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Report on ONR Workshop on Fracture Scaling

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Abstract. The paper reports on the discussions at the ONR Workshop on Fracture Scaling, held at University of Maryland in June 1999, under the chairmanship of Z.P. Bažant and Y.D.S. Rajapakse. The workshop dealt with size effects in structural failure and scale bridging in mechanics of materials. The lectures at the Workshop were published in Volume 95 of this Journal. The objective of this paper is to present records and interpretations of the extensive discussions prepared by invited specialists. The records show which are the areas of disagreement among leading researchers and which are those where consensus has been reached.

Key words: Fracture, scaling, size effect, failure, structural analysis, micromechanics, composites, concrete, damage, nonlocal models, gradient models.

1. ONR Workshop

A special issue of this Journal featured the invited papers presented at the Workshop on Fracture Scaling (Bažant and Rajapakse, Editors, 1999), which was chaired by Bažant and Rajapakse and was held at the University of Maryland, College Station, during June 10–12, 1999. This Workshop, which was scientifically co-sponsored by RILEM Technical Committee QFS and attracted 60 participants from 14 countries (see Figure 1), consisted of five half-day

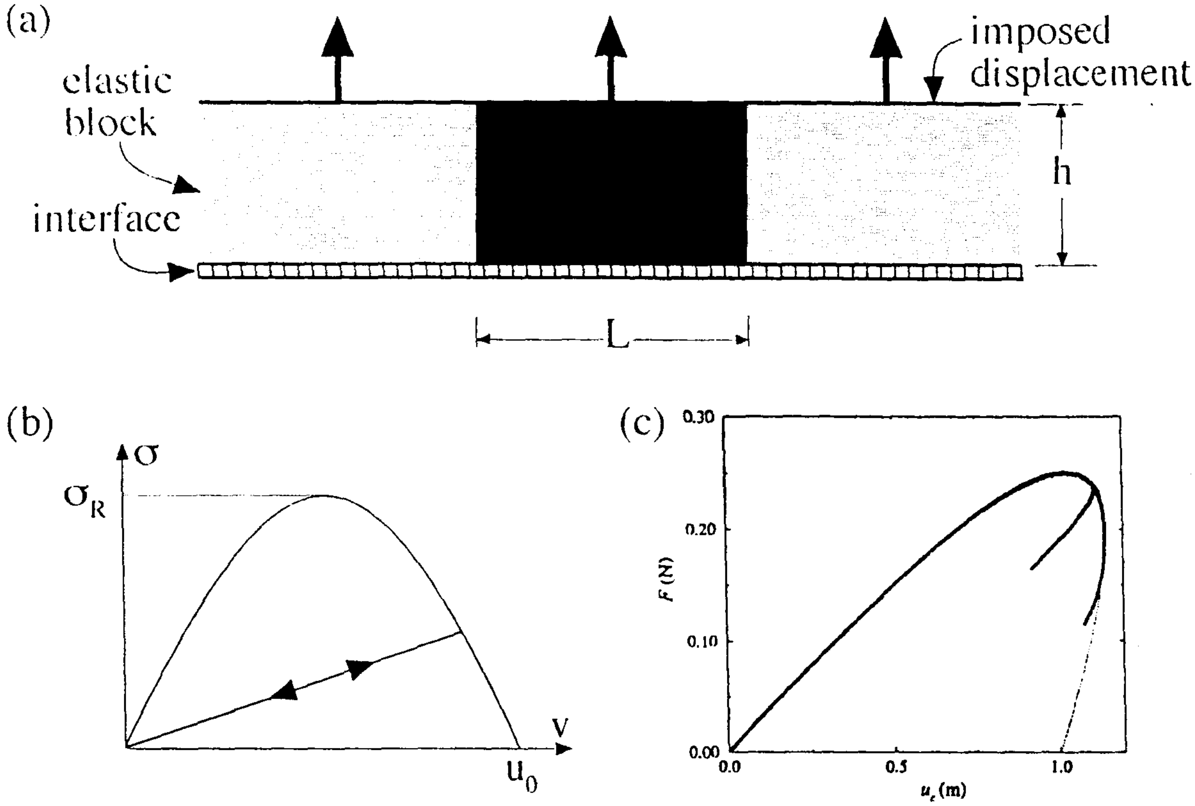

Figure 1. Illustration of the analysis of Delaplace et al. (1999): (a) sketch of elastic blocks and softening interface, (b) stress vs. layer displacement curve: and (c) load vs. displacement curve for two specimen sizes (thick lines) compared to the case of imposed uniform displacement in the layer (dotted line).

sessions. About half of the time of each session was devoted to discussions. The objective of this paper is to present the reports on the discussions, prepared by leading specialists appointed as the session recorders.

Progress in the understanding of fracture is driven by many practical needs, not limited to those of ONR (Office of Naval Research), as explained in the preface of the special issue. As is clear from the reports that follow, fracture scaling as well as the related problems of damage, is a fast evolving subject. Although a consensus has developed on a number of issues, many others remain open, and same are fiercely contested. It is hoped that the present report would help in clarifying the points of agreement and disagreement and stimulate further investigations.

2. Report on Session 1, 'Micromechanical analysis'[1]

The first session focused on the length scales associated with microstructure. J.W. Hutchinson described a unified model of fracture along an interface between a ductile material and a brittle solid. The fracture is modeled using a cohesive zone model and an elastic strip of finite width assumed to separate the plastic zone and the crack tip. Important length scales in the problem include the cohesive zone length d, the elastic strip width D, and the plastic zone

[1]Written by H.D. Espinosa and H. Gao.

height R_0. The limiting cases of $d/D \gg 1$ and $d/D \ll 1$ correspond to the so-called EPZ model (Tvergaard and Hutchinson, 1992) and SSV model (Suo et al., 1993), respectively.

In the discussion of Hutchinson's paper, there was concern about the physical interpretation of the elastic strip parameter D. Is D linked to the TEM observations of dislocation free zones near a crack tip? Hutchinson pointed out that the lack of observable dislocations near a crack tip does not necessarily mean that there is an elastic strip since plastic deformation could occur simply by dislocations emanating from the crack tip and moving to a certain distance away from the tip. There does not seem to be a self-consistent way of determining D. The mechanism leading to an elastic strip is not clear in general. In certain applications, the plasticity free strip is an actual brittle layer between the crack plane and the plastic zone, as in the experiments on multi-layer fracture by Reiner Dauskardt at Stanford University. Here the geometry of the problem clearly defines the parameter D and then the unified model proposed by Hutchinson is particularly useful.

There is agreement that both the cohesive crack modeling and strain gradient plasticity are crucial in the modeling of interface fracture. The strain gradient plasticity model provides another length scale for the problem. This length scale has been used as a fitting parameter (Fleck and Hutchinson, 1997) or has been related to Young's modulus, the yield strength and Burgers vector (see, for example, Nix and Gao, 1998).

It is interesting to note that gradient-based models have also been employed by researchers working on concrete and brittle materials. However, second gradient rather than first gradient models, based on dislocation concepts, are commonly used in describing the constitutive response of quasibrittle and brittle materials. The motivation is to remove mesh dependence and other pathologies in rate independent strain softening models.

From a physical standpoint, the models accounting for kinetics of defects (cracks, voids, transforming volumes) and their interaction are well-posed and naturally lead to non-local constitutive frameworks. In these models, the characteristic length, e.g., distance between active defects, has a clear physical meaning. Within such an interpretation, it can be inferred that the phenomenological strain gradient models can in general be related to nonlocal damage models.

At the present time, it is not clear whether the characteristic parameters used in phenomenological strain gradient theories would hold for all loading regimes. In view of the kinetics of plasticity and damage, dynamic loading may in fact activate a broader distribution of defects than slow loading and therefore the material characteristic lengths could depend on the loading rate. For recent advances in the modeling of fragmentation kinetics at various size scales, see, e.g., Espinosa et al. (1998), or Zavattieri et al. (1999).

