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WEAKLY SINGULAR INTEGRAL FOR CREEP RATE OF CONCRETE

Zdeněk P. Bažant and Tatsuya Tsubaki
Department of Civil Engineering, Northwestern University,
Evanston, Illinois 60201, U.S.A.

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Introduction

Despite decades of intensive research, an entirely satisfactory constitutive relation for creep in concrete is still unavailable, even for working stress levels [1]. The rate-type creep laws with internal variables [2], although rather effective for numerical creep analysis of large structural systems, seem to be incapable of describing the aging phenomena without at least partially violating some thermodynamic requirements [2]. This suggests that, for a complete description of creep in concrete, an integral-type law might be inevitable.

Further challenge is presented by the recently discovered nonlinearities at working stress levels consisting mainly of delayed stiffening of response due to sustained compressive preload [3,4,5,6]. So far, these nonlinearities, which may be called adaptation, have been modeled by modifying the linear aging creep law based on the principle of superposition [7]. The result has, however, been a relatively complex set of equations.

The purpose of this brief communication is to make a preliminary presentation of a new creep law for concrete which, with a simple form, appears to give a qualitatively correct picture of these nonlinear traits. In essence, we propose to abandon the linearity of a creep law even for low stress levels. We do so, however, in a manner which preserves proportionality of response to an arbitrary load history. This property of proportionality is essential in modeling creep in concrete at working stress levels. Indeed, measurements of creep or relaxation at constant low stress or strain confirm the proportionality quite well [3,4], while for a multi-step loading or creep recovery significant stiffening nonlinearity is observed, as compared to the prediction of the linear superposition principle [3,4,5,6,7].

It will be shown that, in spite of nonlinearity, this creep law reduces, for a single step-load, to the double power law or the triple power law the validity of which has already been extensively verified by test results [8-10]. Similar to these laws, the proposed creep law gives an infinite creep rate right after any sudden change of stress. This reflects the creep mechanism in concrete [11], which probably consists in migration of cement gel particles to new equilibrium positions due to stress changes and involves breaking and

Work in Progress and Preliminary Results re-establishment of bonds. The rate of particle migrations (and bond breakages) is initially very large (hence the infinite creep rate at the beginning) and gradually declines as the migrating particles are getting exhausted.

Proposed Creep Law

We consider a uniaxial creep law of the form

$$\frac{d}{dt}\left[\gamma(t)^p\right] = \int_0^t R(t,\tau)\, d\left[\sigma(\tau)^r\right] \tag{1}$$

in which

$$R(t,\tau) = F\, t^{-k}\tau^{-m}(t-\tau)^{-u}[\kappa(t)^s - \kappa(\tau)^s]^{-v} \tag{2}$$

$$\kappa(t) = \int_0^t |d\gamma(\tau)| \tag{3}$$

Here σ = uniaxial stress; γ = creep strain; t = time = age of concrete; κ = path-length of creep strain (intrinsic time); k, m, p, r, s, u and v are non-negative material constants (s > 0); $R(t,\tau)$ = creep kernel; F = function of $\sigma(t)$ and $\gamma(t)$ which models the creep increase beyond proportionality at high stress. Since we are not interested in this phenomenon at high stress [12,13, 14, 15], we will consider only the case F = constant, which is sufficient for working stress levels. The integral in Eq. (1) is a Stieltjes integral. For continuous and differentiable $\sigma(t)$ this integral may be replaced by the usual Riemann integral, substituting $d[\sigma(\tau)^r] = r\,\sigma(\tau)^{r-1}[d\sigma(\tau)/d\tau]\,d\tau$.

If we consider a single-step load history ($\sigma = 0$ for $t < t'$, σ = const. > 0 for $t \geq t'$), under the assumption r = p + sv, Eq. (1) reduces to the form

$$\gamma(t) = \sigma\left[\frac{F}{(t')^m}\,\frac{r}{p}\,B(t,t')\right]^{\frac{1}{r}} \tag{4}$$

in which

$$B(t,t') = \int_{t'}^{t} \frac{d\tau}{\tau^k(\tau - t')^u} \tag{5}$$

For $t - t' \ll t'$, the following asymptotic expression holds:

$$\gamma(t) = \sigma\left[\frac{Fr}{(1-u)p}\,\frac{(t-t')^{1-u}}{(t')^{m+k}}\right]^{\frac{1}{r}} \tag{6}$$

Discussion on the Proposed Creep Law

Eq. (1) has the following noteworthy features.

(1) For $p = r = 1$ with $v = k = 0$, it reduces to the linear integral-type creep law based on the double power law [8,9] and, with $v = 0$ and $k > 0$, it reduces to the one based on the triple power law [10] which has been verified as a slight refinement of the well-substantiated double power law.

