A note on limitations of a certain creep function used in practice [1]

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The creep function of Aleksandrovskii, widely used in some parts of the world, is compared to long-range creep test data available in the literature, using computer optimization techniques. It is found that this creep function is inherently incapable of modeling creep for large creep durations and high ages at stress application.

INTRODUCTION

From extensive creep test data that have been accumulated over the years it becomes evident that creep of concrete is a delayed phenomenon of extraordinarily broad time range, spanning from seconds to decades. Prediction of the long-time values is, of course, of particular concern for structural design. However, in the literature on concrete creep it used to be commonplace to show comparisons of the creep formulas with test data in the actual time scale, which is capable of picturing only about one decade in log-time, say from 10 to 100 days, and obscures eventual misfit for the times out of this middle range. Thus, it is not surprising that upon examination of some practical creep functions in two previous papers ([1], [2]), precisely this was found to happen. In this note, which is intended as an addition to a preceding study [2], we will subject to scrutiny one further creep function which is widely used by designers in some parts of the world.

ALEKSANDROVSKII'S CREEP FUNCTION

The creep function, $J(t, t')$, is defined as the strain at time t caused by a unit uniaxial sustained stress that has benn acting since time τ. Time is measured from the moment of setting of concrete. According to Aleksandrovskii (*see* equations IV.131, IV.161, IV.162 in [3]) this creep function may be approximated as

$$J(t, \tau) = \frac{1}{E(\tau)} + C(t, \tau), \tag{1}$$

$$C(t, \tau) = \Psi(\tau) - \Psi(t)\left(\frac{e^{\gamma\tau} - A_2}{e^{\gamma t} - A_2}\right) + \Delta(\tau)\left[1 - e^{-\alpha(t-\tau)}\right], \tag{2}$$

in which $\alpha \gg \gamma > 0$, $0 \leqq A_2 \leqq 1$, and

$$\Psi(\tau) = C_3 + \frac{A_3}{\tau}, \qquad \Delta(\tau) = C_1 - C_3 + \frac{A_1 - A_3}{\tau}, \tag{3}$$

$$E(\tau) = E_0(1 - e^{-\beta\tau}). \tag{4}$$

For the values of material parameters, Aleksandrovskii (equation IV.163 of [3]) proposed

$$A_1 = 4.62 \times 10^{-5} \frac{\text{day}}{\text{kp/cm}^2},$$

$$C_1 = 0.975 \times 10^{-5} \frac{1}{\text{kp/cm}^2},$$

$$A_3 = 3.42 \times 10^{-5} \frac{\text{day}}{\text{kp/cm}^2},$$

$$C_3 = 0.756 \times 10^{-5} \frac{1}{\text{kp/cm}^2},$$

$$\alpha = 6 \frac{1}{\text{day}}, \qquad \beta = 0.206 \frac{1}{\text{day}}, \qquad \gamma = 0.03 \frac{1}{\text{day}}.$$

and also for figure 1,

$$E_0 = 25{,}500 \text{ N/mm}^2 \ (1 \text{ kp/cm}^2 = 0.098\,07 \text{ N/mm}^2).$$

[1] This work has been carried out as part of a project sponsored by the U.S. National Science Foundation under Grant, No. ENG 75-14848 (A 00 and A 01).