The second paper by R. Ballarini and co-workers dealt with crack growth in micro-size specimens with a finite number of grains. The grain size is of the same order of magnitude as the crack size or the width of the specimen, and so the statistical effects of grain orientations become important.

In the discussion of this paper, there were comments about the effects of residual stresses and crack propagation along grain boundaries which have not been accounted for in the analysis given in the paper. Other fundamental mechanics issues that need further research include three-dimensional effects and inelasticity such as phase transformation and microcracking ahead of the crack tip. It remains to be seen whether these effects could also be modeled from a stochastic point of view.

In the third paper, E. Smith presented a cohesive model of crack initiation from an elliptical notch, thus bringing out the root radius of curvature of a blunted crack as an important length parameter. His analysis of a mode I crack was based on analytical solutions for mode III.

In the discussion of this paper, there was concern whether plastic deformation near a blunted notch can still be represented effectively by a Dugdale type model. For example, it is not clear whether the height of the plastic zone could be ignored compared to the root radius of curvature. This type of cohesive model could potentially provide a condition for crack nucleation at a sharp stress concentration. However, the general validity of such an approach has not been clearly demonstrated.

J. Dempsey presented the fourth paper by T. Atkins on scaling laws of fracture in self-similar structures. This work provides some very simple rules of how the fracture stress and energy scale with the overall size of the sample. A particular contribution of the paper is the inclusion of large scale plasticity in the scaling law. A number of critical questions were raised during the discussions. There was concern that the assumption of constant fracture resistance R might not be adequate. It is not clear whether or not the validity of the analysis is limited to the specific geometry considered in the paper. It remains to be seen whether models calibrated for one geometry can provide accurate estimates for other geometries and loading conditions.

An alternative to the approach described in Atkins' paper is the use of micromechanical models, e.g., void models that explicitly take into account the growth and interaction of defects within the solid. In that approach, it is expected that the structure size scale is an outcome of nonlinear analysis incorporating geometry, loading and boundary conditions. It has been demonstrated through such void models that large scale yielding does not necessarily give higher toughening. Moreover, fracture testing of smaller specimens does not always give conservative estimates for fracture in larger specimens. The advantage of void models is that void spacing is built in as an intrinsic length scale and the effect of stress triaxiality on void growth rate is explicitly considered.

From the foregoing discussion of the various presentations in Session 1, it is clear that the current mathematical modeling of the response of solids presents important limitations and uncertainties due to the need for micromechanical models simple enough to be conducive to analytical or computational solutions. With the increase in computational power and the formulation of multi-scale models, from atomic to continuum, assumptions such as the plasticity free strip, or the use of phenomenological length scales in gradient theories, will not be necessary or will be given a true physical meaning.

The challenge is to properly establish the size scales at which atomistic, grain level and homogenized continuum models are needed in the various applications of interest. In this respect, design of experiments that can provide insight into these size scales is urgently needed. Same advances have been made through nano-indentation tests, in-situ AFM and TEM testing, fracture testing and tensile testing of small samples (see Nix and Gao, 1998; Sharpe et al., 1998; Kahn et al., 1998; Espinosa et al., 1999).

As mentioned earlier, consideration of the loading rate may also be quite useful in assessing material size scales because defect kinetics and failure mode transitions have been successfully examined under dynamic loading (see, e.g., Kalthoff, 1990; Rosakis et al., 1999; Fischer et al., 1992). In this context, wave loading is used to examine the response of the material with high time and spatial resolutions.

3. Report on Session 2, Size effect in fiber composites[2]

The four papers presented in session 2 dealt with the scaling of strength and fracture of composites made of long fibers embedded in a polymer or metal matrix. N. Fleck discussed the effect of scale on the strength of joints and on laminates with random waviness of fibers. G.J. Dvorak's presentation dealt mainly with the scaling of fracture in laminates.

Z.P. Bažant reviewed the asymptotic scaling for quasibrittle fracture, discussed the approximate transitional scaling laws obtained by asymptotic matching, outlined their derivations from the J-integral and from linear elastic fracture mechanics, and described applications to fiber composites failing in tension or in compression due to propagation of a kink band with fiber micro-buckling. Finally, I. Daniel followed by presenting experimental analysis of composite laminates, dealing with experiments aimed at capturing the influence of the ply thickness on the strength of the laminates in compression.

Due to their inherent heterogeneity, composite materials can exhibit several types of size effects. Same as for other quasi-brittle materials, it may be convenient to separate the scaling properties of the strength of specimen from those involved in the fracture process.

The latter scaling properties apply to perfectly homogeneous materials and determine the structural effect. For instance, it is well known that the maximum stress of similar fracture specimens of brittle materials (with similar notches) scales as their size to the power of $-1/2$. As explained in Bažant and Planas (1998), this simple scaling law is valid only when the process zone ahead of the crack tip is small, with an energy release dissipated only in this small region. The exponent $-1/2$ is independent of the specimen geometry. When the size of the fracture process zone is not negligible compared to the structure size, the scaling exponent becomes geometry and load dependent.

The scaling properties of the strength of material, as measured on small unnotched specimens, are a more intricate issue. In fact, it is very much tempting to analyze the strength either with the help of micromechanics of fracture (which is an argument reflecting the aforementioned kind of scaling) or to interpret the scaling properties in probabilistic terms, Weibull theory being the most popular technique. The present report is organized according to these two kinds of scaling properties, although it is expected that they may be related.

3.1. Scaling strength

The sources of the strength scaling discussed in the session were of two kinds: (a) The presence of interfaces (e.g. between two joined elastic bodies of different elastic properties); and (b) the presence of defects such as fiber waviness. In the case of an interface, a singularity of the stress field occurs. It results in a tensile stress field that scales with the interface length approximately as

$$\sigma \propto H w^{\lambda - 1}, \tag{1}$$

where H depends on the load and geometry, λ is the order of the singularity (Liu and Fleck, 1999), and w is the distance from the crack tip measured along the interface. Aside from the load and geometry dependence of H, the order of singularity (which also depends on the orientation of the interface with respect to the load applied on the specimen) may be greater

[2]Written by Gilles Pijaudier-Cabot and David H. Allen.

than 0.5, which is considered to yield a mild deterministic scale effect, weaker than in fracture mechanics. It was also noted that the near tip singular field in the interface is not confined to a negligible region but extends to about 10% of the interface length (at a measurable level). However, Bažant pointed that if λ, or its real part, were greater than 0.5, the flux of energy into a propagating crack tip would vanish, which is a problem with this kind of argument.

The fiber waviness, and consequently the effect of the thickness of laminate, is more difficult to analyze since it can hardly be considered as a purely determinist ic problem. Although it can be tackled with the help of the Cosserat model, which is a method for taking into account the bending stiffness of the fibers in a homogenized fashion, or on the basis of stress redistribution due to a sizable fracture process zone at fracture initiation, as described by Bažant, the two approaches presented, i.e. the numerical one by N. Fleck and the experimental one by I. Daniel (Daniel and Hsiao, 1999), end up with evaluating the scaling properties according to Weibull theory, which indicates that the ratio of the mean nominal strengths F of two specimens having volumes V_1 and V_2 should be

$$\frac{F_{V1}}{F_{V2}} = \left(\frac{V_2}{V_1}\right)^{1/m}, \tag{2}$$

where m is the well-known Weibull modulus. It should be noted that, in the materials presented in the session as well as in the literature, the Weibull modulus is always quite high (Daniel computed a modulus of 40); hence the inherent scale effect is very mild, which poses the problem of adequacy of the tests aimed at verifying the theory. In order to obtain experimentally a large enough variation of strength, specimens of very different sizes would be required (some of them cannot be manufactured for practical reasons). Furthermore, is it necessary to consider such a mild effect for practical purposes? Probably it is in the case of proposed very large ships with very thick laminated shells.

3.2. SCALING OF FRACTURE

Due to their inherent heterogeneities, composite materials can exhibit several types of size effects. Same as for other quasi-brittle materials, it can be convenient to separate the scaling properties of the material strength in the specimen from the scaling properties of the nominal strength of specimen governed by the fracture process.

