDEC=NO. DECADES IN LOG(T-TPRIME) AND JDEC=NO. OF STEPS PER
C        DECADE (FOR COMPUTING RELAXATION FUNCTION IN SUBROUTINE RELAX ).
C      NA-NO. OF DECADES IN LOG(TPRIME) AND JA=NO. OF STEPS PER DECADE
C     FOR CHARACTERIZING THE CREEP COMPLIANCE FUNCTION.
C      W1,W2=WEIGHTS FOR FITTING MAXWELL CHAIN.
C     NI,NJ=NO. OF DURATIONS AND AGES FOR INPUT AND PRINT OF CREEP FUNC.
C     NMU=NO. OF MAXWELL UNITS IN THE CHAIN MODEL.
C     NTSHR=NO. OF DISCRETE VALUES TO SPECIFY THE SHRINKAGE VARIATION
C     IN TIME.
C      JOPT = OPTIONS FOR INPUT OF CREEP AND SHRINKAGE.
C      N1,N2=NO. OF TIMES FOR PRINTING CALCULATED CREEP FUNCTION AT
C        REGULAR INTERVALS.
C     PHYSICAL DIMENSIONS ARE TIMES IN DAYS , MODULI IN PSI.
C     CREEP FUNCTION(=COMPLIANCE) IN 1./PSI ,HOWEVER,FOR OPTIONS WHICH
C     DETERMINE NOTHING FROM COMPOSITION PARAMETERS(JOPT=1,2,3,6,10)
C     THEIR UNITS CAN BE USED INSTEAD OF PSI,THE CORRESPONDING DIMENSION
C     THEN APPLIES TO THE OUTPUT.
C
C     LIST OF INPUT OPTIONS
C     JOPT=1  J(T,TPRIME) GIVEN BY AN ARRAY, NO DRYING
C                   USE DATA CARDS A,B,C,D,E,F(NJ-TIMES).
C     JOPT=2  J(T,TPRIME) GIVEN BY AN ARRAY, WITH DRYING.
C                   USE DATA CARDS A,B,C,D,E,F(NJ-TIMES),G,H.
C     JOPT=3  J(T,TPRIME) GIVEN BY DOUBLE POWER LAW, NO DRYING, ALL
C     PARAMETERS GIVEN.
C                   USE DATA CARDS A,B,C,I.
C     JOPT=4  SAME AS 3 BUT ALL PARAMETERS EXCEPT E28 PREDICTED FROM
C     STRENGTH AND COMPOSITION (E28 GIVEN).
C                   USE DATA CARDS A,B,C,I,K.
C     JOPT=5  SAME AS 3 BUT ALL PARAMETERS PREDICTED FROM STRENGTH AND
C     COMPOSITION.
C                   USE DATA CARDS A,B,C,K.
C     JOPT=6  J(**) GIVEN BY DOUBLE POWER LAW WITH DRYING TERMS   AND
C     SHRINKAGE, ALL PARAMETERS IN THE FORMULAS APE GIVEN.
C                   USE DATA CARDS A,B,C,I,J.
C     JOPT=7  SAME AS 6 BUT ALL PARAMETERS EXCEPT E28 AND FINAL
C     SHRINKAGE PREDICTED FROM STRENGTH AND COMPOSITION
C                   USE DATA CARDS A,B,C,I,J,K,L
C     JOPT=8  SAME AS 6 BUT ALL PARAMETERS EXCEPT E28 PREDICTED FROM
C     STRENGTH AND COMPOSITION.
C                   USE DATA CARD A,B,C,I,J,K,L
C     JOPT=9  SAME AS 6 BUT ALL PARAMETERS PREDICTED FROM STRENGTH AND
C     COMPOSITION.
C                   USE DATA CARDS A,B,C,J,K,L
C     JOPT=10 BEST FIT OF DOUBLE POWER LAW TO LIMITED CREEP DATA INPUT
C     AS ARRAY AJ(I,J).  NO DRYING.CREEP FORMULA PARAMETERS OTHEP THAN
C     E0 AND PHI1 ARE GIVEN AS IN OPTION 3.
C                   USE DATA CARDS A,B,C,D,E,F(NJ-TIMES),I
C     JOPT=11 SAME AS 10 BUT PARAMETERS OTHER THAN E0 AND PHI1 ARE
C     DETERMINED FROM STRENGTH AND COMPOSITION AS IN OPTION 5.
C                   USE DATA CARDS A,B,C,D,E,F(NJ-TIMES),I,K
C     JOPT=12 SAME AS 10 BUT DRYING INCLUDED.
C                   USE DATA CARDS A,B,C,D,E,F(NJ-TIMES),G,H,I,J,K
C     JOPT=13 SAME AS 11 BUT DRYING INCLUDED.
C                   USE DATA CARDS A,B,C,D,E,F(NJ-TIMES),G,H,I,J,K,L
C     THE FOLLOWING DATA CARDS WERE USED IN TESTING THE PROGRAM
C     CARD A
C   9    5    5    7    4    7    6
C     CARD B
C    0.1        5.0         0.    0.01        0.05
C     CARD C
C     12.
C     CARD D
C        5  .01       1.0       10.0       100.0    3000.0
C     CARD E
C        3  7.0       90.0     730.0
C     CARDS F(3 CARDS IN THIS CASE)
C   .26E-6    .45E-6    .59E-6    .77E-6    1.1E-6
C   .18E-6    .29E-6    .37E-6    .47E-6    .71E-6
C   .12E-6    .20E-6    .26E-6    .33E-6    .48E-6
C     CARD G
C8  0.1    1.0        10.0       100.0      300.0      1000.0     3000.0     1
C     CARD H
C   11.E-6     35.E-6    108.E-6    300.E-6    424.E-6    526.E-6    572.E-6
C     CARD I
C     4.E6  1.5        0.135       0.33        .05
C     CARD J
C  14.   0.0006  0.5       1.       300.      10.     0.03      1.4       0.83
C     CARD K
C 50[illegible]     0.45       5.0        2.0        145.0            1
C     CARD L
C 320.0
C
C*********************************************************************
C
      COMMON/CRPAR/JOPT,E28,E0,PHI1,EXPN,EXPM,ALFA,DRYSTA,AVHUM,FINSHR,
     1ESHINF,TAUSH,PHID,CP,CD,AKS,VS,C7,C1T0,EPSKH,AKHPP,AKHP,EXPM2,
     2FKE,TAU10,TAU100,CDN,CPK,E01,CO
C
      COMMON/JFUNCT/AJ(16,8),ATL(16),ATPL(8),NI,NJ,AT(16),ATP(16),
     1SHR(16),TSHR(16),TLSHR(16),NTSHR
      COMMON/STARTV/T1,TP1,TAU1
      COMMON/MAXW/EMU(10,8),TPL(8),TP(8),NA
      DIMENSION DDEF(30),T(30),TT(30),ER(30,8),TAU(10)
C
C     INPUT OF PARAMETERS NEEDED FOR ANY INPUT OPTION JOPT
      WRITE(6,602)
 602  FORMAT(//19H0 INPUT DATA FOLLOW  )
      READ(5,3001) JOPT,NDEC,JDEC,NA,JA,NMU,NTSHR
C     FOR OPTIONS WITH NO DRYING USE SOME DUMMY VALUE,E.G. NTSHR=0
      WRITE(6,3002) JOPT,NDEC,JDEC,NA,JA,NMU,NTSHR
 3001 FORMAT(16I5)
 3002 FORMAT(//6H0JOPT=I2,5X5HNDEC=I2,5X5HJDEC=I2,5X3HNA=I2,5X3HJA=I2,
     15X4HNMU=I2 ,5X6HNTSHR=I2)
      IF(NDEC.GT.(NMU-2)) WRITE(6,3016)
 3016 FORMAT(79H0NO.OF RELAX TIMES IS INSUFFICIENT TO GIVE GOOD ACCURACY
     1OVER ENTIRE TIME RANGE   /28H SHOULD BE NMU .GE .(NDEC+2)     //)
      READ(5,3010)T1,TP1,TAU1,W1,W2
      WRITE(6,602)
      WRITE(6,3009) T1,TP1,TAU1,W1,W2
 3010 FORMAT(5F10.3)
 3009 FORMAT(/4H0T1=F12.5,8X4HTP1=F12.5,8X5HTAU1=F12.5,8X3HW1=F8.3,
     18X3HW2=F8.3/)
C     THERMAL PROPERTIES
      WRITE(6,602)
      READ(5,3003)ALFTEM
      WRITE(6,3004) ALFTEM
 3003 FORMAT(8F10.3)
C     COEFFICIENT OF THERMAL DILATATION USED ONLY IN SUBROUTINE TEMDIL.
C     IF THE USER REPLACES TEMDIL BY HIS OWN SUBROUTINE WITH VARIABLE
C     ALFTEM , THEN ANY DUMMY VALUE MAY BE USED HERE .
 3004 FORMAT(//31H0COEF. OF THERMAL DIL. ALFTEM= ,8F10.3)
C
C     CHECK DIMENSIONS
```

