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A TIME-DEPENDENT MICROPLANE MODEL FOR CREEP OF COHESIVE SOILS

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ABSTRACT: *The microplane model is used in this paper to describe the time-dependent (creep) behavior of cohesive soils (clays). The constitutive equations are defined on each of a set of microplanes which cover all possible spatial orientations, with a kinematic constraint to the macroscopic response. The total strain components are assumed to be the sum of a time-independent and a time-dependent (creep) contribution. The creep evolution laws are defined within the framework of the rate process theory (activation energy principle), in which the strain rates depend on the current stress level, temperature and time. The model can reproduce instantaneous as well as time-dependent behavior, for drained or undrained conditions. Numerical results show good qualitative agreement with laboratory test data taken from the literature. The paper closes by discussing the implementation of the model in finite element codes.*

1. INTRODUCTION

This paper attempts to model the time-dependent behavior of cohesive soils, principally clays, by a simplified micromechanics approach that consists of specifying the stress-strain relation on planes of various orientations within the material (called the *microplanes*). The macroscopic stress–strain relation is then obtained by simple superposition or, as in the present work, using the principle of virtual work. The microplanes may be imagined to characterize the planes of grain contacts (for granular materials) or the slip planes between particles (for clay materials). A true micromechanics model would describe the material behavior by means of the laws governing the interactions of the particles at the contact points on such planes. In our approach, however, the microplanes represent the behavior at the contacts only in an average sense.

The formulation of the material model requires several hypotheses which simplify the complex micromechanical interactions, reduce computational costs and allow the implementation in finite element codes. These are:

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1. The strains on a microplane are the resolved components of the macroscopic strain tensor, ϵ_{ij}:

$$\epsilon_N = n_j \epsilon_j^n = n_j n_k \epsilon_{jk}, \quad \epsilon_{T_i} = (\delta_{ij} - n_i n_j) n_k \epsilon_{jk}, \quad \epsilon_T = \|\boldsymbol{\epsilon}_T\| = \sqrt{\epsilon_{T_i} \epsilon_{T_i}} \tag{1}$$

in which δ_{ij} is the Kronecker's delta tensor.

2. The response on each microplane depends explicitly on the volumetric strain (ϵ_V), in the sense that the microplane equations must include a separate treatment of the volumetric (spherical) and deviatoric normal components of strain. This hypothesis means that we need to consider on a microplane three components of strain: $\epsilon_V = \epsilon_{kk}/3$, $\epsilon_D = \epsilon_N - \epsilon_V$, and $\boldsymbol{\epsilon}_T$, the last being a vector on any direction within the microplane.

3. The volumetric response, deviatoric normal response and the shear response on each microplane are mutually independent, i.e. decoupled.

4. The vector of shear stress ($\boldsymbol{\sigma}_T$) and the vector of shear strain ($\boldsymbol{\epsilon}_T$) acting on a microplane are parallel, i.e. $\sigma_{T_i} \sim \epsilon_{T_i}$ (this is an approximation which seems sufficient for most purposes, introduced in the interest of reducing the number of unknowns).

5. The microplane stress–strain relations for monotonic loading histories (i.e. histories with no unloading) are path-independent, that is, they can be written as total rather than incremental stress–strain relations.

6. The time-dependent behavior on each microplane can be described by means of the rate process theory (Glasstone et al., 1941) applied to strain rate.

2. MICROPLANE EQUATIONS

According to hypotheses 2 and 3, the set of three equations for each microplane, relating volumetric, normal deviatoric and shear stresses and strains can be written as:

$$\begin{aligned} \sigma'_V &= \mathcal{F}_V(\epsilon_V^{\mathrm{i}}) = \mathcal{F}_V(\epsilon_V - \epsilon_V^{\mathrm{cr}}) \\ \sigma'_D &= \mathcal{F}_D(\epsilon_D^{\mathrm{i}}) = \mathcal{F}_D(\epsilon_D - \epsilon_D^{\mathrm{cr}}) \\ \sigma'_T &= \mathcal{F}_T(\epsilon_T^{\mathrm{i}}) = \mathcal{F}_T(\epsilon_T - \epsilon_T^{\mathrm{cr}}) \end{aligned} \tag{2}$$

where superscritps 'i' and 'cr' indicate time-independent and creep contributions respectively. $\mathcal{F}_V$, $\mathcal{F}_D$ and $\mathcal{F}_T$ are empirical material functions defined below (for a more detailed description see Prat & Bažant, 1990). For simplicity, superscript 'i' is omited in the following equations.