The scaling properties of the strength of material characterize the stress needed for material separation, as observed in quasi-homogeneously loaded specimens. The material characteristic obtained is the maximum average stress of the representative volume that the material can support, independently from how this stress may progressively decrease in the course of the full material separation. It may be analyzed with the help of micromechanics of fracture.

Scaling of the nominal strength of structure in a fracture process is a structural effect with the inherent geometry and boundary condition sensitivity. It depends not only on the initial separation but also on how the material cohesion is getting exhausted. Since the analysis must be carried out on a scale that is much larger than the heterogeneity size, the material can be considered homogeneous. Characteristic quantities such as scaling exponents or fracture energy are averages of the local fracture processes that govern material separation at various locations.

For instance, it is well known that the maximum-nominal stress of very large notched tensile specimens scales to their size to the power $-1/2$. As explained in Bažant and Planas (1998), this scaling property is valid when the process zone ahead of the crack tip is small, with an energy release that is dissipated in the small crack-tip region. For such large sizes, the exponent is independent of the specimen geometry. When the size of the fracture process zone is not negligible compared to the structure size, the apparent scaling exponent becomes geometry and load dependent. These aspects of the scaling problem are not restricted to the field of mechanics, as shown by Dvorak's analysis of heat conduction problems in coated fiber composites with imperfect interfaces.

In the fracture problem, the size effect, which is totally deterministic, is due to energy release and its flow into a fracture process zone of a relatively large size. This kind of interaction between the region that undergoes fracture and the rest of the specimen can be quite complex – for instance in the case of composites loaded in compression, discussed by Bažant, in which axial splitting leads to kink bands or the crack interacts with a region of intense plastic straining (as in the case of H-cracks).

What is the expected scaling exponent for notched specimens of very large sizes? The answer to this question lies in the results of linear elastic fracture mechanics. The exponent is usually $-1/2$, which is related to the order of crack tip singularity (but it seems it might be of larger magnitude, down to -1, in the case of an interfacial crack). An asymptotic expansion of the energy balance expression for large notched specimens (in which the crack growth resistance R is equated to the fracture energy G for quasi-static crack growth) can provide this scaling exponent (see, e.g., Bažant, 1997).

The case of very small notched specimens is more controversial because the size of the fracture process zone is comparable to the specimen size. The same kind of asymptotic expansion of the energy balance expression yields the scaling properties of small notched specimens. This technique assumes that it is possible to expand the energy release expression in a Taylor series as a function of the inverse of structure size, i.e., that the function expanded is smooth enough so that its derivatives exist, with finite values. Under this assumption, one can demonstrate the absence of size effect (i.e., the independence of nominal strength of the structure size) in the small size limit, as can be intuitively expected.

Interactions with the statistical size effect on strength must be expected. In the case of Bažant's size effect law, this interaction could be schematically represented according to Figure 1 which shows only the two asymptotic curves in the case of large and small tensile-like specimens. For small specimens, a statistical size effect is observed. This kind of effect was also observed numerically by Mazars et al. (1991) who used a nonlocal damage model combined with a probabilistic distribution of damage threshold. For practical purposes, e.g., for extending laboratory test results to large-scale situations, the statistical size effect might not be of great importance because one needs to extrapolate the test data for notched specimens toward larger sizes where the statistical size effect is overcome by the deterministic size effect.

One might also address the question of relevance of standard continuum models in the case of very small specimens and the relevance of the scale effect as predicted by the theories. This is an important issue in the context of nanomechanics where materials at the scale of a small number of atomic layers are of concern. Their fracture properties (e.g., those of thin films) still remain to be understood and experimental data discriminating between various scales do not exist.

3.3. DYNAMICS AND RATE EFFECTS

All the results presented during the session pertained to quasi-static loading. Nevertheless, the important issue of rate effect on the scaling properties and ultimately the issue of dynamic fracture have to be raised.

For concrete, and in the range of low rates of loading, Bažant and Gettu (1992) performed some tests from which the rate effect can be observed. These results might be of some use in the case of polymer matrix composites. The rate effect induces a shift of the size effect curve down (lower strength at a lower rate) as well as to the right in the bi-logarithmic size effect plot in Figure 1 (increase of brittleness). When the rate of loading decreases, the fracture process zone gets smaller. Due to rate effects, the stresses are locally relaxed too. This shift of brittleness is also observed when moving into fast loading rates, in the opposite direction: the higher the loading rate, the smaller the scale effect. Supposedly these observations can be explained by means of viscoelasticity and activation energy theories. In the case of H-cracks in composites, it is observed that the plastic crack-tip regions do not develop in dynamics.

Predictions for arbitrary structures require implementing general constitutive relations. Today, it is generally admitted that the quasi-static size effect can be predicted with the help of enriched continuum models such as nonlocal, gradient or Cosserat models (see Muhlhaus, 1995; de Borst and Van der Giessen, 1998). Aside from the Cosserat model where the meaning of the inertia terms involved with the rotational degrees of freedom needs to be clarified, a straightforward generalization of the nonlocal and gradient dependent models to dynamics is not entirely satisfactory.

Nonlocality is due to interaction among defects, which can be assumed instantaneous for quasi-static loading. In dynamics however, such interaction should incorporate a time scale that interacts with the fracture and damage process. Enough time is needed in order for one defect to influence another. Theoretically, for very high loading rates, these interactions should not develop just because there is not enough time. It follows from this very simplistic reasoning that the internal length (at least) exhibited by those materials should be a function of the loading rate. As a result, the effects of scale on fracture and strength would become time dependent, on top of the time scaling due to the inherent material rate sensitivity. These aspects of the prediction of size effects on fracture and damage of quasi-brittle materials are still an open issue.

4. Report on Session 3, Scaling and heterogeneity[3]

Five papers were presented and discussed in Session 3: Delaplace et al. (1999), Fairbairn et al. (1999), van Vliet and van Mier (1999), Borri-Brunetto et al. (1999) and Borodich (1999). The subjects of the papers were quite different except for the last two papers that had in common the fractal nature of certain physical and geometrical entities. As a result, a large part of the discussion in this session dealt in one or another way with fractals and their problems.

It will be helpful to first introduce each of these five papers by a brief summary emphasizing the aspects relevant to the discussion. The discussion itself will then be summarized. This will be done in a subject-oriented manner, rather than according to chronology of the questions raised.

[3]Written by Jaime Planas and Roberto Ballarini.

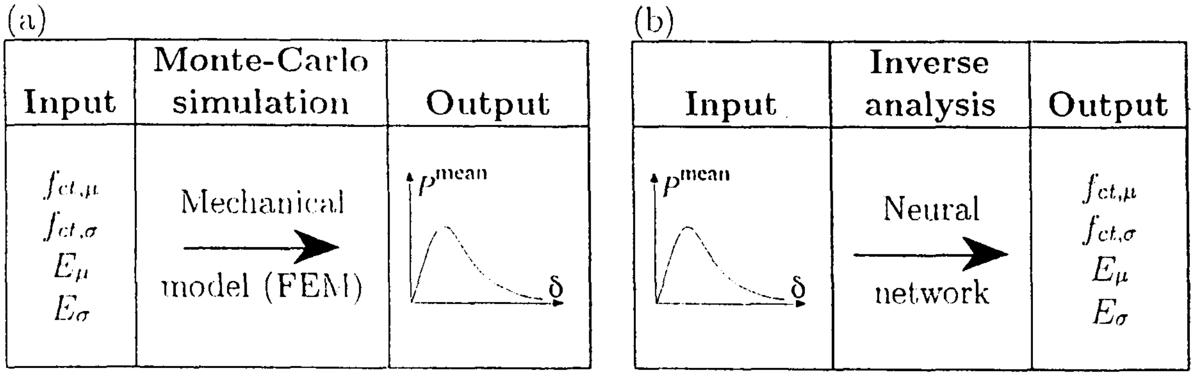

Figure 2. Illustration of the approach of Fairbairn et al. (1999): (a) prediction of mean load-displacement curves from statistical material data; and (b) determination of statistical material properties from experimental load-displacement curves.