```
      IF(NA.LE.8.AND.NMU.LE.10) GO TO 33
      WRITE(6,32)
 32   FORMAT(//47H0NA OR NMU OR BOTH EXCEED DIMENSION OF EMU.STOP
      STOP
 33   CONTINUE
C
      T(1)  = T1
      TP(1) = TP1
      TT(1) = 0.
C
C     DETERMINE TIME STEP AND LAST DISCRETE TIME
C
      DTR  = 10.**(1./FLOAT(JDEC))
      NT   = NDEC*JDEC
      TMAX = T(1)*(DTR**(NT-1))
      WRITE(6,2995)NT,TMAX
 2995 FORMAT(//4H NT=I4,10X,5HTMAX=,E11.4/)
      IF(NT.LE.30)GO TO 72
      WRITE(6,71)
 71   FORMAT(///52H STOPPED IN MATPAR BECAUSE DIMENSIONS OF T,TT,ER ARE
     1,12H TOO SMALL.    ,/40H EITHER INCREASE DIMENSIONS OR DECREASE
     115H NDEC OR JDEC           //)
      STOP
 72   CONTINUE
C
C     CHECK CHOSEN FIRST RELAXATION TIME TAU1
C
      IERR = 0
      IF (TAU1 .EQ. 0.)  GO TO 50
      TAU(1) = TAU1
      IF (TAU(1) .LE. T(1)/0.3)  GO TO 60
   50 TAU(1) = T(1)/0.3
      IERR = 1
      WRITE(6,1005)
 1005 FORMAT(///5X,34H   ***   WARNING FROM MATPAR   ***)
      WRITE(6,1050)  TAU(1)
 1050 FORMAT(/5X,54H   SMALLEST RELAXATION TIME UNASSIGNED.CHOOSE - TAU1
     1= ,E10.3)
C
C     IMPLIED MAX. RELAXATION TIME
C
   60 TAUM = TAU(1)*(10.**(NMU-2))
      IF(TAUM.GE.0.5*TMAX.AND.IERR.EQ.0) GO TO 70
      WRITE(6,1005)
      WRITE(6,1060)  TAUM
 1060 FORMAT(/5X,36H   HIGHEST RELAXATION TIME : TAUM = ,E10.3)
      IF (IERR .EQ. 0)  GO TO 70
      TMI  = 0.5*TAU(1)
      TMA  =0.50*TAUM
      TPMI = 3.0*T1
      WRITE(6,1065)  TMI,TMA,TPMI
 1065 FORMAT(/5X,49H   NOTE : FOR GIVEN RELAXATION TIMES,CREEP CAN BE
     1/5X,28H   REPRESENTED ONLY FROM T1=,E10.3,10H  TO TMAX=,E10.3,
     1/5X,50H.  MAKE SURE THAT THE SMALLEST AGE TP1 IS .GE.    ,E10.3,
     1/5X,53H   OR ELSE AGING OF YOUNG CONCRETE DISTORTS LONG-TIME
     2/5X,11H   RESPONSE)
C     THE REMAINING RELAXATION TIMES TAU(MU) ARE SET IN MAXWL1
   70 CONTINUE
C
C     NOW INPUT FOR VARIOUS OPTIONS
C
      IF (JOPT .GT.2 .AND. JOPT.LT. 10) GO TO 605
C     HERE JOPT=1 OR 2, OR .GE.10.INPUT OF CREEP FUNCTION BY AN ARRAY
C     OF VALUES.
C     FOR OPTION JOPT=1 ARRAYS OF J-VALUES AND CORRESPONDING TIMES MUST
C     BE READ
C     HERE THE NUMBER OF ALL CARDS TO  READ MUST BE NJ+2.  HOWEVER, FOR
C     OPTIONS OTHER THAN JOPT=3 TO 9 THESE CARDS MUST BE OMITED.
C
      WRITE(6,603) JOPT
 603  FORMAT(//29H0 INPUT DATA FOR OPTION JOPT=  ,I3,2X,6HFOLLOW  /)
      WRITE(6,3013)
 3013 FORMAT(// 41HINPUT VALUES OF CREEP FUNCTION J PER PSI /)
      READ(5,3011)NI,(AT(I),I=1,NI)
 3011 FORMAT(I10, 7F10.1/(8F10.1))
      WRITE(6,3008) NI
 3008 FORMAT (/50H NO. OF DISCRETE TIMES FOR INPUT AND PRINT...NI = ,I3)
      READ(5,3011)NJ,(ATP(J),J=1,NJ)
      WRITE(6,3007) NJ
 3007 FORMAT(/50H NO. OF DISCRETE AGES FOR INPUT AND PRINT....NJ = ,I3/)
      WRITE(6,3012) (AT(I),I=1,NI)
 3012 FORMAT(//13X,6HAT(I)=,8F13.4/26X,8F13.4)
      DO 3040 J=1,NJ
      READ(5,3015) (AJ(I,J),I=1,NI)
 3015 FORMAT(8F10.1)
 3040 WRITE(6,3014) J,ATP(J),(AJ(I,J), I=1,NI)
 3014 FORMAT(5H ATP(,I2,2H)=F9.4,4X,8E13.4/21X8E13.4)
      IF(NI.LE.16.AND.NJ.LE.8) GO TO 3032
      WRITE(6,3031)
 3031 FORMAT(//48H0N1 OR NJ OR BOTH EXCEED DIMENSION OF AJ. STOP    )
      STOP
 3032 CONTINUE
C
C     CHECK RANGE OF T AND TP.  IT SHOULD NOT EXCEED RANGE OF AT AND ATP
C
C     CHECK STARTING VALUES OF  T,TP
C
      IF (T(1) .GE. 0.5*AT(1))  GO TO 10
      WRITE(6,1005)
      WRITE(6,1010)  T(1)
 1010 FORMAT(/5X,57H   STARTING TIME TOO SMALL FOR GIVEN CREEP DATA :  T
     11  = ,E10.3, 8X, 27H MUST EXTRAPOLATE(LEFTWARD))
C
   10 IF (TP(1) .GE. 0.5*ATP(1))  GO TO 20
      WRITE(6,1005)
      WRITE(6,1020)  TP(1)
 1020 FORMAT(/5X,57H   STARTING AGE TOO SMALL FOR GIVEN CREEP DATA :  TP
     11  = ,E10.3, 8X, 27H MUST EXTRAPOLATE(LEFTWARD))
   20 CONTINUE
C
      DO 901 I=1,NI
  901 ATL(I)=ALOG10(AT(I))
      DO 902 J=1,NJ
  902 ATPL(J)=ALOG10(ATP(J))
C
C     CHECK FINAL VALUES OF  T,TP
C
      IF (TMAX .LE. 2.*10.**ATL(NI) )  GO TO 30
      WRITE(6,1005)
      WRITE(6,1030)  TMAX
 1030 FORMAT(/5X,56H FINAL AGE IS BEYOND RANGE OF GIVEN CREEP DATA; TPM
     1AX = ,E10.3,8X,17H MUST EXTRAPOLATE  )
C
   30 XX    = 10.**(1./FLOAT(JA))
      TPMAX = TP(1)*(XX**(NA-1))
      IF (TPMAX .LE. 2.*10.**ATPL(NJ))  GO TO 40
      WRITE(6,1005)
      WRITE(6,1040)  TPMAX
 1040 FORMAT(/5X,56H FINAL AGE IS BEYOND RANGE OF GIVEN CREEP DATA; TPM
     1AX = ,E10.3,8X,17H MUST EXTRAPOLATE  )
   40 CONTINUE
C
C
  605 IF(JOPT .NE.2 .AND. JOPT.LT. 12) GO TO 610
C     HERE JOPT=2 OR 12 OR 13
      WRITE(6,606) JOPT
  606 FORMAT (//30H606 MUST BE JOPT=2 OR 12 OR 13, 3X5HJOPT=I2 //)
C
C     INPUT OF SHRINKAGE AS  AN ARRAY OF VALUES
```

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* Note that in all statements COMMON/CRPAR/the variable CITO contains the digit "zero", not the letter "0".

