

Practical prediction of time-dependent deformations of concrete

Part I: Shrinkage (1)

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Proposed is a practical model for predicting creep and shrinkage of concrete from the composition of concrete mix, strength, age at loading, conditions of environment, size and shape, etc. The main features are: double power law for basic creep, square-root hyperbolic law for shrinkage, diffusion-type size dependence of humidity effects, additive drying creep term related to shrinkage, and activation energy treatment of thermal effects. Optimization techniques are used to fit numerous test data available in the literature. The work is a continuation of previous investigations and consists of several parts. This first part deals with shrinkage.

INTRODUCTION

Creep deformations are normally two to three times larger than the instantaneous deformations of concrete. So, if the engineer is concerned at all about deflections, stiffness, and redundant internal forces due to permanent loads, he must primarily look into creep.

Scanning the literature on creep and shrinkage of concrete, one cannot fail to observe that a wealth of experimental data has been accumulated. Yet, the existing code provisions and standard recommendations, even some that have just been introduced in an international model code, have been based only on a very narrow and quite arbitrary selection among the available data sets. Consequently, it is not surprising that they are not in the best possible agreement with the overall experimental reality. This state of affairs is understandable in view of the use of traditional methods of data fitting. However, the computer methods and optimization techniques that have been

recently developed ([2], [4]) make it now possible to analyze large sets of test data with ease, which is bound to bring about a revolution in our mathematical modeling of material behavior. Thus, the time is ripe to tap the vast pool of test data on creep and shrinkage scattered through the literature.

Among mathematical models for time-dependent deformations of concrete, two types may be distinguished:

(a) constitutive relations [1], [5] which describe the behavior of a small element of the material and must satisfy the well-known invariance and objectivity requirements of continuum mechanics;

(b) models for practical prediction of material behavior in simple basic situations, such as uniaxial creep or relaxation test or mean deformation of the whole cross section that may be in a nonuniform state.

The latter type of models, which is the sole type of models to be considered in this study, is far less complicated and is quite satisfactory for many problems in design practice. It is one purpose of this study to develop and propose an improved code-type formulation for such practical problems.

In addition, however, and we must emphasize it, realistic forms of probabilistic models for statistical scatter of creep and shrinkage will be placed within reach once a good deterministic model of the latter type (b) is developed, whereas they are only a distant

(1) The work presented in this series of papers has been sponsored under National Science Foundation Grant ENG 75-14848. The second author also received a Fellowship of The Alumnae of Northwestern University and Walter P. Murphy Fellowship from The Technological Institute of Northwestern University.

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possibility with general constitutive relations (a). Thus, from a fundamental viewpoint, the ultimate albeit longer-range goal of our endeavor is to set the stage for developing a complete stochastic process model for the time-dependent deformations of concrete. Part of this has already been accomplished – the previously developed double-power law [2] has been shown to be compatible with a probabilistic model based on a realistic statistical concept of the physical mechanism of creep [3].

This study, which consists of a sequence of papers that will deal essentially with (I) shrinkage, (II) creep at constant humidity and temperature, (III) creep at variable humidity, and (IV) creep at variable temperature, represents a direct continuation of the preceding work ([2], [4]), in which the time-dependent deformations were characterized basically by the following concepts:

- (1) square-root hyperbolic law for shrinkage ([5], [4]);
- (2) double power law for creep (introduced in [5]) ([6], [2]);
- (3) diffusion-type size dependence of humidity effects [4];
- (4) additive drying creep term related to shrinkage [4];
- (5) activation energy concept for thermal effects.

Development of a mathematical model of material behavior naturally proceeds in two stages. First, it must be demonstrated that the model is capable of adequately representing diverse concretes taken individually, even though the material coefficients for various concretes may have rather different, and seemingly quite random, unrelated values. This was done in the preceding papers ([2], [4]). Second, it is necessary to find consistent rules by which the material coefficients can be correlated and evaluated from the composition parameters of concrete mix, and eventually introduce some nuances in the model that would allow establishing such consistent rules. This second, last stage of development is the main goal of this study, in which primarily the composition effects on material parameters will be quantified.

It must be emphasized that our formulation is guided in many respects by previous works on the effect of composition upon creep and shrinkage; see, e. g., those of L'Hermite *et al.*, Kesler *et al.*, Neville *et al.*, Branson *et al.* ([7], [12], [35], [35 a], [35 b]) as the outstanding sample among many valuable contribution.

Finally, it should be mentioned that the present work serves at the same time the need of organizing, summarizing, and comparing the existing test data on creep and shrinkage. This would save much tedious work to anyone who intends to evaluate an existing model or develop more advanced models for predicting creep and shrinkage and study the statistical scatter.

FORMULAS FOR SHRINKAGE

The formulas which allowed a consistent representation of various sets of data on shrinkage of unsealed concrete specimens and members are essentially those

from reference [4], with only slight modifications:

$$\varepsilon_{sh}(\hat{t}, t_0) = \varepsilon_{sh,\infty} k_h S(\hat{t}); \quad \hat{t} = t - t_0; \quad (1)$$

square-root hyperbolic law in time:

$$S(\hat{t}) = \sqrt{\frac{\hat{t}}{\tau_{sh} + \hat{t}}}; \quad (2)$$

humidity dependence:

$$\left. \begin{array}{l} \text{for } h \leq 0.98 : k_h = 1 - h^3; \\ \text{for } h = 1.00 : k_h = -0.2. \end{array} \right\} \quad (3)$$

size-dependence (of diffusion type):

$$\tau_{sh} = 600 \left(\frac{k_s}{150} D \right)^2 \frac{C_1^{\text{ref}}}{C_1(t_0)}, \quad D = 2 \frac{v}{s}; \quad (4)$$

age-dependence:

$$C_1(t) = C_7 k'_7 (0.05 + \sqrt{6.3/t}), \quad (5)$$

$$\varepsilon_{sh,\infty} = \varepsilon_{s,\infty} \frac{E(7 + 600)}{E(t_0 + \tau_{sh})}, \quad (6)$$

$$E(t') = E(28) \sqrt{\frac{t'}{4 + 0.85 t'}}; \quad (7)$$

temperature dependence:

$$k'_7 = \frac{T}{T_0} \exp \left(\frac{5,000}{T_0} - \frac{5,000}{T} \right). \quad (8)$$