4.1. Bifurcation in a softening interface

Delaplace et al. (1999) analyze the behavior of an array of mutually connected, identical elastic blocks of width L and height h, bonded to a rigid layer through a softening interface (Figure 2a). The problem is solved using a discretized form of an integral equation for the displacements within a half-space together with imposed periodic boundary conditions (having spatial period L). The tangential stresses are assumed to be zero along the interface, which implies either that there is free sliding over the rigid substrate, or that the interface lies along a symmetry plane. The normal stress transferred through the interface is assumed to depend on the displacement jump normal to the interface. In the paper, the force-displacement relation for the discrete elements is given, but this can be easily recast as a relation of stress σ versus jump displacement v which, for monotonic stretching, has a parabolic shape as depicted in Figure 2b. A damage variable is introduced to make the interface unload to the origin (it is a stiffness-degradation model, see Figure 2b).

Note that with the $\sigma(v)$ relation it is easy to identify the tensile strength of the interface σ_R and the critical jump u_0 for which the interface is fully broken (Figure 2b). The fracture energy of the interface, although not used in the paper, can be computed as the area under the $\sigma(v)$ curve, and thus $G_F = \frac{2}{3}\sigma_R u_0$. Especially relevant for the discussion are the results in Figure 2c, showing the load-displacement curve for two different sizes.

During the discussion, same other topics were addressed, not directly related to the paper (decay of microcrack interaction, gradient vs. implicit gradient models). But the discussion finally concentrated on the following, mutually related, questions: (1) Is there a characteristic length in this problem? – and (2) is there a size effect? The answer was not clear at the time of the discussion, in spite various thoughtful contributions (Bažant, de Borst, Jirásek, Barsoum, Willam, Planas).

After the session, a discussion between Pijaudier-Cabot and Planas led to the following tentative conclusions: (1) There is a characteristic length 'hidden' in the compliance of the layer, namely u_0 in Figure 2b, or alternatively in Irwin-type (or Hillerborg-type) characteristic length, which is $EG_F/\sigma_R^2 = (2E/3\sigma_R)u_0 \propto u_0$; (2) there is no size effect in the pre-bifurcation regime (which includes the peak stress for the cases analyzed); (3) there is size effect in the post-bifurcation regime, as shown in Figure 2c. The last two conclusions are related to the issue addressed by Planas in the general discussion, namely that a size effect must be expected only when stress gradients are present; for this case the stress state is uniform

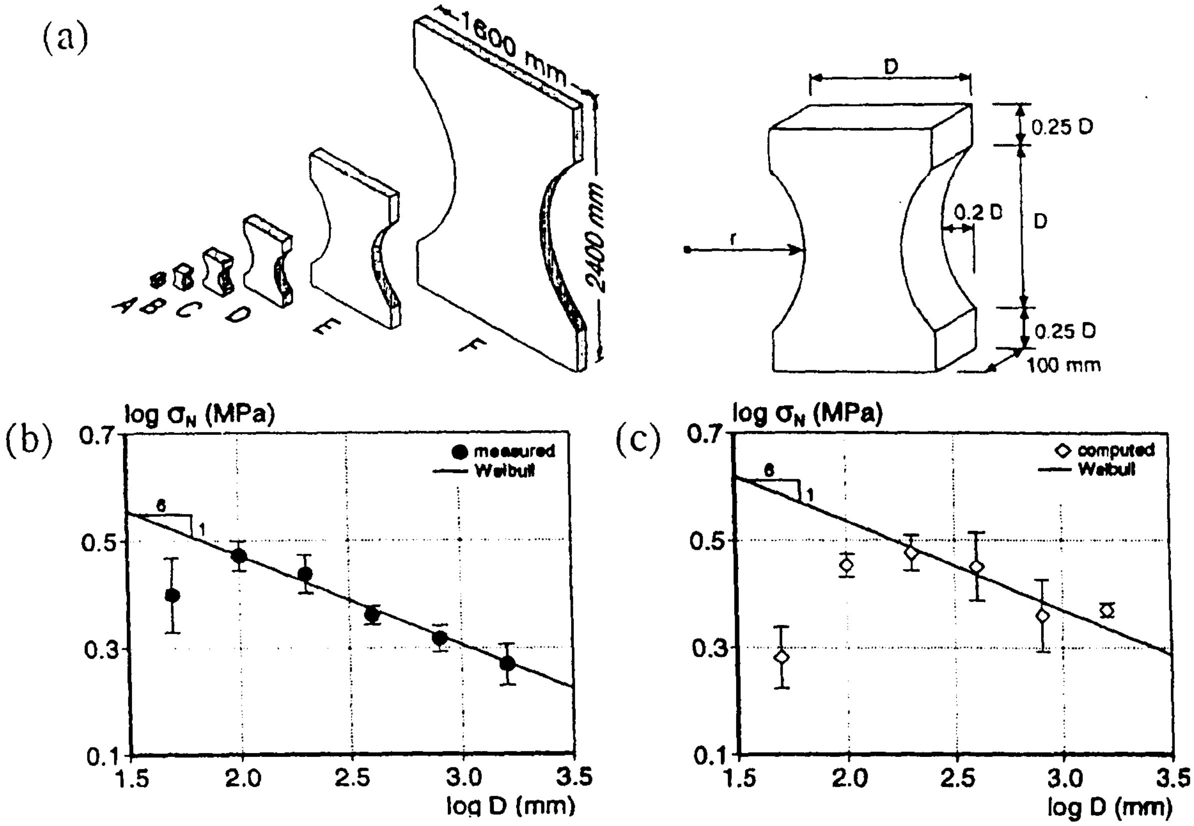

Figure 3. Illustration of van Vliet and van Mier's (1999) tests: (a) specimens; (b) measured size effect; and (c) size effect computed from a brittle-elastic model with macroscopic nonuniformity.

in the pre-bifurcation regime and a size effect will occur only in the post-bifurcation regime. Further work is needed to ascertain whether the bifurcation point itself is size-dependent or not.

4.2. Neural networks and probabilistic scaling

Fairbairn et al. (1999) present a finite element based neural network methodology for calculating, from the given response of a concrete structure (the load-displacement curve, for example), the statistical distributions of material strength and elastic modulus. They assume that these properties are normally distributed, with mean values $f_{ct,\mu}$ E_μ and standard deviation $f_{ct,\mu}$, E_σ. In the first part of the paper, they summarize the direct Monte Carlo finite element approach following the sketch shown in Figure 3a.

The input parameters include the mean values and standard deviations of strength and elastic modulus, which depend on the volume of the coarsest aggregate according to the empirical formulae included in the paper. These are used together with a random number generator to prescribe, within each element of the discretized structure, random material properties. The finite element method is then used, for n samples, to calculate the progression of damage evolution and the associated load-deflection curves, from which an average curve is then established. The authors apply the direct approach to a notched bend specimen tested at TU Delft (Hordijk, 1991) and find that while the softening region of the load-deflection curve is not predicted accurately, the peak load and the onset of softening is predicted quite well.

The authors propose that the results of these types of calculations be used in conjunction with neural networks to solve the inverse problem. As shown schematically in Figure 3b, the load-displacement curves generated using the direct approach, along with their statistical material property parameters, are used as training data for the neural network. The trained neural network represents a curve fit that relates load-deflection curves to the means and standard deviations of strength and elastic modulus, and can therefore yield the statistical parameters associated with any additional load-deflection curve provided as the input.

The following points were raised as part of the discussion. Xi suggested that the standard deviations of material properties should decrease with decreasing aggregate size. Ulm replied that since this approach is of the Weibull type, a correlation length does not exist; the neural network cannot capture this physics. Willam asked whether the model captures the effect that the bending tensile strength of normal size laboratory beams is typically 80% greater than the direct tensile strength. Ulm said yes, and added that the statistical effect is even greater in structures of a very large volume such as containment vessels.