```
      WRITE(6,603)JOPT
      READ(5,3100) NTSHR,(TSHR(I),I=1,NTSHR)
      WRITE(6,3101) NTSHR, (TSHR(I),I=1,NTSHR)
 3100 FORMAT(I2,1X,F7.1,(7F10.1))
 3101 FORMAT(1X//23H GIVEN SHRINKAGE VALUES ,3X,6HNTSHR= ,I2//13H TIMES(
     1DAYS)= ,(8E14.4))
      READ(5,3105) (SHR(I),I=1,NTSHR)
      WRITE(6,3107)(SHR(I),I=1,NTSHR)
 3105 FORMAT(8E10.3)
 3107 FORMAT(13H0SHR.VALUES= 8E14.4/(15X8E14.4))
      DO 3109 I=1,NTSHR
 3109 TLSHR(I)=ALOG10(TSHR(I))
C
C
  610 IF ((JOPT . NE . 3 . AND . JOPT . NE . 4) . AND. (( JOPT . LT . 6
     1. OR  . JOPT . GT . 8) . AND .JOPT .LT. 10)) GO TO 620
C
C      FOR JOPT=3,4,6,7,8,10,12 FUNCTION J IS DEFINED BY DOUBLE POWER LAW
C      AND ITS COEFFICIENTS ARE SPECIFIED.  HOWEVER, FOR JOPT .EQ. 10 OR
C      12 USE SOME DUMMY VALUES FOR E28 AND E028 BECAUSE E0 AND PHI1 ARE
C      LATER REDEFINED, AND FOR JOPT=11 OR 13 PLACE A BLANK (DUMMY) CARD
C      FOR JOPT=8 OR 7 ONLY E28 IS NEEDED AND THE REST CAN BE BLANK SINCE
C      IT IS LATER REDEFINED BY INPUT FROM DATA CARD K .
C
      WRITE(6,603)JOPT
C
C      DATA INPUT FOR DOUBLE POWER LAW.
C      NOTE THAT ON DATA CARD THE EXPONENT OF E-FORMAT MUST BE RIGHT-
C      ADJUSTED.
C
      READ(5,4401) E28,E0E28,EXPN,EXPM,ALFA
      WRITE(6,4402)E28,E0E28,EXPN,EXPM,ALFA
 4401 FORMAT(E10.3,4F10.1)
 4402 FORMAT(1X///34H GIVEN DOUBLE POWER LAW PARAMETERS//1X22H28-DAY EL.
     1MODULUS E28=E15.5,4H PSI//5X,6HE0E28=,F8.4,5X,5HEXPN=F7.4,5X,
     15HEXPM=F7.4,5X,5HALFA=F8.5//)
      IF((JOPT.NE.3.AND.JOPT.NE.6).AND.JOPT.NE.10) GO TO 620
      E0=E28*E0E28
      PHI1=(E0E28-1.)/((0.1**EXPN)*(28.**(-EXPM)+ALFA))
      WRITE(6,4403)E0,PHI1
 4403 FORMAT(17H0 CALCULATED E0 = ,E15.5,10X,5HPHI1=,F8.5//)
      E01=1./E0
C
C
  620 IF((JOPT.LT. 6 .OR. JOPT .GT. 9).AND.JOPT.LT.12) GO TO 630
C
C      HERE JOPT=6 TO 9 OR 12 OR 13
C      FUNCTION J IS DEFINED AS DOUBLE POWER LAW, DRYING IS
C      CONSIDERED,AND SHRINKAGE AND DRYING CREEP IS GIVEN BY FORMULA.
C      DATA INPUT FOR SHRINKAGE AND CREEP TERM  DUE TO DRYING
C
      WRITE(6,623) JOPT
  623 FORMAT(//37H623   MUST BE JOPT=6 TO 9 OR 12 OR 13,3X,5HJOPT=I2//)
      CP=0.
      C7 =0.
      CD=0.
      PHID=0.
      AKS=0.
      WRITE(6,603)JOPT
      READ(5,4501) DRYSTA,FINSHR,AVHUM,AKS,VS,C7,PHID,CD,CP
 4501 FORMAT(9F8.1)
C      IF UNSURE ABOUT INPUT DATA USE ZEROS ON THE ABOVE CARD.
C      THE FOLLOWING AUTOMATIC ASSIGNMENTS ARE THEN MADE--.
      IF(AKS .EQ. 0.) AKS=1.
      IF(C7  .EQ .0.) C7 =10.
      IF(PHID .EQ. 0.) PHID=0.03
      IF(CD .EQ. 0.) CD=1.4
      IF(CP .EQ. 0.) CP=0.83
C
C      OPTIONS 8 AND 9-FOR C7,PHID,CP AND FINSHR ANY DUMMY VALUES CAN BE
C      USED SINCE THEY ARE LATER DETERMINED FROM COMPOSITION.
C      FOR OPTION 7 THE SAME EXCEPT FINSHR IS NOT LATER REDEFINED .
C
      WRITE (6,4502) DRYSTA,FINSHR,AVHUM,AKS,VS,C7,PHID,CD,CP
 4502 FORMAT(1X///55H GIVEN PARAMETERS FOR SHRINKAGE AND CREEP DUE TO DR
     1YING    ///14H DRYING START= ,F8.1,4HDAYS,10X,16HFINAL SHRINKAGE= ,
     1F9.6,10X,18HAVERAGE REL.HUM.=  ,F7.3//5X,4H KS=,F5.2,5X,17HVOL.-SU
     1RF. RATIO= ,F9.3,10HMILIMETERS ,9X,3HC7=,F12.5,14HMM-SQUARE/DAY  ,
     1//1X,5HPHID=,F7.3,5X,3HCD=,F7.4,5X,3HCP=,F7.4//)
C
  630 IF (JOPT .LT. 4 .OR. (JOPT .EQ.10 .OR. JOPT .EQ. 6)) GO TO 640
      IF (JOPT.EQ. 12) GO TO 640
C
C      PREDICT PARAMETERS OF DOUBLE POWER LAW WITHOUT DRYING TERMS FROM
C      STRENGTH AND COMPOSITION, USING FORMULAS OF BAZANT-PANULA (1978)
C      HERE JOPT=4 OR 5 OR 7 OR 8 OR 9 OR 11 OR 12 OR 13
C
      WRITE(6,632) JOPT
  632 FORMAT(//52H0    MUST BE JOPT=4 OR 5 OR 7 OR 8 OR 9 OR 11 TO 13
     1,3X,5HJOPT=,I2//)
      WRITE(6,603)JOPT
      READ(5,891) FCP,WC,AC,SC,RO,ITYPE
      WRITE (6,892) FCP,WC,AC, SC,RO,ITYPE
  891 FORMAT(5F10.1,I10)
  892 FORMAT(//1X,4HFCP=F10.3,5X3HWC=F10.3,5X3HAC=F10.3,5X3HSC=F10.3,
     15X3HRO=F10.3,11HPOUND/CU-FT,5X,6HITYPE=,I5)
      AG=1./(1.-SC/AC)
      A1=1.0
      IF(ITYPE .EQ. 3) A1=0.93
      IF(ITYPE .EQ. 4) A1=1.05
      ALFA=0.025/WC
      EXPM=0.28+1./(FCP*FCP)
      X=((2.1*AC)*(SC**(-1.4))+(0.1*FCP**1.5)*((WC**0.3333333)*AG**2.2))
     1*A1-4.
      IF(X .LE. 0.) GO TO 810
      EXPN=0.12+0.07/(1.+5130./(X**6))
      GO TO 811
  810 EXPN=0.12
  811  X=ALFA+28.**(-EXPM)
      PHI1=(0.5*(1000.**EXPN))/X
      IF(JOPT.EQ.5.OR.JOPT.EQ.9) GO TO 813
C      HERE JOPT=4 OR 7 OR 8 OR 11 OR 12 OR 13
      E0=1.5*E28
      GO TO 814
  813  Z1=(0.00005*(RO*RO))*FCP
      E0=1.E6/(0.09+0.59824/(Z1*Z1))
      E28=E0/(1.+(PHI1*(0.001**EXPN))*X )
C      FOR CHECK
      X=E0/E28
      WRITE(6,6789)X
 6789 FORMAT(//1X,10H CHECK IF  ,F8.4,27H .NE.1.5 -ERROR- CORRECT IT //)
C
  814  CONTINUE
C
      E01=1./E0
      WRITE(6,895)E0,PHI1,EXPN,EXPM,ALFA,E28
  895 FORMAT(///58H0DOUBLE POWER LAW PARAMETERS PREDICTED FROM FCP AND