Here ε_{sh} = shrinkage strain; t = time (in days), representing age of concrete, t_0 = age when drying begins, \hat{t} = duration of drying, $\varepsilon_{sh,\infty}$ = ultimate shrinkage, E = Young's modulus, h = relative humidity of the environment ($0 \leq h \leq 1$), τ_{sh} = shrinkage square half-time, D = effective cross section thickness in millimetres, v/s = volume-to-surface ratio, C_1 = drying diffusivity of nearly saturated concrete at reference temperature T_0 (chosen as 23°C for all data), k'_7 = temperature coefficient, which is based on activation energy of moisture migration [13] and must be calculated from temperature T and T_0 in Kelvin (absolute temperatures). Furthermore, $C_1^{\text{ref}} = 10 \text{ mm}^2/\text{day}$.

C_7 = given or assumed value of C_1 at age 7 days. The temperature correction [equation (8)] was applied even when the test temperature was rather close to $T_0 = 23^\circ\text{C}$. Finally, k_s = shape factor which equals 1.0 for an infinite slab, 1.15 for an infinite cylinder, 1.25 for an infinite square prism, 1.30 for a sphere, and 1.55 for a cube [4]. The k_s -value for a cylindrical or prismatic specimen of finite length was obtained by interpolating (linearly in length) between the value for a sphere or cube and the value for a specimen of length three times its width, for which k_s was taken the same as for an infinitely long specimen. For $h = 1.0$, function k_h gives swelling under water and for $h = 0.98$, it gives roughly the autogeneous shrinkage of standard size specimens, assuming that 0.98 is the typical pore humidity caused by self-desiccation in sealed specimens.

Although the formulas are semi-empirical, many aspects of their structure are theoretically justified; see reference [4]. For example, assuming that shrinkage is proportional to the pore humidity drop, diffusion

theory (linear as well as nonlinear) indicates that the shrinkage half-time should be proportional to D^2 , thickness-square, as well as to diffusivity with its age-dependence. Nonlinear diffusion calculations also yield the values of k_s . Imagining shrinkage to result from a tensile force that is generated in pore water and is resisted by solid microstructure, formula (6) ensues. It should be noted that the basic form of equations (1) and (2) is rather similar to that of Branson ([35], [35 a], [35 b], [35 c]).

There exists a more accurate formula than equation (7) for the age-dependence of elastic modulus, $E(t)$; see equations (12) and (15)-(18) in Part II which follows. However, these equations were established only after the fitting of the shrinkage data was completed, and it was not possible to recalculate all diagrams, especially since the differences in shrinkage are too small to justify it.

PROPOSED DEPENDENCE ON COMPOSITION AND STRENGTH

The following empirical dependence on strength and the composition parameters of concrete mix has been identified:

Reference diffusivity (mm^2/day):

$$C_7 = \frac{1}{8} \frac{w}{c} - 12; \quad (9 a)$$

$$\left. \begin{array}{l} \text{if } C_7 < 7 \text{ set } C_7 = 7, \\ \text{and if } C_7 > 21 \text{ set } C_7 = 21. \end{array} \right\} \quad (9 b)$$

Final shrinkage (in 10^{-6}):

$$\varepsilon_{s_z} = 1,210 - 880 y, \quad y = (390 z^{-4} + 1)^{-1}, \quad (10 a)$$

$$z = \left[[1.25 \sqrt{a/c} + 0.5(g/s)^2] \times \left(\frac{1+s/c}{w/c} \right)^{1/3} \sqrt{f'_c} \right] - 12$$

$$\text{if } z \geq 0, \text{ else } z = 0. \quad (10 b)$$

Here c = cement content in kg/m^3 , w/c = water-cement ratio, a/c = aggregate-cement ratio, g/s = gravel-sand ratio, s/c = sand-cement ratio (all ratios by weight), and f'_c = 28-day cylinder strength in ksi (1 ksi = 6.895 N/mm^2). Sand is defined as aggregate of less than 4.7 mm size (sieve No. 4), the rest being considered as gravel.

The qualitative trends reflected in the preceding formulas agree with what is generally known ([7], [8], [10], [11], [12], [9]) or may be intuitively expected: (a) Diffusivity increases as water content increases, and decreases as the cement content increases. (b) Stronger concrete shrinks less. (c) Increase of the water-cement ratio at the same strength results in a higher shrinkage (e. g., Rostasy [34]). (d) Ultimate shrinkage at the same strength decreases as the aggregate content increases, and the effect of gravel content is more pronounced than the effect of sand content.

Other well-known influences ([7], [8], [10], [11], [12]) are hidden in these expressions. The mineralogical type of aggregate enters through its elastic modulus, which affects the elastic modulus of concrete and in turn the

strength of concrete, and also through the unit weight of aggregate, which affects f'_c , a/c , s/c . However, all aggregate could not be examined; only granite, gneiss, basalt, quartz, limestone and dolomite were included among the data available. The aggregate size distribution is partly reflected in the gravel-sand ratio. Only a continuous size distribution (within the normal range) was considered. The gap-graded aggregate is not covered by the formulas, and the aggregate size distribution curve must lie within the bounds specified by codes.