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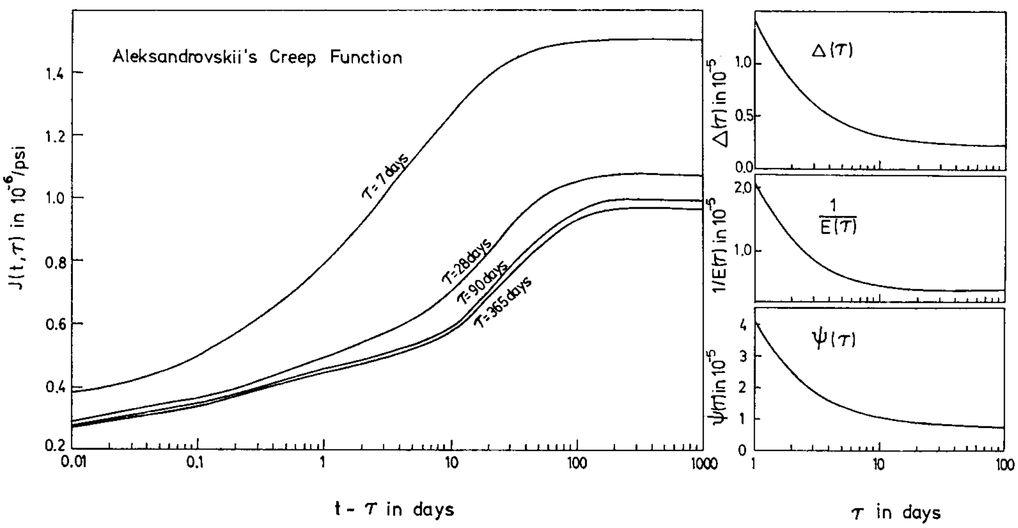
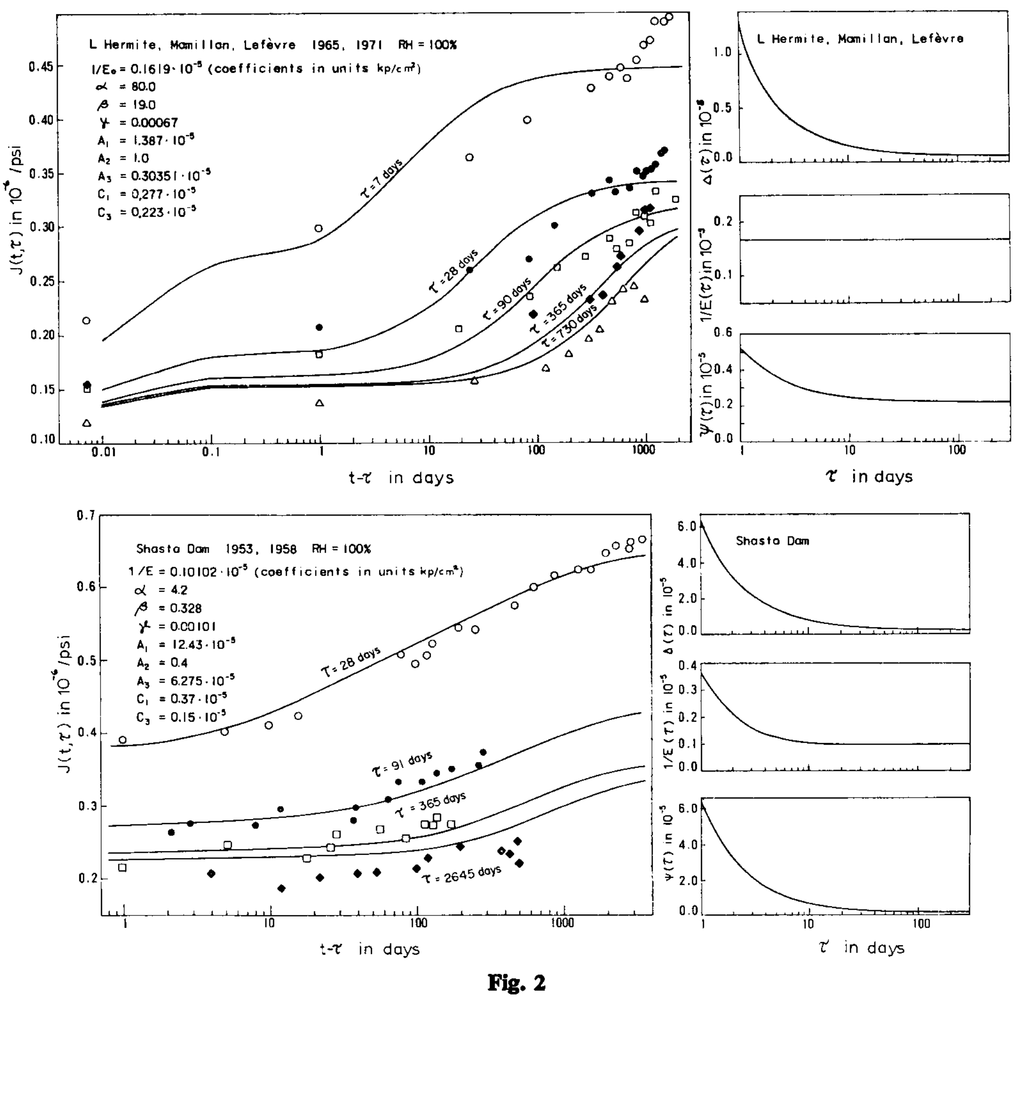


Aleksandrovskii's Creep Function
τ=7 days
τ=28 days
τ=90 days
τ=365 days
J(t,τ) in 10⁻⁶/psi
t - τ in days
Δ(τ)
Δ(τ) in 10⁻⁵
1/E(τ)
1/E(τ) in 10⁻⁵
ψ(τ)
ψ(τ) in 10⁻⁵
τ in days