     1COMP./ 4H0E0=E14.4, 5X,5HPHI1=F8.3, 5X,5HEXPN=F8.3, 5X,5HEXPM=F8.3
     1, 5X,5HALFA=F8.3,5X,4HE28= ,E14.4 /80H HOWEVER FOR JOPT=7 OR 8 THE
     1 ABOVE E28 IS NOT DETERMINED FROM COMPOSITION.                    /)
C
  640 IF (JOPT.LT. 7-.OR. (JOPT .GT. 9 .AND. JOPT .NE. 13))  GO TO 650
C
C      HERE JOPT=7 OR 8 OR 9 OR 13
C      PREDICT PARAMETERS FOR SHRINKAGE AND DRYING CREEP FROM STRENGTH
C      AND COMPOSITION.  DOUBLE POWER LAW PARAMETERS MUST ALREADY BE
C      ASSIGNED.
C
      WRITE(6,643) JOPT
  643 FORMAT(//35H643  MUST BE JOPT=7 OR 8 OR 9 OR 13 ,3X,5HJOPT=I2//)
      WRITE(6,603)JOPT
      READ(5,713)CEM
      WRITE (6,714) CEM
  713 FORMAT(1F10.1)
  714 FORMAT(16H0CEMENT CONTENT=F8.1,2X18HKG PER CUBIC METER)
C
C      PARAMETERS FOR SHRINKAGE
C
      C7=(0.125*WC)*CEM-12.
      IF(C7 .LT. 7.) C7=7.
      IF(C7 .GT. 21.) C7=21.
      GS=AC/SC-1.
      IF(JOPT .EQ. 7) GO TO 723
      Z=(1.25*SQRT(AC) + 0.5 * (GS*GS))*((((1.+ SC)/WC) **0.33333333)*
     1SQRT(FCP)) - 12.
      IF(Z .LT. 0.) Z=0.
      Y=1./(390.*(Z**(-4)) + 1.)
      FINSHR=(1210.-880.*Y)*1.E-6
      WRITE(6,603)JOPT
      WRITE(6,712) FINSHR,C7,GS
  712  FORMAT(//19H CALCULATED FINSHR= , E13.4,10X,3HC7=,E13.4,10X,3HGS=,
     1F7.3/)
C
C      PARAMETERS FOR DRYING  CREEP
C
  723 R=(56000.* ((FCP*(SC/AC))**0.3))*((GS**1.3)*((WC/(FINSHR*1.E6))
     1**1.5))-0.85
      IF (R .LE. 0.) GO TO 715
      U=1./(1.+0.7*(R**(-1.4)))
      PHID=0.008+0.027*U  <——— CD = 2.8 - 7.5 * EXPN
      GO TO 716
  715 PHID=0.008
  716 CP=0.83
      WRITE(6,718) PHID,CP
  718  FORMAT(//17H CALCULATED PHID=, F8.4,10X,3HCP=,F7.3/)
  650 IF(JOPT .LT. 6 .OR. (JOPT .GT. 9 .AND. JOPT .LT. 12)) GO TO 660
C
C      HERE JOPT=6 OR 7 OR 8 OR 9 OR 12 OR 13
C      CALCULATE SHRINKAGE AND CREEP PARAMETERS OF BAZANT-PANULA'S
C      FORMULAS (MATERIALS & STRUCTURES, VOL.11,1978,P.308).
C      PARAMETERS FOR SHRINKAGE
C
      WRITE(6,653) JOPT
  653 FORMAT (//42H0MUST BE JOPT=6 OR 7 OR 8 OR 9 OR 12 OR 13, 3X5HJOPT=
     1I2//)
      C1T0=C7*(.05+SQRT(6.3/DRYSTA))
      TAUSH=(600.*(AKS*(VS/75.)**2))*(10./C1T0)
      ESHINF=FINSHR*SQRT(1.16742*(.85+4./(DRYSTA + TAUSH)))
      EPSKH =ESHINF*(1.-AVHUM**3)
C
C      PARAMETERS FOR CREEP TERMS DUE TO DRYING
C
      AKHPP=1.-AVHUM**2
      AKHP=1.-AVHUM**1.5
      EXPM2=-EXPM*.5
      FKE=(AKHP/E0)*ESHINF * 1.E6
      TAU10=10.*TAUSH
      TAU100=100.*TAUSH
      CDN=-CD*EXPN
      CPK=CP*AKHPP
      E01=1./E0
      WRITE(6,603)JOPT
      XX=10.**(1./FLOAT(JA))
      DO 4009 IA=2,NA
 4009 TP(IA)=TP(IA-1)*XX
      DTR=10.**(1./FLOAT(JDEC))
      WRITE(6,3990)(TP(IA),IA=1,NA,2)
 3990 FORMAT(////13H0      TP(IA)=,8E12.3/(13X,8E12.3))
      WRITE(6,4000)
 4000 FORMAT(//70H0CREEP FUNCTION VALUES INTERPOLATED FROM GIVEN DISCRET
     1E ARRAY AJ(I,J)        /55H TO BE APPROXIMATED BY DOUBLE POWER LAW.
     1 UNITS (1./PSI)  /42H IT  T(IT)        AJ(IT,1)    AJ(IT,2)        )
      DO4020 IT=1,NT
      IF(IT.GT.1) T(IT)=T(IT-1)*DTR
      DO 4010 IA=1,NA,2
 4010 DDEF(IA)=CREEP(T(IT),TP(IA))
 4020 WRITE(6,4030)IT,T(IT),(DDEF(IA),IA=1,NA,2)
 4030 FORMAT(1X,I2,F10.3,10E12.3/(13X,10E12.3))
      WRITE(6,657) C1T0,TAUSH,ESHINF,EPSKH,AKHPP,AKHP,CDN,CPK
  657  FORMAT(//6H C1T0=,E12.4,5X,6HTAUSH=,E12.4,5X,7HESHINF=,E12.4,5X,
     16HEPSKH=,E12.4/7H AKHPP=,E12.4,5X,5HAKHP=,E12.4,5X,4HCDN=,F8.4,
     15X,4HCPK=,F8.4/)
  660 IF(JOPT .LT. 10) GO TO 670
C
C      JOPT .GT. 10.  MODIFY E0 AND PHI1 IN DOUBLE POWER LAW TO GIVE BEST
C      FIT OF LIMITED DATA SET AJ(I,J)
C
      WRITE(6,603) JOPT
      E01=1.
      PHI1=1.
      A12=0.
      A22=0.
      B1=0.
      B2=0.
      A11=NI*NJ
      DO 672 J=1,NJ
      DO 672 I=1,NI
      XIJ=CREEPB(AT(I),ATP(J))-1.
      Y=AJ(I,J)
      IF(JOPT.GT.11) Y=Y-DRTERM(AT(I),ATP(J))
      A12=A12+XIJ
      A22=A22+XIJ*XIJ
      B1=B1+Y
  672  B2=B2+Y*XIJ
      DET=A11*A22-A12*A12
      E01=(B1*A22-B2*A12)/DET
      E0=1./E01
      PHI1=E0*((A11*B2-A12*B1)/DET)
      WRITE(6,671) E0,PHI1
  671  FORMAT(//49H0JOPT.GT.10 OPTIMIZED FROM GIVEN DISCRETE AJ, E0=   ,
     1E15.5,5X,5HPHI1=,F8.4,22H(FOR DOUBLE POWER LAW)  //)
  670 CONTINUE
C
C      PRESCRIBED STRAIN INCREMENTS FOR RELAXATION
C
      DDEF(1) = 1.
      DO 110 I=2,NT
  110 DDEF(I) = 0.
      WRITE(6,9999)
 9999 FORMAT(//53H0 END OF INPUT AND OF GENERATION OF INPUT PARAMETERS )
C
C      COMPUTE DISCRETE VALUES  ER  OF THE RELAXATION FUNCTION
C
      CALL RELAX(NDEC,JDEC,JA,DDEF,T,TT,NT,ER)
C
C      COMPUTE DISCRETE VALUES OF ELASTIC MODULI EMU OF MAXWELL CHAIN
C
      CALL MAXWL1(ER,T,TT,NT,W1,W2,TAU,  NMU)
C
C      IN DEBUGGING ARGUMENT NDEC AND JDEC AFTER TPL IN MAXWL1 WERE
C      REMOVED.  CHECK AGAIN LATER.
C
      WRITE(6,1100)
      WRITE(6,1110)  (TAU(MU),MU=1,NMU)
C
 1110 FORMAT(/5X,10(E10.3,2X))
 1100 FORMAT(///23H      RELAXATION TIMES :)
```