METHOD OF OPTIMIZING DATA FITS FOR TIME-DEPENDENT DEFORMATIONS

To find the preceding formulas [equations (9), (10)], a computer program which uses Marquardt optimization algorithm to minimize the sum of squared deviations from test data has been written. First, all data sets and their mean trends obtained by hand smoothing have been stored in the program. The optimization subroutine enabled it to quickly obtain the optimum fit of any given data by equations (1)-(8). It must be emphasized, however, that the optimization subroutine could be utilized only after one has gained, by trial-and-error approach, the material coefficient values which give a qualitatively correct behavior, not far away from the data points. This is because, as a consequence of the nonlinearity of the optimization problem, the sum of squared deviations as a function of material parameters exhibits not a single minimum but many local minima, only one of which is the sought after overall minimum. So, one must start closer to this overall minimum than to any other local minimum, in order to achieve proper convergence. For the same reasons, it was usually more effective to optimize only for some material coefficients, leaving few others to be determined by a trial-and-error approach. Even the trial-and-error part, however, was made partly automatic by running the optimization program for a great number of trials of the remaining material coefficients, calculating (and printing) the sum of square deviations (with proper weights) for each trial and automatically selecting among this discrete set the best trial as the one which gives the smallest sum. All fits were also automatically drawn by Calcomp plotter (for more details, see reference [14]). Furthermore, because most existing data sets do not give complete information on the test conditions, some material coefficients had to be fixed at values similar to those for other data sets.

Using the aforementioned procedure, the best fits of various data sets considered individually have been determined as the first step. Subsequently, all available data were analyzed collectively. First, each of the material coefficients was changed by about $\pm 10\%$, one by one, to ascertain the sensitivity of the combined objective function, i. e., the sum of squared deviations for all data sets combined (with a proper weight on each data set and on each point within the data set). The values of the less sensitive coefficients for various data sets (here C_7) were then plotted against various composition parameters of individual data sets, such as f'_c , w/c , a/c , g/s and many other possible parameters which do not appear in equations (9)-(10). The type of plot which revealed the most consistent and

most pronounced trend was then approximated by a formula, which was kept simple, such as equation (9). This left only a few parameters to be further studied for a more sophisticated and more pronounced dependence on composition (here only $\epsilon_{s,c}$; see fig. 1). Again, diagrams of this (or these) parameter(s) for various data sets against all kinds of conceivable mix parameters were automatically plotted (here only $\epsilon_{s,c}$ was treated in this manner). Among these plots, the one which revealed the most consistent and most pronounced trend was selected. The degree of consistence of the plot was then improved by optimizing certain non-dimensional coefficients, such as the exponents in equation (10 b).

In this manner, the formula in equation (10 b) was finally obtained. Although this formula certainly cannot be expected to be the absolutely best possibility, it nevertheless represents the best one among a large number of possibilities. Except for the general trends spelled out before, the formula is, of course, empirical.

The same procedure as just described has been used in fitting the creep data analyzed in the sequel.

COMPARISON WITH SHRINKAGE TEST DATA

Fits of eleven different comprehensive data sets available in the literature ([15], [16], [17], [18], [19]-[24]) are exhibited in figures 2-11 (where R.H. = h = relative humidity). Two different types of fits are shown in each figure.

The dashed line fits represent the case when both ϵ_s and C_7 are predicted from composition [equations (9) and (10)]; the deviations of the dashed lines from test data give an idea of the error committed by a designer using the formulas. Although in some figures the error is quite substantial, it is overall far less than that with the more primitive formulas available before.

The solid line fits represent the case where only diffusivity C_7 is predicted from the composition of

concrete mix [equation (9)], and the final shrinkage ϵ_s is optimized individually for each data set, rather than being obtained from equation (10). The deviations from the data give a picture of the magnitude of the error that is committed when a short-time shrinkage test of the concrete for a given structure is performed and is then used by the designer to find the value of $\epsilon_{s,c}$ which fits best the short-time shrinkage test. In this case the overall agreement with test data is much better.

The most comprehensive and consistent data set is that of Keeton [20], and its fitting has been assigned priority. On the size effect, the data of L'Hermite, Mamillan [16] and of Hansen, Mattock [15] are most relevant. The misfit of initial deformation of the massive specimens in L'Hermite and Mamillan's shrinkage data (1968, [16]) in figure 3 is due to the fact that the thermal expansion due to hydration heat, which is large in massive specimens, is not included in equations (1)-(10). The tests of Hummel *et al.* [19] also included w/c ratios other than 0.38 and 0.55, which are not shown in figure 10 because the trend of shrinkage change with w/c in these data was opposite to that generally observed. The data of Rüsç *et al.* [17] include many further cases in addition to the one shown in figure 11; their fits are rather poor, but these are mostly concretes of rather extreme properties (e.g., $f'_c = 8,300$ and 2,900 psi) for which the present formulation apparently does not always apply. Moreover, some of these data [17] could not be fitted because they exhibit an increase rather than a decrease of ϵ_s with f'_c , contrary to most others. The tests of Dreux and Gorisse [24] dealt with reinforced prisms. Before fitting, the corresponding shrinkage ϵ_{sh} for unreinforced prisms was estimated from these data as $\epsilon_{sh} = \epsilon / (1 + pE_s/E'_c)$ where ϵ = measured strain, p = reinforcement ratio, E_s = elastic modulus of steel, and E'_c = age-adjusted effective modulus (given by equation (5) of [26], [27]). The curve from [24] was adjusted in this manner to obtain figure 9 (the ratio ϵ/ϵ_{sh} was about 0.97, so any possible inaccuracy of correction was insignificant).