Fig. 1



L Hermite, Mamillan, Lefèvre 1965, 1971 RH = 100%
1/E₀ = 0.1619·10⁻⁵ (coefficients in units kp/cm²)
α = 80.0
β = 19.0
γ = 0.00067
A₁ = 1.387·10⁻⁵
A₂ = 1.0
A₃ = 0.30351·10⁻⁵
C₁ = 0,277·10⁻⁵
C₃ = 0,223·10⁻⁵
τ=7 days
τ=28 days
τ=90 days
τ=365 days
τ=730 days
J(t,τ) in 10⁻⁴/psi
t-τ in days
L Hermite, Mamillan, Lefèvre
τ in days
Shasta Dam 1953, 1958 RH = 100%
1/E = 0.10102·10⁻⁵ (coefficients in units kp/cm²)
α = 4.2
β = 0.328
γ = 0.00101
A₁ = 12.43·10⁻⁵
A₂ = 0.4
A₃ = 6.275·10⁻⁵
C₁ = 0.37·10⁻⁵
C₃ = 0.15·10⁻⁵
τ=28 days
τ=91 days
τ=365 days
τ=2645 days
t-τ in days
Shasta Dam
τ in days

Fig. 2

The creep function for these parameters is plotted in figure 1 (in which 1 psi = 0.006 895 N/mm^2).

Recalling the plots of creep test data in log-time scale, exhibited in [1] and [2] and especially [4], it becomes apparent that the general trend of the creep curves in figure 1 does not agree well with experiments. Aleksandrovskii used for experimental validation only his own test data (*fig.* 53 of reference [3]) which were of rather limited time range and did not justify extrapolation to the usual lifetime of structures. Aleksandrovskii's creep curves reach a horizontal asymptote at about 200-day creep duration, while according to creep test data in general they should continue rising steadily up to at least 10,000 days. Moreover, the curves for loading ages $\tau = 90$ days and $\tau = 365$ days are much too close, which reveals that the function is inherently incapable of modeling the aging effect for ages at loading from 100 to 10,000 days.

In view of this conclusion, one is tempted to ask whether equation (2), whose form has been chosen so as to allow converting the integral stress-strain relation to a second-order differential equation, might be applicable if some other values of material parameters defining functions $\Psi(\tau)$, $\Delta(\tau)$ and $E(\tau)$ were introduced. This question can be easily answered using computer optimization algorithms for nonlinear sum-of-squares objective functions [2]. In this manner, optimum fits or various test data sets available in the literature (*cf.* [1], [2], [4]) have been run, allowing parameters A_1, A_2, A_3, C_1, C_3, α, β, and γ to have arbitrary values. Some typical optimum fits of test data (referenced and described in [2] and [4]) are demonstrated in figure 2 along with graphical plots of the functions $\Psi(\tau)$, $\Delta(\tau)$ and $E(\tau)$ that gave the best fits. Curiously, for the data of L'Hermite *et al.*, the optimum function for $E(\tau)$ came out to be a constant.

The fits in figure 2 reveal poor agreement. The separation between the curves for $\tau = 365$ and 730 days as well as 365 and 2,645 days is much too close. The bumpy rise of the optimum creep curves in figure 2 *a* is unacceptable, and the long-time creep is not properly described. Yet, the case in figure 2 *b* is one of the more favorable ones for the formula of Aleksandrovskii's type because these data do not include small ages at loading and short creep durations.

It must be emphasized, however, that there is no objection against using this creep function when prediction of the long-time response is not needed. It must be also remembered that, at the time when this function was first introduced (about 1961), the form of the creep function was dictated by the need o enabling conversion of the integral stress-strain relation for variable stress to a differential equation. Along with Glanville's rate-of-creep type formulas (Dischinger's formulations), equation (2) was one of the functions which fulfilled that need. Today, this need no longer exists, because of one other recently developed method of creep structural analysis (*cf.* [1], [2]).

CONCLUSION

Similarly to other practically used creep functions analyzed in [1] and [2], the creep function of Aleksandrovskii has a rather limited time range and is inherently incapable of modeling creep for long creep durations and for high ages at stress application.

REFERENCES

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RÉSUMÉ

Note sur les limitations d'une fonction de fluage entrée dans la pratique. — *La fonction de fluage d'Aleksandrovskii largement utilisée dans certaines parties du monde est confrontée à l'aide des techniques d'optimisation informatisée aux résultats d'essais de fluage à long terme publiés. On établit que cette fonction de fluage ne peut se prêter à une mobilisation pour les études de fluage sur de grands intervalles de temps et sous des sollicitations appliquées à des âges avancés.*