```
C     NOW PRINT SHRINKAGE VALUE AS PRESCRIBED ABOVE.
      IF(JOPT.LT.6)STOP
      IF((JOPT.GT.9).AND.(JOPT.LT.12))STOP
      WRITE(6,1415)
 1415 FORMAT(//39HOPRINT SHRINKAGES VALUES AS PRESCRIBED  //
     144X,9HAGE(DAYS) ,11X,6HSTRAIN /)
      DO 1420 IT=1,NT
      A2=T(IT) +DRYSTA
      B2=SHRINK(A2)
 1420 WRITE(6,1425)A2,B2
 1425 FORMAT(1X,39X,F12.2,5X,F12.6)
      STOP
      END

      FUNCTION CREEP(X,Y)
C
      COMMON/CRPAR/JOPT,E28,EO,PHI1,EXPN,EXPM,ALFA,DRYSTA,AVHUM,FINSHR,
     1ESHINF,TAUSH,PHID,CP,CD,AKS,VS,C7,C1TO,EPSKH,AKHPP,AKHP,EXPM2,
     2FKE,TAU10,TAU103,CDN,CPK,E01,CO
C
      IF(JOPT .GT. 2) GO TO 10
      CREEP=CREEPA(X,Y)
      RETURN
   10 CREEP = CREEPB(X,Y)
      RETURN
      END

      FUNCTION CREEPA(X,Y)
C
C***********************************************************************
C
C     CREEP RETURNS THE CREEP COMPLIANCE FUNCTION.  J(X+Y,Y)=CREEP(X,Y)
C     IS EVALUATED BY DOUBLE LINEAR INTERPOLATION OF DISCRETIZIZED DATA
C     AJ  IN LOG-SCALE. INTERPOLATION OR EXTRAPOLATION IS DONE LINEARLY
C     IN LOGARITHMIC SCALES OF TIMES AT LOADING AND LOAD DURATIONS
C
C        X    : TIME ELAPSED FROM LOADING
C        Y    : AGE AT LOADING
C
C     INPUT
C      AJ(16,8)=DISCRETE VALUES OF COMPLIANCE FUNCTION
C      ATL(K)=VALUES OF DEC. LOGARITHMS OF LOAD DURATIONS
C      ATPL(L)=VALUES OF DEC. LOGARITHMS OF AGES AT LOADING
C      N1=NO. OF AGES AT LOADING
C      N2=NO. OF LOAD DURATIONS
C
C     AFTER BAZANT, MATERIALS AND STRUCTURES (PARIS) VOL.5,NO. 7, 1972
C     P.139
C
C***********************************************************************
C
      COMMON/JFUNCT/AJ(16,8),ATL(16),ATPL(8),NI,NJ,AT(16),ATP(16),
     1SHR(16),TSHR(16),TLSHR(16),NTSHR
C
      TMIN = 10.**(ATL(1))
      YL = ALOG10(Y)
      L = 1
   10 L = L + 1
      A = ATPL(L) - YL
      IF (A .LT. 0. .AND. L .LT. NJ)  GO TO 10
      AA = YL - ATPL(L-1)
      IF (X .GT. TMIN)  GO TO 20
      CREEPA= (AJ(1,L-1)*A + AJ(1,L)*AA)/(ATPL(L) - ATPL(L-1))
      RETURN
C
   20 XL = ALOG10(X)
C
      K = 1
   30 K = K + 1
      B = ATL(K) - XL
      IF (B .LT. 0. .AND. K .LT. NI)  GO TO 30
      BB = XL - ATL(K-1)
      CREEPA= ((AJ(K-1,L-1)*A + AJ(K-1,L)*AA)*B +
     1         (AJ(K,L-1)*A  + AJ(K,L)*AA)*BB)/
     2         ((ATL(K) - ATL(K-1))*(ATPL(L) - ATPL(L-1)))
C
  100 RETURN
      END