The basic information on the test data used is summarized in Appendix I, so that the data shown can

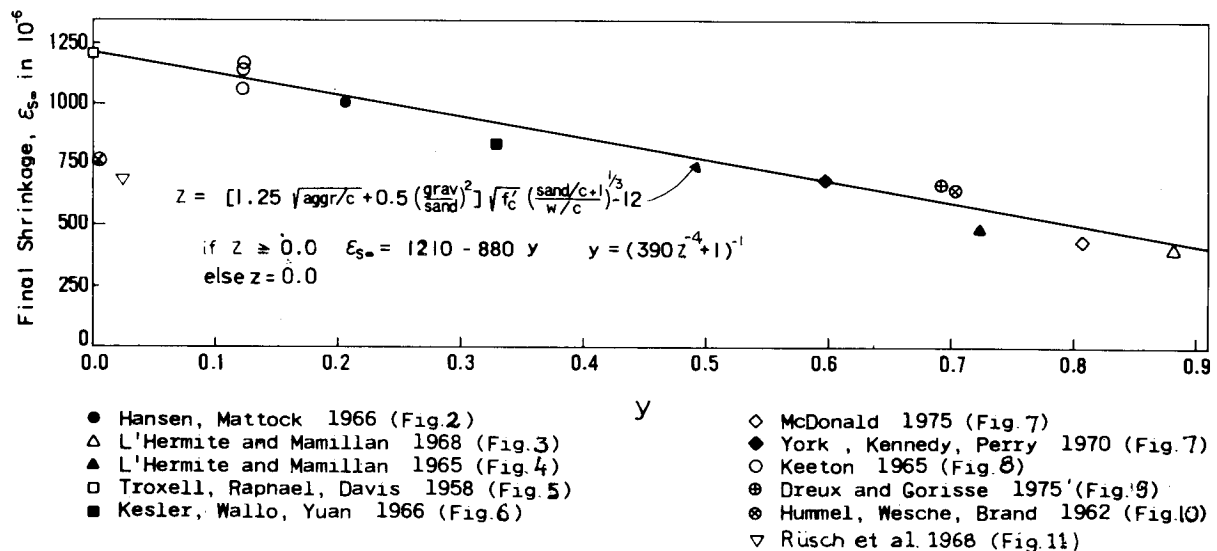


Fig. 1. - Final Shrinkage as a Function of Composition and Strength.

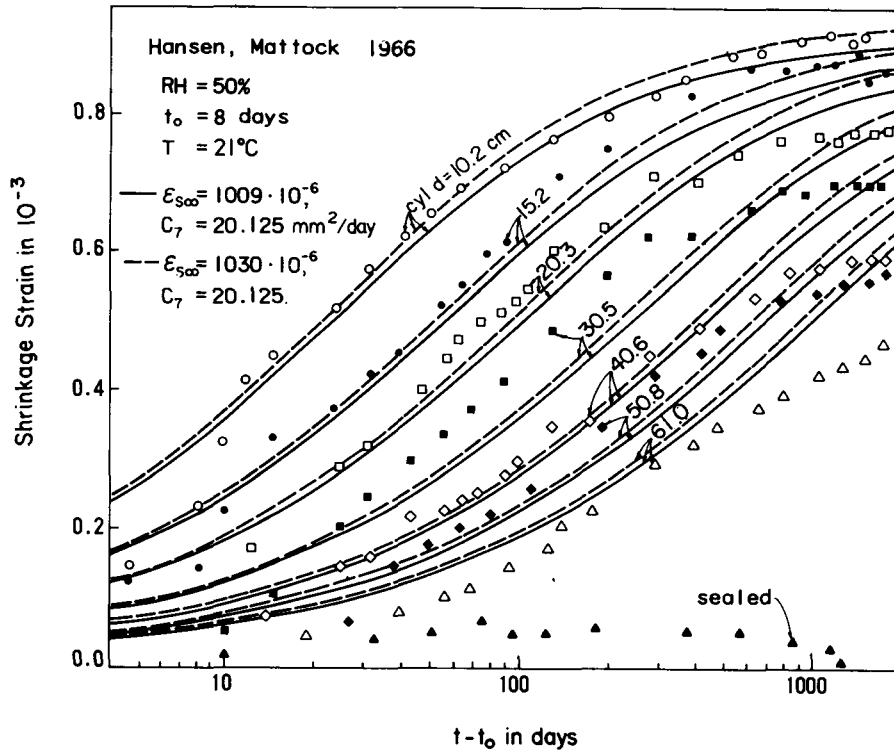


Fig. 2. — Fits of Hansen and Mattock's Tests of Shrinkage (1966) [15].