Jirásek inquired whether the tensile strength used is actually a parameter of the model, and Ulm replied that indeed it is. Jirásek then commented that, in the model presented, the fracture energy is actually controlled indirectly through the standard deviation parameters. Karihaloo commented that inverse problems are ill posed, and that one may want to address uniqueness of the calculated solutions.

Pijaudier–Cabot asked why Gaussian rather than Weibull distributions were assumed. Jirásek replied that the experiments indicated Gaussian behavior. Pijaudier–Cabot disagreed, stating that, in order to capture the extreme values of failure probability, the tails of the distributions are essential. Finally Planas inquired as to whether the neural network method could be used for nonprobabilistic models, such as the standard cohesive zone models. Ulm replied that this is in principle possible and that there are other possible applications; for example, he is currently interested in predicting crack spacing.

4.3. STRAIN GRADIENTS AND SIZE EFFECT IN UNIAXIAL TENSION

Van Vliet and van Mier (1999) presented a detailed experimental analysis of size effect on concrete dog-bone shaped specimens (Figure 4a), subjected to slightly eccentric uniaxial tension. During the discussion, Reinhardt pointed out that the combination of the taper and the load-eccentricity mimic a notch; van Mier replied that, even if the stress concentrations could be similar in both cases, the relative volume subjected to high stresses is much larger in the dog-bone shaped specimens. The specimen size D ranged from 50 to 1600 mm, while the specimen thickness was constant and equal to 100 mm. The maximum aggregate size was 8 mm. Although many other interesting details are included in the paper, the 'raw' size effect results in Figure 4b were basic for the discussion.

The discussions addressed several experimental details, such as a possible influence of the stiffness of the frame (Perdikaris). According to van Mier, this stiffness was not important except for small instabilities when testing small specimens in larger machines (three machines were used because no single machine could accommodate all the sizes; the specimens of some sizes were tested in two machines to provide overlap checking).

Another detail was the size of the smallest specimen, whose minimum dimension might be too small (Carpinteri). Van Mier indicated that this was probably true, and that according to his experience the minimum size of the specimen should be 8–10 times the maximum aggregate size, and that one needs to be very careful in specimen preparation. In the general

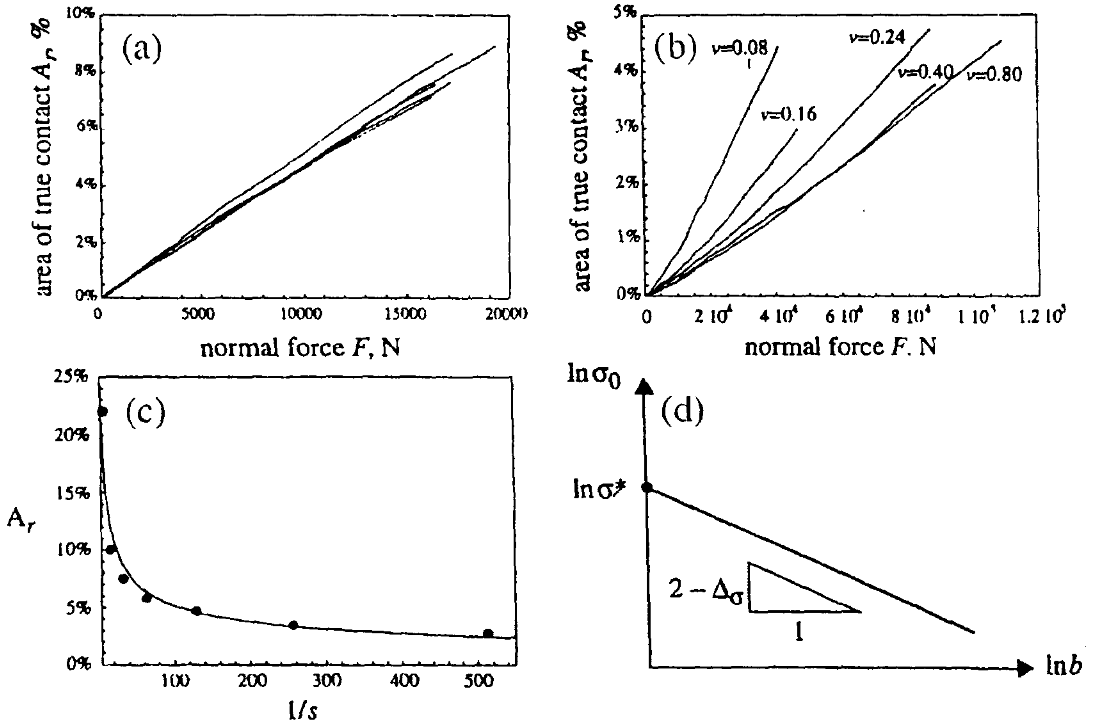

Figure 4. Illustration of the analysis of Borri-Brunetto et al. (1999): 'true' contact area vs. normal load for (a) rock and (b) concrete; (c) power-law decrease of real contact area with increasing resolution; and (d) power-law decrease of 'true' pressure with size.

discussion Bažant pointed out that, since all the specimens were equally thick, a pronounced three-dimensional action was to be expected in the smallest specimens; van Mier agreed.

The greatest interest, however, was generated by van Vliet and van Mier's analysis of the results in Figure 4b. Additional experimental work showed that, due to the manufacturing process, a through-the-thickness variation of material properties was present in the specimens, and subsequent elastic analyses using three-dimensional finite elements, as well as simplified beam-theory, showed that the material non-uniformity caused a substantial modification of the theoretical strain and stress profiles (as calculated for a homogeneous material).

The main conclusion is that the essential trends of the observed size effect (Figure 4b) can be reproduced using a simple elastic-brittle model with a critical-strain fracture criterion if the nonuniformity of the elastic modulus and the experiment al curvatures are taken into account (Figure 4c). This implies that the statistical (Weibull's) size effect, which would apparently be a good explanation of the size effect (dashed lines in Figures 4b and 4c) is actually secondary. This was one of the main subjects of the discussions (Carpinteri, Planas, Belytschko, Jirásek), and the answers of van Mier were categoric – he asserted that the observed size effect is mainly due to the property gradients, not to the randomness of strength (Weibull). Karihaloo pointed out that if this is true, then other gradients such as, e.g., shrinkage eigenstresses, should also induce or modify the size effect; van Mier agreed and indicated that this was demonstrated by the different behavior of wet and dry specimens.

4.4. Fractal contact

Borri-Brunetto et al. (1999) analyzed the contact between fracture surfaces considering the topography of the actual fracture surfaces for rock and concrete, discretized on a 0.1 mm spacing square grid. Using the discretized geometry, the authors carried out a numerical analysis of the process of separating the two fracture lips, displacing them a distance v parallel to the mean fracture surface (v being the relative shear displacement), and pressing the surfaces against each other with an increasing force. The development of contact areas as a function of the applied normal force was computed using an influence function method.

The results completely characterize the mechanical response of the joint under normal force for any fixed shear displacement, and show, among other things, that the 'real' contact area A_r is proportional to the applied force (Figures 5a and 5b). However, the contact domain has a fractal structure (pre-fractal, according to Carpinteri's denomination, because the minimum resolution is finite and equal to the spacing of the discretization grid). Thus the 'true' contact area depends on the resolution of the measurement (the length s of the measuring stick; Figure 5c).

Therefore, the 'true' contact stress, too, depends on the resolution, and a fractal mean pressure σ^* can be defined by the sequence of equalities

$$F = \sigma_0 A_0 = \sigma_1 A_1 = \ldots = \sigma_n A_n = \ldots = \sigma^* \mathcal{H}_{\mathcal{D}}, \tag{3}$$

where F is the applied normal force, A_0 the apparent contact area, σ_0 the apparent pressure, $A_1, \ldots, A_n, \ldots$ the areas measured for increasingly refined resolutions $s_1, \ldots, s_n, \ldots$, and $\mathcal{H}_{\mathcal{D}}$ the fractal measure of the contact domain. From this, a power law is obtained for the plot of the apparent stress versus the size (Figure 5d, where $b \propto \sqrt{A_0}$). This is used by Carpinteri to conjecture that the apparent shear stress should also decrease with size, and that this would make the friction coefficient size dependent, as it appeared to be the case in experiments.