      FUNCTION CREEPB(X,Y)
C
C***********************************************************************
C
C     DETERMINES CREEP FUNCTION J FROM DOUBLE POWER LAW WITHOUT OR WITH
C     DRYING TERMS.
C     Y=T-PRIME=AGE, X=T MINUS T-PRIME=LOAD DURATION.
C
C***********************************************************************
C
      COMMON/CRPAR/JOPT,E28,EO,PHI1,EXPN,EXPM,ALFA,DRYSTA,AVHUM,FINSHR,
     1ESHINF,TAUSH,PHID,CP,CD,AKS,VS,C7,C1TO,EPSKH,AKHPP,AKHP,EXPM2,
     1FKE,TAU10,TAU100,CDN,CPK,E01,CO
C
      IF(JOPT .LT. 3) WRITE(6,99) JOPT
   99 FORMAT(6HOJOPT=I5,6H=WRONG)
C     DOUBLE POWER LAW (BAZANT-PANULA, MATERIALS AND STRUCTURES, 1978)
      CO = (PHI1* ((ALFA+Y**(-EXPM))*X**EXPN))*E01
      CREEPB=E01+CO
C     ADD THE DRYING TERMS
      IF(JOPT .GT. 5 .AND. (JOPT .NE. 10 .AND. JOPT .NE. 11)) CREEPB=
     1CREEPB+DRTERM(X,Y)
      RETURN
      END

      FUNCTION DRTERM(X,Y)
C
C***********************************************************************
C
C     CALCULATE DRYING TERM TO BE ADDED TO DOUBLE POWER LAW.
C
C***********************************************************************
C
      COMMON/CRPAR/JOPT,E28,EO,PHI1,EXPN,EXPM,ALFA,DRYSTA,AVHUM,FINSHR,
     1ESHINF,TAUSH,PHID,CP,CD,AKS,VS,C7,C1TO,EPSKH,AKHPP,AKHP,EXPM2,
     2FKE,TAU10,TAU100,CDN,CPK,E01,CO
C
      PHIDP1=SQRT(1.+(Y-DRYSTA)/TAU10)/PHID
      CAPCD=((Y**EXPM2)/PHIDP1)*(FKE*(1.+TAU10/X)**CDN)
      Z=X+Y-DRYSTA
      IF(Z.LT.1.E-6) Z=1.E-6
      DRTERM=CAPCD-(CPK*CO)*((1.+TAU100/Z)**(-EXPN))
      RETURN
      END

      FUNCTION SHRINK(T)
C
C***********************************************************************
C
C     COMPUTES THE SHRINKAGE STRAIN FOR GIVEN DRYING DURATION T.
C
C***********************************************************************
C
      COMMON/CRPAR/JOPT,E28,EO,PHI1,EXPN,EXPM,ALFA,DRYSTA,AVHUM,FINSHR,
     1ESHINF,TAUSH,PHID,CP,CD,AKS,VS,C7,C1TO,EPSKH,AKHPP,AKHP,EXPM2,
     2FKE,TAU10,TAU100,CDN,CPK,E01,CO
C
      IF((JOPT .EQ. 2).OR.(JOPT.GT.11))GO TO 10
      IF (JOPT .LT. 6 .OR. JOPT . EQ. 10) WRITE(6,11) JOPT
   11 FORMAT(6HOJOPT=I5,17H  = WRONG.CORRECT)
      SHRINK =  EPSKH*SQRT(T/(TAUSH+T))
      RETURN
   10 SHRINK = SHRDAT(T)
      RETURN
      END

      FUNCTION SHRDAT(T)
C
C***********************************************************************
C
C     INTERPOLATES OR EXTRAPOLATES THE VALUES OF SHRINKAGE FROM A
C     PRESCRIBED ARRAY SHR OF SHRINKAGE VALUES
C
C***********************************************************************
C
      COMMON/JFUNCT/AJ(16,8),ATL(16),ATPL(8),N1,N2,AT(16),ATP(16),
     1SHR(16),TSHR(16),TLSHR(16),NTSHR

      TL=ALOG10(T)
      L=1
   10 L=L+1
      A=TLSHR(L)-TL
      IF (A .LT. 0 .AND. L .LT. NTSHR) GO TO 10
      AA=TL-TLSHR(L-1)
      SHRDAT=(SHR(L-1)*A+SHR(L)*AA)/(TLSHR(L)-TLSHR(L-1))
      RETURN
      END

      FUNCTION TEMDIL(TEM)
C
C***********************************************************************
C
C     COMPUTES THERMAL DILATION
C
C***********************************************************************
C
      COMMON/ALFT/ALFTEM
C
      TEMDIL=ALFTEM*TEM
      RETURN
      END

      SUBROUTINE RELAX(NDEC,JDEC,JA,DDEF,T,TT,NT,ER)
C
C***********************************************************************
C
C     RELAX  COMPUTES STRESS RESPONSE TO A GIVEN STRAIN HISTORY ,
C     IN GENERAL , AND DISCRETE VALUES  ER  OF THE RELAXATION FUNCTION ,
C     IN PARTICULAR.
C
C     FUNCTION CREEP(X,Y) MUST BE SUPPLIED.  CREEP(X,Y)=COMPLIANCE FUNC-
C     TION J(X+Y,Y) ,  WHERE Y=AGE AT LOADING,X=LOAD DURATION(T-TPRIME),
C      X+Y=CURRENT AGE.
C     CAUTION...NOTE THAT IN SOME OTHER PROGRAMS ARGUMENT
C     X IS USED FOR THE AGE.
C
C     INPUT
C      NDEC=NO. OF DECADES OF TIME IN LOG-SCALE
C      JDEC=NO. OF STEPS PER LOG(10)
C      NA=NO. OF AGES AT LOADING
C      JA=NO. OF AGES PER LOG(10)
C      T(I)=FIRST DISCRETE TIME
C      TP(I)=FIRST AGE AT LOADING
C      DDEF(IT)=PRESCRIBED STRAIN INCREMENT PER STEP FROM IT-1 TO IT
C        MINUS PRESCRIBED SHRINKAGE INCREMENT PER STEP
C      (FOR RELAXATION SET DDEF(I)=1., ALL OTHER DDEF(IT)=0.) TT(I)=0.
C
C     OUTPUT
C      ER(IT,IA)=DISCRETE VALUES OF RELAXATION FUNCTION
C      T(IT)=CORRESPONDING DISCRETE TIMES FROM THE INSTANT OF IMPOSING
C        UNIT STRAIN AND TP(IA)=DISCRETE AGES AT THIS INSTANT
C      NT=NO. OF DISCRETE TIMES FOR EACH AGE AT LOADING
C
C     DIMENSIONS MUST BE AT LEAST DSTR(NT),DDEF(NT),T(NT),TT(NT),TP(NA),
C        TPL(NA),ER(NT,NA)
C
C
C     FROM - BAZANT AND ASGHARI - CEMENT AND CONCRETE RESEARCH,BUT TWO
C     MISPRINTS IN PUBLISHED LISTINGS ARE CORRECTED HERE VOL.4,1974,
C     PP.567-579.
C
C***********************************************************************
C
      COMMON/MAXW/EMU(10,8),TPL(8),TP(8),NA
      DIMENSION DDEF(30),T(30),TT(30),ER(30,8),DSTR(30)
C
      XX = 10.**(1./FLOAT(JA))
      DO 10 IA=2,NA
      TP(IA)  = TP(IA-1)*XX
   10 TPL(IA) = ALOG10(TP(IA))
      TPL(1)  = ALOG10(TP(1))
C
      DTR = 10.**(1./FLOAT(JDEC))
      NT   = NDEC*JDEC
      DO 20 IT=2,NT
      T(IT)   = T(IT-1)*DTR
   20 TT(IT) = SQRT(T(IT)*T(IT-1))
C
      WRITE(6,1000) (TP(IA),  IA=1,NA)
C
      DO 50 IA=1,NA
C
C     COMPUTE THE FIRST STRESS VALUE USING EFFECTIVE MODULUS.
C
      DSTR(1)   = DDEF(1)/CREEP(T(1),TP(IA))
      ER(1,IA) = DSTR(1)
C
C     DEPP = INELASTIC STRAIN INCREMENT FROM JT-1 TO JT
C     DSTR(JT) = STRESS INCREMENT FROM JT-1 TO JT
C
      DO 40 IT=2,NT
      I1 = IT - 1
      DEPP = 0.
      DO 30 JT=1,I1
   30 DEPP = DEPP + DSTR(JT)*(CREEP(T(IT)-TT(JT),TP(IA)+TT(JT)) -
     1CREEP(T(I1)-TT(JT),TP(IA)+TT(JT)))
      DSTR(IT) = (DDEF(IT) - DEPP)/CREEP(T(IT)-TT(IT),TP(IA)+TT(IT))
   40 ER(IT,IA) = ER(I1,IA) + DSTR(IT)
   50 CONTINUE
      WRITE(6,1010)
C
      DO 60 IT=1,NT
   60 WRITE(6,1020)  IT, T(IT)  ,(ER(IT,IA),  IA=1,NA)
C
C
 1000 FORMAT(50HOTABLE OF CALCULATED RELAXATION FUNCTION ER(IT,IA)/7X7HT
     1P(IA)=10E12.4/(14XE12.4))
 1010 FORMAT(41HOIT     T(IT)      ER(IT,1)     ER(IT,2) ...)
 1020 FORMAT(1XI2,F11.3,10E12.4/(13X10E12.4))
C
  100 RETURN
      END