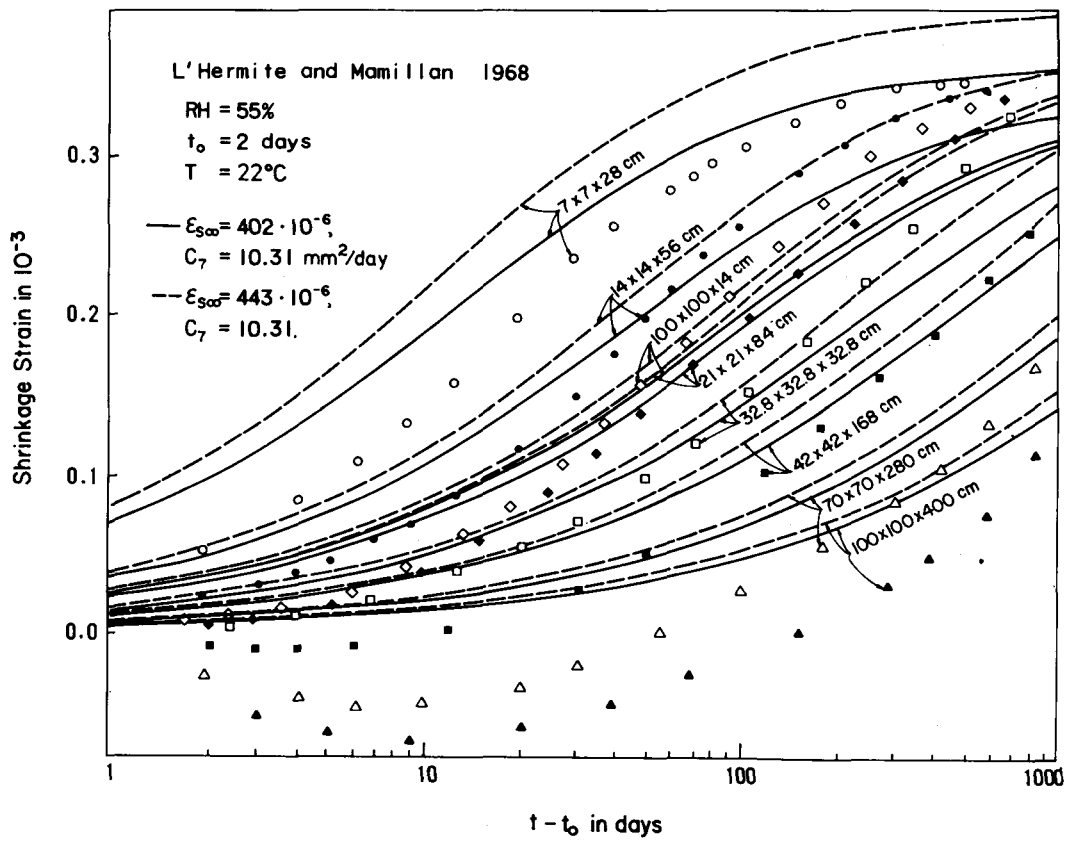


Fig. 3. — Fits of L'Hermite and Mamillan's Tests of Shrinkage (1968) [16].

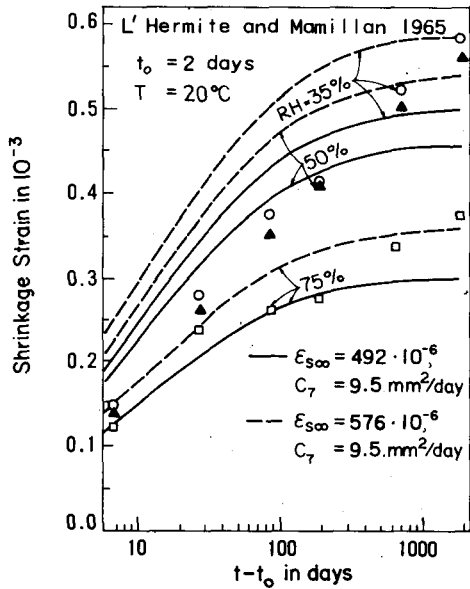


Fig. 4. — Fits of L'Hermite and Mamillan's Tests of Shrinkage (1965) [16].

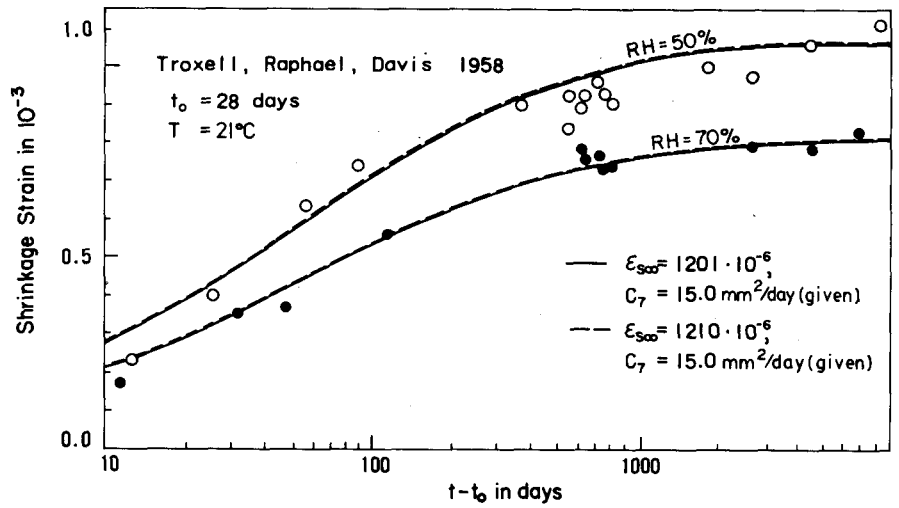


Fig. 5. — Fits of Troxell, Raphael and Davis' Tests of Shrinkage (1958) [18].