(2) If any one of the conditions $p = 1$, $r = 1$, and $v = 0$ is violated, this creep law ceases to be linear and, therefore, the principle of superposition does not apply. In particular, the stiffening nonlinearity is obtained for $p > 1$.

(3) However, if at the same time

$$r = p + sv \tag{7}$$

this nonlinear creep law exhibits proportionality in the sense that if $\gamma(t)$ corresponds to $\sigma(t)$ then $k\gamma(t)$ corresponds to $k\sigma(t)$. Observing the test results [3,4], the fact that a nonlinear creep law can be obtained without violating proportionality seems useful for modeling the creep in concrete at working stress levels.

(4) It is also necessary that

$$u + sv < 1 \qquad \text{and} \qquad u + v < 1 \tag{8}$$

for the creep kernel to be weakly singular and, consequently, integrable. The second of these conditions must be added with regard to the second and the subsequent steps of a multistep loading history and prevails when $0 < s < 1$.

(5) As observed in Eq. (4) for a single-step load history with the proportionality condition (Eq. 7), for $k = 0$, Eq. (1) still leads to the well-verified double power creep law and, for $k > 0$, to the triple power law. Therefore, using the previously obtained results on these power laws, it is possible to estimate some parameters involved in the present model.

(6) A further important property is that not only the term $(t - \tau)^{-u}$, which is present in the previous completely linear integral expressions for the creep rate, but also the term $[K(t)^s - K(\tau)^s]^{-v}$ yields an infinite creep rate $\dot{\gamma}$ (singularity) right after any sudden change in stress σ [7,11]. If this term were omitted (i.e., $v = 0$ with $s > 0$), the strength of the singularity of $\dot{\gamma}$ would be given solely by $(t - \tau)^{-u}$, i.e., independent of γ, and would not contribute to expressing the nonlinearity.

(7) With $s = 1$, the strength of the singularity at each stress jump is the same. Analysis of available test data [e.g., 3,4] suggests, however, that $s > 1$. This has an interesting consequence for a two-step stress history, i.e., $\sigma = 0$ for $t < t'$, $\sigma = \sigma_1 =$ const. (> 0) for $t' < t < t''$, and $\sigma = \sigma_2 =$ const. $(> \sigma_1)$ for $t > t''$. If we let $\sigma_1 \to 0$ at constant σ_2, the history approaches a one-step history with a jump of σ_2 at t'', but the singularity strength $(u + v)$ in the limit is not the same as that for a one-step history $(u + sv)$ if $s \neq 1$.

(8) The fact that the integral in Eq. (1) expresses the creep rate $\dot{\gamma}$ rather than the total creep strain γ is appropriate for modeling the nonlinear creep properties at high stress as shown previously [7].

(9) Asymptotic approximations as well as numerical integration of the creep law have further revealed that at low stress levels the creep law generally gives qualitatively correct deviations from linear superposition principle. For a two-step increasing load the response is after the second step lower than the prediction of the superposition principle. For creep recovery after a period of creep at constant stress, the recovery response is and remains higher than the recovery curve predicted from the superposition principle. In both cases, the deviation vanishes as the duration of the first load step tends to zero. These properties represent the essential nonlinear features of concrete creep at low stress levels.

(10) Function $\kappa(t)$ is needed for the case of unloading. This function which is analogous to the well-known intrinsic time assures the definiteness of $R(t,\tau)$. Without excluding the case of creep recovery, it is impossible to use $\kappa(t) = \gamma(t)$ because $R(t,\tau)$ would be undefined for unloading.

From the foregoing discussion, it appears that Eq. (1) is qualitatively capable of capturing all the significant traits of the nonlinear creep behavior of concrete at working stress levels. It is also encouraging that the proposed creep law is compatible with a realistic picture of the creep mechanism. We imagine that creep in concrete consists of a vast number of small particle migrations within the cement paste microstructure. Any sudden change of stress, $\Delta\sigma$, is assumed to activate a number of potential migration sites, the number of which, N_s, is very large. This points to an infinite strain rate right after any stress jump, which in turn suggests the existence of kernel singularity, resulting from the term $[\kappa(t)^s - \kappa(\tau)^s]^{-v}$. The subsequent growth of this term reflects the gradual exhaustion of potential particle migration sites, thus causing a reduction in creep rate. The exhaustion rate must decrease as the creep strain already caused by stress jump $\Delta\sigma$ increases, i.e., it must decrease as $[K(t)^s - K(\tau)^s]$ and $(t - \tau)$ grow, as reflected in Eq. (1).

The creep rate must also decrease due to the continuing hydration of the cement paste while it carries the load. The hydration results in formation of further bonds in the microstructure, which reduces the number of potential migration sites. This reduction depends strictly on time and proceeds at a gradually decreasing time rate, as modeled by the term $\tau^{-m}(t - \tau)^{-u}$ in Eq. (1).

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