      SUBROUTINE MAXWL1(ER,T,TT,NT,W1,W2,TAU,NMU)
C
C***********************************************************************
C
C     MAXWL1  COMPUTES RELAXATION SPECTRA (VALUES OF ELASTIC MODULI  EMU
C     IN MAXWELL CHAIN) FOR VARIOUS AGES AT LOADING TP(IA).
```

```
C
C     CALCULATES DISCRETE VALUES EMU(MU,IA) FOR VARIOUS RELAXATION
C     TIMES TAU(MU) AND AGES TP(IA).
C     INPUT
C      ER(IT,IA),T(IT),TT(IT),TP(IA),TPL(IA),NDEC,JDEC,NT,NA-ALL FROM
C         SUBROUTINE RELAX(SEE DEFINITIONS THERE).
C      W1,W2=CHOSEN WEIGHTS(SUITABLE VALUES W1=0.01,W2=0.08)
C      NMU=NO. OF RELAX. TIMES TAU(MU).
C     OUTPUT
C      EMU(MU,IA)=MAXWELL CHAIN MODULI FOR TAU(MU) AT AGES TP(IA)
C      TAU(MU)=RELAXATION TIMES OF MAXWELL CHAIN
C
C     (FROM : BAZANT AND ASGHARI : CEMENT AND CONCRETE RESEARCH,
C                                  VOL.4,NO.4,1974,PP.567-579)
C
C***********************************************************************
C
      COMMON/MAXW/EMU(10,6),TPL(8),TP(8),NA
      DIMENSION ER(30,8),T(30),TT(30),TAU(10)
      DIMENSION NK(4),D(8),CE(4),AAA(10),MUEX(10),IXEX(10)
      DIMENSION A(8,8),WKAREA(8),B(8,1),NJ(8),A4(4,4),B4(4,1)
C
      WRITE(6,777)W1,W2
 777  FORMAT(//32H WEIGHTS USED IN MAXWL1 ARE W1=  ,F8.4,4X,7HAND W2=  ,
     1F8.4//)
C
C     CHOOSE RELAXATION TIMES  TAU
C
      NMU = NMU - 1
      IF (TAU(1) .EQ. 0.)  TAU(1) = T(1)/0.3
      DO 10 MU=2,NMU
   10 TAU(MU)  = TAU(MU-1)*10.
      TAU(NMU) = 1.E30
C
C     PRINT RELAXATION TIMES
C
      WRITE(6,401)(TAU(MU),MU=1,NMU)
  401 FORMAT(//26H0RELAXATION TIMES TAU(MU)=7E14.4//)
C
C     COMPUTE COEFFICIENTS  A  AND  B  OF LEAST SQUARE METHOD
C
      DO 100 IA=1,NA

      DO 50 NU=1,NMU
      DO 30 MU=1,NMU
      SUM = 0.
      DO 20 IT=1,NT
      X=T(IT)/TAU(MU)+T(IT)/TAU(NU)
 20   IF(X.LE.50.) SUM=SUM+EXP(-X)
   30 A(NU,MU) = SUM
      SUM      = 0.
      DO 40 IT=1,NT
      X=T(IT)/TAU(NU)
 40   IF(X.LE.50.) SUM=SUM+ER(IT,IA)*EXP(-X)
   50 B(NU,1) = SUM
C
C     ENFORCE A SMOOTH SPECTRUM BY ADDING WEIGHTS  W1,W2  ON 1-ST
C     AND 2-ND DIFFERENCES OF  EMU  AS FUNCTION OF  MU
C
      W22     = 2.*W2
      W24     = 4.*W2
      A(1,1) = A(1,1) + W1
      A(1,2) = A(1,2) - W1
      A(2,1) = A(2,1) - W1
      A(2,2) = A(2,2) + W1
      NMU1    = NMU
      NMU2    = NMU - 2
      NMU3    = NMU - 3
C
      DO 60 NU=1,NMU3
      A(NU,NU) = A(NU,NU) + W2
      MU       = NU + 1
      A(NU,MU) = A(NU,MU) - W22
      MU       = NU + 2
   60 A(NU,MU) = A(NU,MU) + W2
C
      DO 70 NU=2,NMU2
      MU       = NU - 1
      A(NU,MU) = A(NU,MU) - W22
      A(NU,NU) = A(NU,NU) + W1 + W24
      MU       = NU + 1
   70 A(NU,MU) = A(NU,MU) - W1 - W22
C
      DO 80 NU=3,NMU1
      MU       = NU - 2
      A(NU,MU) = A(NU,MU) + W2
      MU       = NU - 1
      A(NU,MU) = A(NU,MU) - W1 - W22
   80 A(NU,NU) = A(NU,NU) + W1 + W2
C
C     SOLVE THE EQUATIONS BY A LIBRARY SUBROUTINE  LEQT1F.
C
      CALL LEQT1F(A,1,NMU,8,B,0,WKAREA,IERR)
C
C     ARG. NO. 2 - NO. OF RHS
C     ARG. NO. 3 - NO. OF EQS.
C     ARG. NO. 4 - DIMENSION OF MATRIX A
C     ARG. NO. 5 - RHS, ON OUTPUT THE SOLUTION
C     ARG. NO. 6 - 0 FOR NO ACCURACY TEST
C     ARG. NO. 7 - WORK AREA, DIMENSION .GE. NO. OF EQS.
C     ARG. NO. 8 - IERR = 128 + N, N = 1 SINGULAR.
C
      DO 90 MU=1,NMU
   90 EMU(MU,IA) = B(MU,1)
C
  100 CONTINUE
C
      WRITE(6,1000)  T(1),T(NT)
      DO 110 IA=1,NA
  110 WRITE(6,1010)  IA,TP(IA), (EMU(MU,IA),  MU=1,NMU)
C
C     AS A CHECK , COMPUTE CREEP FUNCTION VALUES FROM  EMU
C
      CALL CRCURV(T,TT,TAU,NT,NMU   )
      RETURN
C
 1000 FORMAT(43H0MAXWELL CHAIN MODULI FOR TIME RANGE FROM  E11.3,3H TO ,
     1E11.3,12H (UNITS PSI)/40H0IA   TP(IA)   EMU(1,IA)  EMU(2,IA) ... )
C                 USE DATA CARDS A,B,C,K,L,M
 1010 FORMAT(1XI2,F10.3,(8E11.3))
      END