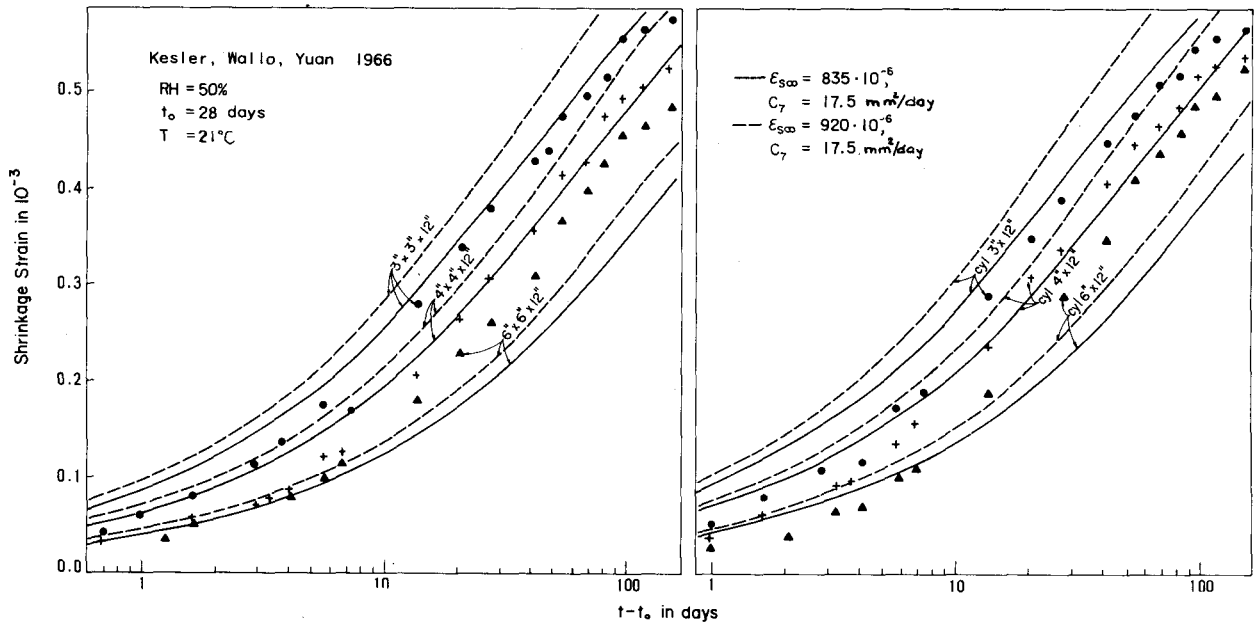


Fig. 6. — Fits of Kesler, Wallo and Yuan's Test of Shrinkage (1966) [21].

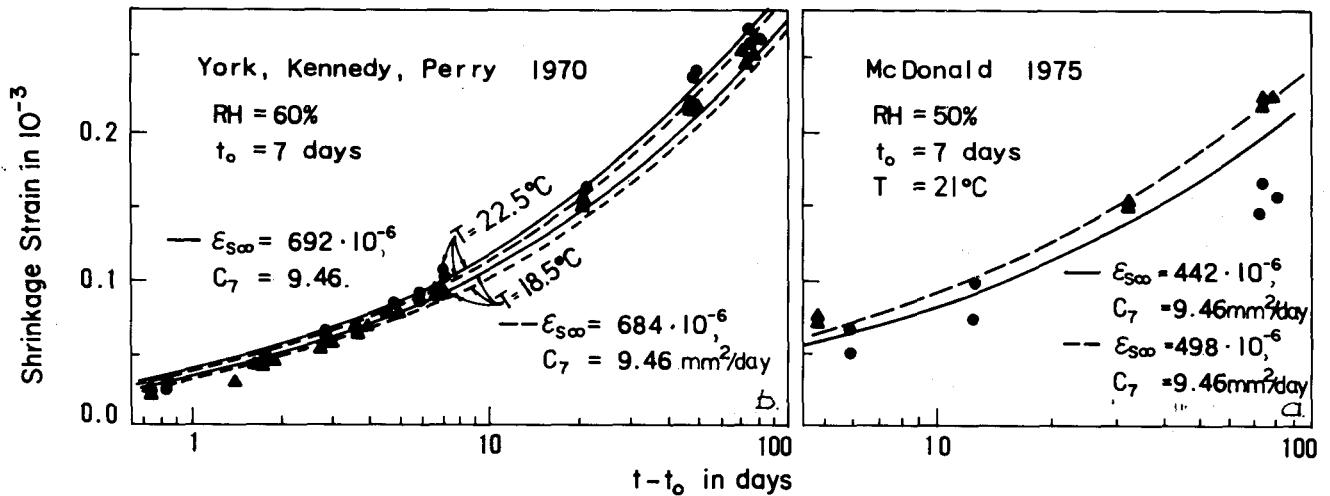


Fig. 7. — Fits of a) McDonald's (1975) [22] and b) York, Kennedy and Perry's (1970) [23] Tests of Shrinkage.