The conclusion that fractality of the contact surface explains the size dependence of friction coefficient was refuted during the discussion by Jirásek based on the fact that an equation similar to (3) can be written for the shear components (substituting T, τ_i and τ^* for F, σ_i and σ^*, respectively). Combining the equation for shear with Equation (3) it follows that

$$\mu = \frac{T}{N} = \frac{\tau_0}{\sigma_0} = \ldots = \frac{\tau_n}{\sigma_n} = \ldots = \frac{\tau^*}{\sigma^*} \tag{4}$$

and since τ^* and σ^* are independent of size, so is μ at any level of observation.

The various remaining contributions to the discussion were associated with general aspects of the fractal approach rather than to the contact problem. Same of these were taken up again in Borodich's presentation and in the general discussion. To avoid repetitions, they will be summarized in the following together with similar observations.

4.5. Fractal scaling in fracture

Borodich's (1999) paper discusses fractal concepts and their application to fracture. One of the basic points of the paper is that fractal features can be treated by using (1) purely mathematical models (true fractal objects), or (2) physical, or experimental, fractals, which display a fractal behavior over a limited range of measurement resolution. For both types of formulation an expression for the fracture energy absorbed over a region of size X is given. In both cases the fracture energy has a power-law form, but in the later case the range of applicability is limited, i.e.:

$$\langle W_F \rangle = w_f \left(\frac{X}{h}\right)^D \quad \text{for} \quad h < X < \Delta_1, \tag{5}$$

where, in this approach, h and Δ_1 are the lower and upper cutoffs of the fractal law, respectively, D is the fractal dimension, and W_f is the work of fracture required to break a cube of material of side h. All these parameters are assumed to be material constants. Outside the fractal range the foregoing equation is not valid. Borodich gives the equations to extend the calculation over a second fractal range $\Delta_1 < X < \Delta_2$, with fractal dimension D_2.

During the discussions of the presentations of Carpinteri and Borodich, and in the subsequent general discussion, the following points relative to the fractal approaches were addressed. (1) The physical fractals follow the power law on ranges that span typically between 1.5 and 3 orders of magnitude (Borodich). (2) Same published plots of surface roughness fractality appear to span 6 orders of magnitude (as Pijaudier–Cabot pointed out) but these are compilations of various overlapping measurements on different surfaces (Borodich). (3) The fractal approach is directly applicable to energy concepts but is difficult or not applicable at all to stress concepts because a fractal surface is nowhere differentiable and the normal to it is not defined (Ulm, Borodich).

A debate took place, spanning the three discussion periods, regarding the importance and the necessity of the fractal approach. The ubiquity and importance of fractal phenomena was emphasized by Carpinteri. Borodich agreed; however, he pointed out several of its present limitations – the stress not being defined on a fractal surface, or the impossibility of using the classical approaches to boundary value problems.

Several other participants (Bažant, Planas) emphasized that in today's state of the question, the fractal approach is more geometrical than mechanical, and that its predictive power is inherently limited (Bažant called attention to the fact that when a fractal based size effect is calibrated for one structure geometry, there is no way to predict it for another geometry). Others indicated that there are alternatives to the fractal approach based on classical statistical formulations or deterministic approaches (Palmer, Bažant). Aifantis pointed out that fractality may be a complementary explanation of classical phenomena, such as softening, and that having a classical explanation for them does not preclude looking for other explanations.

Jirásek raised an important point that did not catch the attention it deserves: There seems to be a confusion between the resolution effect and the size effect, which are not the same. Related questions were raised by Belytschko and Dvorak, regarding the size effect when bigger structures are built by 'pattern repetition' rather than by 'pattern expansion'. Belytschko also commented that, at very small scales, molecular dynamics models may be required to model the physics of the phenomena. These points seem worthy of being addressed in the future by specialists.

5. Report on Session 4, 'Computational aspects and nonlocal or gradient models'[4]

De Borst opened his talk with some examples related to delamination of composites, which were not contained in his journal article. He stressed the importance of the manufacturing process which generates significant initial stresses affecting delamination in ply composites. This type of failure also exhibits a size effect in the sense that the onset of delamination in thicker composites is observed at lower stress levels. Predictive simulations can be carried out with the cohesive crack model. On the other hand, for heterogeneous materials such as concrete, the smeared (continuum-based) approach is considered by de Borst as more appropriate.

[4]Written by Milan Jirásek.

The main part of de Borst's presentation was an overview of continuum-type models for concrete damage and fracture. He emphasized the need for an efficient localization limiter and observed that it can be provided by nonlocal or gradient enhancements of standard constitutive theories. He demonstrated that gradient-enhanced models can capture size effects. The key to success in obtaining the correct curved crack trajectory for the fracture test of Iosipescu geometry was not only a gradient enhancement but also proper modeling of the crack-induced anisotropy of the material.

One important issue addressed in the discussion was the suitability of continuum models at late stages of the material degradation, when a macroscopic, stress-free crack propagates across the specimen. De Borst mentioned that numerical problems appear after the complete loss of strength and stiffness of certain finite elements. One simple remedy is to remove such elements from the mesh, as suggested by Rots. However, as pointed out in the discussion, this is not completely justified, since in reality the material can still transmit stresses parallel to the crack. A more realistic representation of the actual behavior can be obtained by re-meshing and representing the stress-free crack as a geometric discontinuity. The reporter has developed an alternative approach, in which the discontinuity is embedded into the existing finite element(s) and no re-meshing is needed.

Responding to another question, de Borst clarified that the finite element implementation of gradient models typically requires the development of special mixed finite elements with additional global degrees of freedom that describe the distribution of the nonlocal variable(s). Finally, Bažant pointed out an important difference between the so-called explicit and implicit gradient models. In explicit models, the nonlocal variable is directly affected only by an infinitesimally small neighborhood of the given material point, because it is defined by a differential operator applied on the corresponding local variable. On the other hand, implicit gradient models define the non local variable as the solution of a certain differential equation of the Helmholtz type, and so they are equivalent to (and not only approximations of) nonlocal integral formulations with special kernels constructed as Green's functions of the Helmholtz differential equation. In this latter case, some longer-range interactions are automatically included.

The talk of Aifantis on gradient models stimulated a passionate discussion. His central theme was the identification and objectivity of the parameters appearing in such models. Aifantis obtained very good descriptions of the size effect on the stiffness, yield stress and hardening curve of various materials (bones, foams, metals) under various loading conditions (torsion, bending). However, it is not completely clear to which extent his theories, going beyond ad hoc fitting of the available data, have a predictive power. There is no doubt that the gradient-enriched models are inspiring and help to better approximate the mechanical behavior on scales close to the characteristic length of the material structure.

As summarized by Aifantis in a single, concise statement, we cannot live without gradients – in the sense that without enriched theories we cannot reproduce certain systematically observed phenomena. However, it is also legitimate to ask whether it is possible to construct a unified theory that would cover a reasonably wide range of conditions (such as specimen size and geometry, boundary conditions, type of loading) within which the parameters can be considered as constants, provided that the material remains the same.

Of course, no constitutive theory can be expected to be completely universal. It is therefore important to detect and clearly specify the limits of applicability of each theory that is to be used outside the range of measurements from which the parameters were originally identified. In this area, much work still needs to be done, especially on the experimental side.

In Aifantis' paper, each example refers to a different material, with the exception of foams, for which both torsional and bending experiments are available. The theoretical description of both cases is derived by specialization of the same three-dimensional constitutive model that uses only one gradient parameter, c. For polymeric foam, the best fits for torsion and bending are obtained with $c = -3.45$ mm and $c = -2.82$ mm, respectively, and for polyurethane foam with $c = -0.48$ mm and $c = -0.35$ mm. This agreement is quite satisfactory, given that the gradient parameter is extracted from the deviation of the measured effect of size on stiffness from the behavior predicted by the standard elasticity theory, which greatly amplifies the influence of experimental scatter. The order of magnitude of this parameter agrees with the size of the cells (dominant material inhomogeneity), which was 1 mm for both foams.