      SUBROUTINE CRCURV(T,TT,TAU,NT,NMU)
C
C***********************************************************************
C
C     CRCURV  COMPUTES DISCRETE VALUES  BJ OF CREEP COMPLIANCE
C     FUNCTION AJ FROM RELAXATION MODULI OF MAXWELL CHAIN.
C     (FROM : BAZANT AND ASGHARI : CEMENT AND CONCRETE RESEARCH,
C                                  VOL.4,NO.4,1974,PP.567-579)
C
C     INPUT-T,TT,TP,NT,NA,TAU,MMU,NMU, FROM SUBROUTINES RELAX AND
C     MAXWL1 (SEE DEFINITIONS THERE)
C
C     OUTPUT-VALUES BJ(IT,IA) OF CREEP FUNCTION CALCULATED FROM MAXWELL
C     CHAIN, TO BE COMPARED WITH ORIGINALLY GIVEN VALUES DJ(IT,IA)
C
C***********************************************************************
C
      COMMON/MAXW/EMU(10,8),TPL(8),TP(8),NA
      DIMENSION T(30),TT(30),TAU(10),CE(4),AAA(10),MUEX(10),IXEX(10)
      DIMENSION ELAM(10),SMU(10),EX(10),BJ(30,8),DJ(30,8)
      IF(NT.GT.30)GO TO 902
      IF(NA.GT.8)GO TO 902
      IF(NMU.GT.10)GO TO 902
      GO TO 904
 902  WRITE(6,903)
 903  FORMAT(62H0MESSAGE FROM CRCURV..NT OR NA OR NMU EXCEEDS DIMENSION.
     1 STOP       )
      STOP
 904  CONTINUE
C
C     LOOP OVER CREEP CURVES FOR VARIOUS AGES  TP(IA)  AT LOADING
C
      DO 300 IA=1,NA
C
C     NOW INITIALIZE VARIABLES.
C
      DEF = 0.
      DO 10 MU=1,NMU
   10 SMU(MU) = 0.
      DT      = T(1)
C
C     LOOP OVER DISCRETE TIMES  T(IT)  FROM LOADING
C
      DO 200 IT=1,NT
C
      IF (IT .GT. 1)  DT = T(IT) - T(IT-1)
      EPP = 0.
      SUM = 0.
C
      DO 100 MU=1,NMU
C
      X       = DT/TAU(MU)
      IF(X.LT.50.0) GO TO 9031
      EX(MU)=0.0
      GO TO 9041
 9031 EX(MU) = EXP(-X)
 9041 BD      = 1. - EX(MU)
      IF(X.LT.1.E-7) GO TO 9045
      BDA     = BD/X
      GO TO 9046
C
C     WHEN  X  TENDS TO ZERO , LIMIT(BDA) = 0./0.
C     COMPUTE  BDA  FROM TAYLOR SERIES ABOUT X = 0.
C
 9045 BDA=1.-(0.5-0.16666667*X)*X
 9046 ELAM(MU) = BDA*EMUF(MU,TT(IT)+TP(IA))
      EPP      = EPP + ELAM(MU)
      SUM      = SUM + BD*SMU(MU)
C
  100 CONTINUE
C
      IF (IT .EQ. 1)  SUM = SUM + 1.
C
C     HERE 1.=VALUE OF STRESS INCREMENT AT INSTANT OF LOADING (ALL LATER
C     INCREMENTS = 0.).  DE=STRAIN INCREMENT, DEF=STRAIN, SMU(MU)=STRESS
C     IN SPRING NO. MU, BDA=LAMBDA, EPP=E-DOUBLE PRIME, DT=TIME STEP.
C
      DE          = SUM/EPP
      DEF         = DEF + DE
      BJ(IT,IA) = DEF
      DO 150 MU=1,NMU
C
  150 SMU(MU) = SMU(MU)*EX(MU) + ELAM(MU)*DE
  200 CONTINUE
C
  300 CONTINUE
C
      WRITE(6,990)(TP(IA),IA=1,NA  )
 990  FORMAT(////13H0      TP(IA)= 8E12.3/(13X,8E12.3))
      WRITE(6,1000)
 1000 FORMAT(61H0CREEP FUNCTION VALUES COMPUTED FROM MAXWELL CHAIN MODUL
     1I EMU/42H IT  T(IT)        AJ(IT,1)    AJ(IT,2)  ...)
      DO 310 IT=1,NT
  310 WRITE(6,1010) IT,T(IT), (BJ(IT,IA),  IA=1,NA  )
 1010 FORMAT(1XI2,F10.3,10E12.3/(13X10E12.3))
      WRITE(6,1020)
 1020 FORMAT(50H0COMPARE CREEP FUNCTION VALUES AS ORIGINALLY GIVEN/)
C
      WRITE(6,993)
  993 FORMAT(97H HERE T(IT)= CURRENT TIME MINUS T-PRIME=STRESS DURATION
     1S, TP(IA) = T-PRIME = AGE AT LOADING                            /)
C
      DO 330 IT=1,NT
      DO 320 IA=1,NA
  320 DJ(IT,IA) = CREEP(T(IT),TP(IA))
  330 WRITE(6,1010) IT,T(IT), (DJ(IT,IA),  IA=1,NA  )
C
C     CALCULATE DEVIATIONS FROM GIVEN CREEP FUNCTION AND STATISTICS.
C
      J=0
      X=0.0
      DO 410 IT=1,NT
      DO 410 IA=1,NA
      J=J+1
 410  X=X+DJ(IT,IA)
      X=X/J
      WRITE(6,2010)
 2010 FORMAT(47H0 DEVIATIONS DIVIDED  BY MEAN OF GIVEN J VALUES/)
      SUM=0.0
      DO 430 IT=1,NT
      DO 420 IA=1,NA
C     DEVIATIONS
      DE=BJ(IT,IA)-DJ(IT,IA)
      SUM=SUM+DE*DE
 420  BJ(IT,IA)=DE/X
 430  WRITE(6,2015) IT,T(IT),(BJ(IT,IA),IA=1,NA)
 2015 FORMAT(1X,I2,F10.3,10F12.5/(13X,10F12.5))
      DE=(SQRT(SUM/J))/X
      WRITE(6,2020)DE
 2020 FORMAT(51H0COEFF. OF VARIATION FOR THE DEVIATIONS FROM GIVEN ,
     111HJ-VALUES IS  ,F8.4/)
  500 RETURN
      END

      FUNCTION EMUF(MU,TPA)
C
C***********************************************************************
C
C     EMUF RETURNS THE ELASTIC MODULUS EMU OF UNIT NO. MU IN THE MAXWELL
C     CHAIN, CORRESPONDING TO AGE TPA.
C
C***********************************************************************
C
      COMMON/MAXW/EMU(10,8),TPL(8),TP(8),NA
C
      TPLA = ALOG10(TPA)
      DO 20 I =2,NA
      IA=I
      A=TPL(IA)-TPLA
      IF(A)20,30,30
 20   CONTINUE
 30   JA=IA-1
      EMUF=(EMU(MU,JA)*A+EMU(MU,IA)*(TPLA-TPL(JA)))/(TPL(IA)-TPL(JA))
C
      RETURN
      END
```