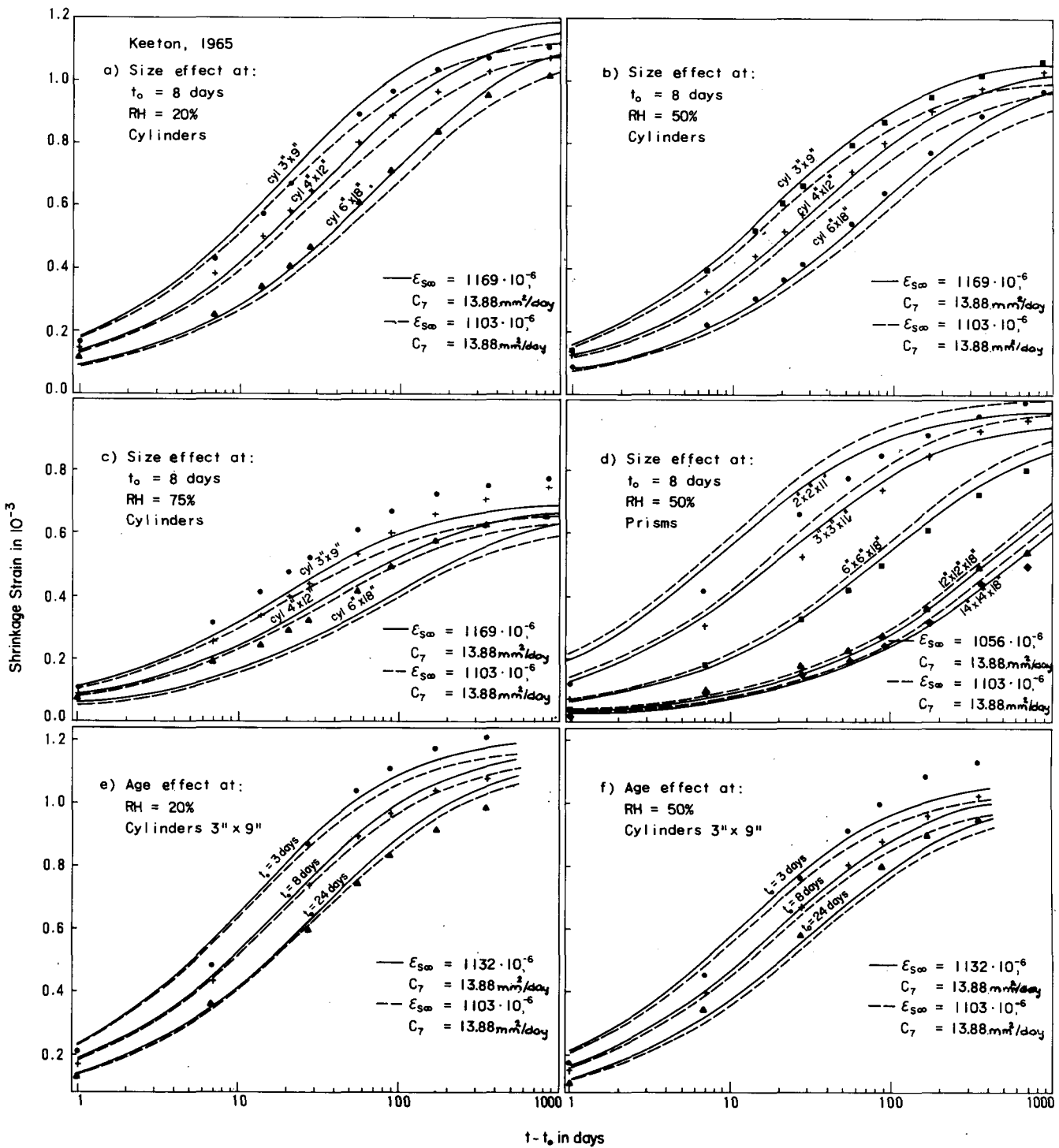


Fig. 8. — Fits of Keeton's Tests of Shrinkage (1965) [20].

be used by other researchers to check the present results, as well as to analyze possible alternative formulas (some of the information included in Appendix I was not included in the original publication and has been kindly supplied by the respective authors upon personal request.) A host of other shrinkage test data which are not shown have been analyzed; these included those from references ([35], [29], [25], [30], [31], [32], [33]). However, various potentially valuable further data sets, especially the Russian ones ([28], [29]),

had to be omitted because it was impossible to obtain all necessary information on the tests.

An idea of the degree of agreement of test results with the proposed formula (10) for composition effects may be obtained from figure 1, in which the ordinate is chosen in such a way that linear regression applies.

The fits cannot be assumed to confirm the temperature factor k'_T because the range of temperatures was very narrow. This factor is strictly theoretical.

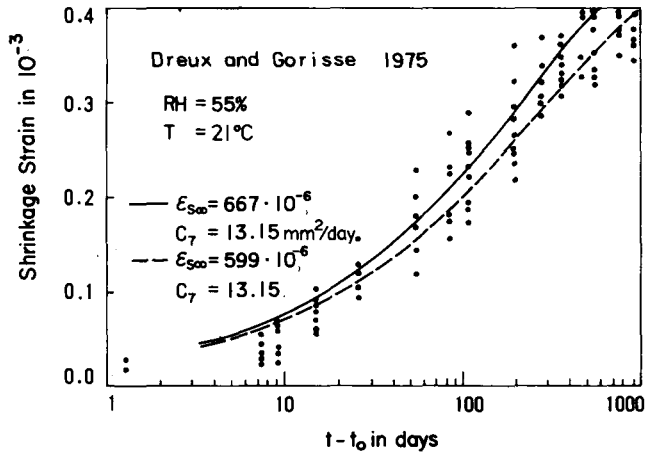


Fig. 9. — Fits of Dreux and Gorisse's Tests of Shrinkage (1975) [24].

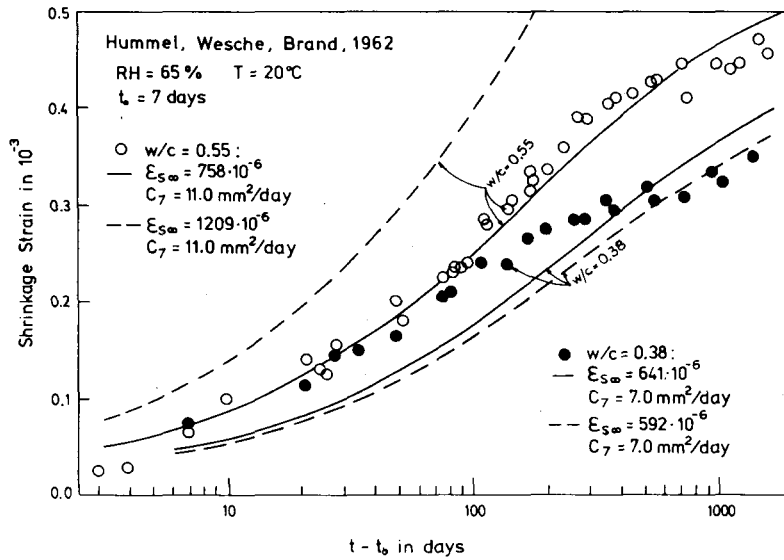


Fig. 10. — Fits of Hummel, Wesche and Brand's Tests of Shrinkage (1962) [19].

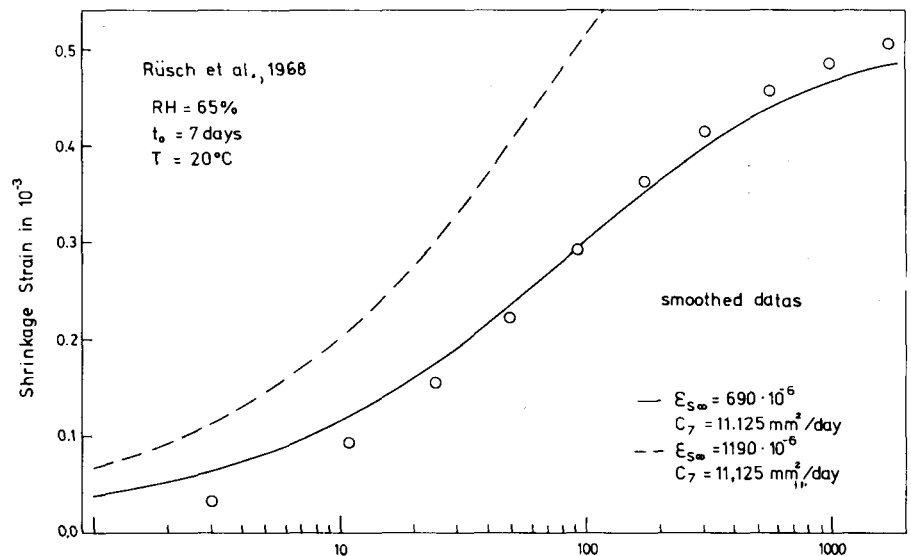


Fig. 11. — Fits of Rüsç et al.'s Tests of Shrinkage (1968) [17].