Later in the general discussion, Gao suggested that it should be possible to develop simple gradient models with no parameter adjustment, starting from a description of the mechanical processes on the microscopic level. He pointed out that the multi-scale framework naturally leads to gradient plasticity based on dislocation models, and he briefly presented his results. Aifantis replied that while Gao's model was certainly very nice, it was not a model that could be used for an arbitrary deformation pattern and on an arbitrary length scale. If the material itself is not constant, it is natural to expect that the model parameters evolve, too.

There were also other questions regarding the theoretical basis and physical justification of gradient models. Several of them were concerned with nonstandard boundary conditions that are usually required by models incorporating higher-order gradients. Ulm was puzzled by the fact that such conditions contain internal variables, which by definition should not be controllable from the outside. What is the physical meaning of the boundary flux of such a variable?

Aifantis suggested that this flux could represent dislocations sent from the boundary, and Gao supported this idea, citing micro-indentation as an example. Bažant and Willam pointed out that if such an interpretation is adopted, one should also take into account the plastic spin, non-symmetric stresses, etc. Aifantis stated that non-symmetric stress should be used only when absolutely necessary, because it complicates the formulation of a meaningful yield condition. Belytschko mentioned that the issue of boundary conditions is, for example, thoroughly explained in the papers by Fleck and Hutchinson, and he emphasized that similar enrichments have been widely accepted in the theories of heat conduction and are nowadays considered as standard, so why is there so much opposition in this community?

After the workshop, Voyiadjis submitted the following written comment: 'Aifantis suggested the gradient approach to deformation in order to describe plastic instabilities including dislocation pattern(s) and spatial characteristics of shear bands. De Borst et al. included the gradient term in plasticity models by means of the yield function. However, these gradient terms are introduced by taking the gradient of the internal state variables defined at the macro-scale. Gradient terms related to damage and plasticity should be first defined at the meso-scale level, in order to provide sufficient details of defects and their interactions to properly characterize the physics of the material behavior. The higher-order internal state variables should then be averaged and converted into their macro-scale counterparts. This involves the additional task of averaging the evolution expressions from the meso-scale in order to obtain the corresponding evolution expressions of the internal variables at the macro-scale'.

Allen presented a rate-dependent model for polycrystalline materials. His model combines viscoplasticity for grains with a rate-dependent cohesive zone model for damage at grain boundaries. Aside from other examples, he showed an application to Wipp salt. The discussion focused mainly on the constitutive description of the grain boundaries. The author explained that the physical origin of rate dependence is to be sought in the viscous behavior of fibrils connecting the neighboring grains, and that the model can be calibrated by fracture toughness tests run for three modes of fracture at three different loading rates. It can be interpreted as a rate-dependent extension of the Tvergaard–Hutchinson model. Consequently, the damaging boundary does not dilate when subjected to pure shear traction. It was pointed out by Sture that this is not completely realistic, since experiments under pure shear typically generate volume density changes, which can be attributed to dilatancy at grain boundaries.

The reporter is not sure on which scale such experiments were performed. It is probably very difficult to impose a state of pure shear directly on a single grain boundary. If an assembly of grains is loaded by pure shear, mixed-mode tractions are generated on randomly oriented grain boundaries. The normal tractions are positive on some boundaries and negative on others, but the normal separation can only be positive. Consequently, the overall volume increases even without the dilatancy effect on individual boundaries.

Willam was wondering about the large stresses in a propagating cohesive zone shown in one of the figures presented in the talk but not contained in the original paper. Contrary to the author's explanation that the large magnitude of these stresses was due to boundary effects, the reporter's opinion is that the primary source is the rate effect. According to the constitutive law used, the cohesive stresses uniquely depend on the history of the separation vector (displacement jump) at the grain boundary. For a typical rate-independent softening law, they cannot exceed the tensile strength. However, the model used involves a rate-dependent evolution law for the damage parameter, and if the separation grows fast, the damage does not have enough time to evolve while the effective traction increases instantaneously. This then results into a nominal traction exceeding the (static) tensile strength.

The paper originally submitted by Pandolfi, Krysl and Ortiz was presented by their coworker Ruiz. In the paper, fragmentation of an aluminum ring in a magnetic field is simulated using a cohesive zone model. Ruiz added a new example of a split Hopkinson bar. The bulk material is represented by a viscoplastic model with mechanical hardening and thermal softening. Cohesive zones are inserted at interfaces between finite elements when the normal traction reaches a certain critical level. The model is conceptually similar to that presented by Allen, however, with certain differences. Here, only the bulk material is rate-dependent, while the cohesive interfaces are not. This means that the traction transmitted by the interface cannot exceed the static strength. In Allen's work, the cohesive interfaces have a physical meaning of grain boundaries, while here they represent only potential crack locations dictated by the geometry of the finite element mesh.

This fact motivated a question regarding the mesh-sensitivity of the results. According to Ruiz, the adopted re-meshing strategy guarantees objectivity of the solution. It was pointed out by Gao that the initiation criterion is based on stress, which may be affected by the mesh size. De Borst added that, if the actual stress field has a singularity, the crack can be initiated at very small load levels provided that the mesh is sufficiently fine. This is of course true, but it does not invalidate the model. The onset of the separation process is controlled by stress, but the process itself is controlled by energy. Upon mesh refinement, the load at which the load-displacement curve starts deviating from linearity converges to zero, but the deviation remains negligible as long as the load level remains sufficiently low, and the entire load-displacement curve converges to a physically meaningful solution.

Additional comments were related to the shear part of the cohesive traction-separation law, especially in relation to the Hopkinson bar simulation. Bažant questioned the fact that the model implies the stress resultant in the cohesive crack to have the same direction as the relative displacement vector, which implies that no normal compressive stress is generated in the case of pure shear slip without normal displacement, while in real materials dilatancy always generates such normal stress. He also recommended that the authors look at the test data for shear of concrete analyzed at Northwestern by Gambarova. Ruiz admitted that they had not calibrated the model for shear. According to him, the initiation in Mode II is excluded due to grain interlock. Willam pointed out that there is certainly some shear crack initiation at the specimen boundary around the indentor, and that this should be included in the model.

Finally, in the general discussion, Belytschko brought the attention of the audience to his recently developed numerical technique that can incorporate an arbitrary crack into a finite element mesh in a simple and efficient manner, without any re-meshing.

6. Report on Session 5, Size effects in concrete, ice and soils[5]

The papers presented in Session 5 dealt, in the broad sense, with demonstrating the importance of the analysis of size and geometry effects on quasibrittle fracture. The arguments for such analyses are basically related to one of the following two complementary issues: The prediction of the behavior of 'real' large-scale structures, and the determination of fundamental material properties. The size effects are of significance in the prediction of large-scale behavior since the techniques used in such prediction can mostly be validated only with data obtained from (smaller) laboratory-scale experiments, and since it is normally assumed in such analyses that the material behavior is size-independent. On the other hand, the relevance of size and geometry effects in the experimental determination of material properties lies in the use of these trends as the means of obtaining size-independent values or the size-dependence of the parameters quantified in standard laboratory tests.

In the first of his two papers (Dempsey et al., 1999a), Dempsey presented the tests of freshwater lake ice over the size range of 1:81 and the analysis of the data obtained. The development of an experimental program in which specimens of different sizes were cut from floating ice sheets and the loads were applied on the floating specimens permitted the wide scale ranges used in this paper and the companion paper. For obvious reasons, the material characteristics were not ideal, which gave rise to two issues that may have influenced the results: The irregularity of the grain size, with grains of 20 cm at the bottom of the sheets and grains 0.5 cm at the top, and the relatively high temperature of the freshwater ice (higher than −5 °C).

Due to practical problems with the usage of the beam specimen, a reversed-taper geometry was used, which was easier to prepare. This geometry is characterized by stable crack extension, i.e., a negative geometry, as designated by Planas and Elices (1990). The data were analyzed using the size effect model (SEM) of Bažant (1984), in which the authors expected to obtain the complete ductile-brittle transition in the failure mode but found that nearly all the results fell close to the linear elastic fracture mechanics (LEFM) asymptote. Three explanations can be given for the observed trend.

First, the size effect model applied by the authors is valid only for positive geometries and not for specimens such as those used here, in which there exists stable propagation of a traction-free crack, which is taken into account explicitly in the analysis. Second, the brittleness number $\beta = L/L_0$ would have to be very small for the ductile failure mode to occur. However, this would require the specimens to be smaller than those that can used in real testing and would in fact be impossible since the corresponding sizes would be of the

[5]Written by Ravindra Gettu and Franz-Josef Ulm.

order of the grain size and smaller than the representative volume of the material. Third, an aspect that may be important is the temperature of the ice, which could influence the behavior considerably, as pointed out in the discussions by Elfgren. Due to the occurrence of creep near the melting point, the resulting stress re-distributions may lead to a smaller process zone and more brittle behavior, as observed in concrete (Bažant and Gettu, 1992).

In the second paper of Dempsey et al. (1999b), results for an even wider size range, 1:160, were reported, this time for sea ice sheets. The relevance of this study is supported by the fact that floating sea ice sheets exist on scales of meters to kilometers, and therefore, the issues of fracture scaling of such sheets has significant implications for the dynamics of the Arctic regions and the exploitation of its resources. Square plate specimens were cut from a 1.8 m thick ice sheet. They ranged from 0.5 m to 80 m in size.

A satisfactory prediction of the test results of the larger specimens was obtained with Bažant's size effect model after calibrating it with the data from smaller specimens. However, the ductile mode of failure that was expected in smaller sizes was, again, not clearly evident. This could be attributed to the small grain size of the ice but the question posed in the discussions remains unanswered: Does the real trend of the ductile-brittle transition in the failure mode correspond to that of the size effect model?

The reason that such a wide range of sizes did not reveal this transition may be that ice specimens of much smaller horizontal dimensions than the ice thickness of 1.8 m are not proper fracture specimens. More importantly, however, it appears that the behavior near the plastic asymptote is more a theoretical issue than a practical one.

On the other hand, the results of these tests show that the fractal scaling model represents the trend inadequately. The results support modeling of the size effect in terms of fracture mechanics.

The paper of Planas et al. (1999) described in detail the use of geometry effects for determining the fracture parameters of concrete. In their inverse analysis procedure, peak loads from Brazilian splitting tension and notched beam tests are used to obtain the initial slope of the tensile stress-separation relation. They state that this part of the curve is sufficient for failure analysis in most practical cases since the process zone is just beginning to develop when the maximum load is reached. In the splitting test, the use of a loading strip of width of 0.08 times the specimen diameter (about one half of the standard width) is recommended in order to reduce size effects. Also, the use of displacement control is indicated as preferable since load control can lead to a sudden failure resulting in higher loads (as also shown by Carmona et al., 1998). Replacement of the splitting tension test with that of an unnotched beam was suggested by Bažant in order to avoid the high compressive stresses generated in the former configuration. However, Planas mentioned that the high non-linearity in the pre-peak response of the latter was a disadvantage, in order to justify the two geometries recommended in their approach.

The paper of Karihaloo (1999) took a different look at the fracture mechanics type size effect in quasi-brittle materials. He postulated the existence of a minimum size for the applicability of scaling laws such as the SEM, and his objective was to derive this minimum size. His underlying idea was that at failure the fracture process zone (FPZ) is fully developed in the specimen. The procedure for obtaining this limit was presented and illustrated for a three-point bend specimen, as a function of the material fracture properties and structural dimensions. However, only the asymptotic limit of linear elastic fracture mechanics (LEFM) was considered in this analysis. The asymptotic matching to the small-size asymptotic limit of plastic fracture, on which other models such as the size effect model are also based, was not taken into account, as pointed out in the discussions by Planas, Bažant and Jirásek. This suggests that the analytical formulae obtained in the paper should be valid only close to the LEFM limit. Thus, Karihaloo's approach appears to lead to an alternative scaling that depends only on the asymptotic near-LEFM trend. Furthermore, the procedure adopted by Karihaloo appears to be recursive since the limiting values are determined from SEM parameters that may have been obtained from data falling much below the LEFM limit. In addition, these values were computed using a function, denoted as $Y(a)$ in Equation (28) of the paper, that gives significant errors for small values of a, as shown by Pastor et al. (1996). Planas also stated that Karihaloo's conclusion regarding the minimum size for the applicability of the size effect law must be significantly revised if all of the relevant asymptotic terms are included.

Furthermore, Planas and others observed that the integral equation of Karihaloo is similar to the approach of Horii et al. (1989) in that only first-order kernels corresponding to infinite size are used. However, as Planas emphasized, the second-order kernels must be included to achieve a consistent two-term asymptotic approximation (see Planas 1989). Also, it was stated that Karihaloo's assumption that the peak load occurs when the stress at the crack-tip reaches zero is true only for infinite-size specimens, as shown by the studies of Petersson (1981) and Planas and Elices (1991, 1993). The statement of Karihaloo that changing of the softening curve (which is nearly linear for large sizes) does not significantly after the numerical factors in the results was also questioned, based on published conclusions from asymptotic analyses (Planas and Elices, 1991, 1992, 1993; Bažant and Planas, 1998). Moreover, Karihaloo's opinion that changes in the shape of the softening curve would affect the numerical results only slightly was questioned (Planas et al., 1999). The asymptotic analysis of Planas and Elices (1991, 1992, 1993) in fact showed that $l_{p\infty}$ and $\Delta\alpha_{c\infty}$ depend strongly on the shape of the softening curve, for a material with a given fixed tensile strength and fracture energy (Bažant and Planas, 1998, Section 7.2.4). As another point, the softening stress-displacement curve implied parametrically by Karihaloo's approach is found to be size-dependent, which means that the material is not defined objectively. Finally it was emphasized that the applicability of SEM through the entire size range is now supported by extensive experimental results as well as extensive numerical simulations of various types (Planas et al., 1999)[6].

In the fourth paper (Ožbolt et al. 1999), Ožbolt reviewed the size effects on the pullout failure of anchors in concrete, and analyzed the test data and the results from smeared crack modeling of the problem. The comparison of test results obtained for different anchor embedment depths and anchor head sizes with LEFM and with the size effect model was made by assuming the initial defect to be proportional to the embedment depth or the size of the anchor head. This aspect is significant since, in the analysis of unnotched elements, the size effect model can be applied by supposing either a defect of constant size, which is a reasonable assumption, or a defect whose size is proportional to the structure size. This difference has often led to misinterpretations of the trends given by the size effect model and their implications (Gettu et al., 1998). During the discussion, Elfgren pointed out the importance of the boundary conditions in the numerical simulations, which can considerably influence the mode of failure and the response (Elfgren et al., 1998).

In the fifth and last paper, by Sture et al. (1999), Sture considered geotechnical implications of the size effects on fracture, especially as related to the analysis of stability of slopes and excavations, where tensile fracture dominates the failure mechanism. The paper presents an extensive review of fracture mechanics applications to problems concerning stiff clays, soft

[6] For a detailed statement of these criticisms, see Planas et al. (2001).

rocks and cemented Sands. Similarities between these problems and those in ice mechanics were pointed out by Dempsey, with examples of crevasses in glaciers. The experimental program discussed in the paper dealt with Mode I fracture tests of beams fabricated with sugar-cemented sand. Sture et al. presented a fracture criterion based on the crack mouth opening, which is geometry and notch dependent. It seems that this aspect could be improved by considering the crack tip opening as the basic parameter, as done for concrete by Jenq and Shah (1985), among others. Also, incorporation of the effects of softening of the shear capacity (Bernander et al., 1989), and of the confinement and pore pressure, were suggested in the discussions by Elfgren and by Voyiadjis, in order to make the approach more appropriate for general applications.

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