APPENDIX I

Basic Information on Shrinkage Data Used

Hansen and Mattock (1966) [15]. — Specimens drying at 50% relative humidity; 28-day cylinder strength 6,000 psi (41.4 N/mm²) Elgin gravel (92% calcite, 8% quartz). Max. size of aggregate = 3/4 inch. ASTM type III cement 362 kg/m³, cured 2 days in mold and 6 days in fog at 70°F (21°C). Specimens exposed to drying at 8 days of age. Water-cement-sand-coarse aggr. ratio = 0.71 : 1 : 3.3 : 2.7. Lengths of cylinders of 10 to 61 cm dia. are 457, 559, 660, 864, 1,067, 1,270, 1,473 mm.

L'Hermite and Mamillan (1968) [16]. — Specimens drying at 55% relative humidity and 22°C temperature. Water-cement-sand-coarse aggregate ratio = 0.51 : 1 : 1.69 : 3.37. Seine gravel (siliceous calcite), max. aggregate size 20 mm; 28 day cylinder strength 5,365 psi (34.8 N/mm²). Cement French type I, 350 kg/m³. Specimens cured in water, then exposed to drying at 2 days. Strain measured along the axis of cylinder.

L'Hermite and Mamillan (1965) [16]. — Specimens drying at 20°C. Cement French type 400/800, 350 kg/m³ of concrete. Water-cement-sand-coarse aggr. ratio = 0.49 : 1 : 1.75 : 3.07. Aggregate Seine gravel (siliceous calcite), 28 day cylinder strength 5,365 psi (34.8 N/mm²). Specimens 7 × 7 × 28 cm cured in water and exposed to drying at age of 2 days.

Troxell, Raphael and Davis (1958) [18]. — Cement type I, granite aggregate, max. size of aggregate 1.5 in., 28 day cylinder strength 2,500 psi (17.2 N/mm²). Water-cement-sand-coarse aggregate ratio 0.59 : 1 : 2 : 3.67. Cylinders 4 × 14 inches (102 × 356 mm). Cured for 28 days and then exposed to drying at 70°F (21°C) temperature.

Kesler, Wallo, Yuan (1966) [21]. — Specimens drying at 50% relative humidity, temperature 70°F (21°C). Water-cement-sand-coarse aggregate ratio 0.5 : 1 : 2 : 2.4. Max. size of aggregate 1.5 in. (38 mm). 472 kg/m³ of type I portland cement, river gravel. Exposed after moist curing for 28 days to drying.

McDonald (1975) [22]. — After 7 days of wet curing, cylinders 6 × 16 inch (152 × 406 mm) were allowed to dry at 73°F (23°C) and 50% relative humidity. Portland cement type II 404 kg/m³, 3/4 inch (19 mm) max. size of limestone aggregate. Water-cement-sand-coarse aggregate ratio 0.425 : 1 : 2.03 : 2.62; 28 day average cylinder strength 7,320 psi (50.5 N/mm²).

York, Kennedy, Perry (1970) [23]. — After 7 days curing cylinders 6 × 16 inches (152 × 406 mm) were allowed to dry at relative humidity of 60%. Portland cement type II 404 kg/m³, limestone aggregate max. size 1/2 inch (13 mm). Water-cement-sand-coarse aggregate ratio 0.425 : 1 : 2.03 : 2.62. 28 day cylinder strength 6,200 psi (42.8 N/mm²).

Keeton (1965) [20]. — Portland cement type III, 450 kg/m³. Water-cement-sand-coarse aggregate ratio 0.46 : 1 : 1.66 : 2.07. Max. size of aggregate 3/4 inch (19 mm). Fine aggregate Saticoy sand (river), coarse aggregate Santa Clara river gravel. 28 day cylinder strength 6,550 psi (45.2 N/mm²). Temperature 73°F (23°C).

Dreux and Gorisse (1975) [24]. — After 66 hours curing, specimens 15 × 40 × 50 cm were allowed to dry at 21°C and 55% relative humidity, 400 kg cement per m³, max. size of aggregate assumed as 20 mm. Water-cement-sand-coarse aggregate ratio 0.50 : 1 : 1.58 : 2.84. 28 day cylinder strength 39.1 N/mm² (average value).

Hummel, Wesche, Brand (1962) [19]. — Cured 1 day in mold, then until age of 7 days under wet rugs at 20°C. After 7 days, cylinders 20 × 80 cm were transferred to an

environment of relative humidity 65% and temperature 20°C. Water-cement-aggregate ratios 0.38 : 1 : 5.4 and 0.55 : 1 : 5.4; 28 day cylinder strength 40.6 N/mm², Cement (PZ225) content 350 kg/m³; 28 day cylinder strength 28 N/mm², Cement content 334 kg/m³.

Rüsch et al. (1968) [17]. — Cured 24 hours in molds, then in 100% relative humidity until age of 7 days. Subsequently, specimens transferred to a room of relative humidity 65% ± 5%, temperature 20°C ± 2. Rhine gravel; limestone and quartz fine aggregate. Water-cement-sand-gravel ratio 0.55 : 1 : 3.08 : 2.27. 28 day strength 34.1 N/mm². Cement content 336 kg/m³.

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