VOLUMETRIC STRESS – STRAIN RELATIONS

$$\begin{aligned} \textit{Compr. Loading}: &\quad \sigma'_V = (\sigma'_V)^{\circ} \left[\frac{(\sigma'_V)^c}{(\sigma'_V)^{\circ}} \right]^{1 - C_S^{\circ}/C_C^{\circ}} e^{\epsilon_V / C_C^{\circ}} \\ \textit{Compr. Unloading/Reloading}: &\quad \sigma'_V = \sigma'^{\,\max}_V e^{(\epsilon_V - \epsilon_V^{\max})/C_S^{\circ}} \\ \textit{Tension Loading}: &\quad \sigma'_V = (\sigma'_V)^{\circ} + E_V^{\circ} (\epsilon_V - \epsilon_a) e^{-\frac{1}{p} \left| \frac{\epsilon_V - \epsilon_a}{\epsilon_p} \right|^p} \\ \textit{Tension Unloading/Reloading}: &\quad \Delta\sigma'_V = E_V^{\circ} \Delta\epsilon_V \end{aligned} \tag{3}$$

DEVIATORIC STRESS – STRAIN RELATIONS

$$\begin{aligned} \textit{Compr. Loading}: &\ \sigma_D = \sigma_{DC}^{\infty}[1 - e^{-k_{DC}|\epsilon_D|}] \\ \textit{Compr. Unloading/Reloading}: &\ \Delta\sigma_D = E_D^{\circ}\Delta\epsilon_D \\ \textit{Tension Loading}: &\ \sigma_D = \sigma_{DT}^{\infty}[1 - e^{-k_{DT}|\epsilon_D|}] \\ \textit{Tension Unloading/Reloading}: &\ \Delta\sigma_D = E_D^{\circ}\Delta\epsilon_D \end{aligned} \tag{4}$$

SHEAR STRESS – STRAIN RELATIONS

$$\begin{aligned} \sigma_T &= \sigma_T^{\infty}[1 + (a\epsilon_T - 1)e^{-k_T\epsilon_T}] \\ a &= a_0(r_{OCR} - 1) \end{aligned} \tag{5}$$

3. PORE WATER PRESSURE

Since the total and effective deviatoric stress tensor are identical, the pore water pressure term, p_w, has to appear only in the volumetric equation, $\sigma_V = \sigma'_V + p_w$. Following Ansal et al. (1979) and the underlying formulation of Bažant & Krizek (1975), we use the following expression to compute the pore water pressure:

$$p_w = \frac{C_w K}{nK + C_w}\left(\frac{\sigma_V}{K} + 3\epsilon''\right) \tag{6}$$

where $K = 2G(1+\nu)/3(1-2\nu)$ = bulk modulus, G is the shear modulus (non-constant along the stress path), ϵ'' the accumulated inelastic strain, C_w the water compressibility, and n the porosity.

4. CREEP LAWS

According to the rate process theory (Glasstone et al., 1941), generally accepted for creep of clays, the strain rates on the microplanes may be expressed as

$$\dot{\epsilon}_V^{cr} = k_1^V \sinh(k_2^V\sigma_V), \quad \dot{\epsilon}_D^{cr} = k_1^D \sinh(k_2^D\sigma_D), \quad \dot{\epsilon}_T^{cr} = k_1^T \sinh(k_2^T\sigma_T) \tag{7}$$

where the k_1's and k_2's are constants depending on temperature, activation energy and time (see Bažant & Prat, 1987). The current creep component of each strain (volumetric, deviatoric or shear) are computed after each time increment Δt as $[\epsilon^{cr}]_{t_1} = [\epsilon^{cr}]_{t_0} + \dot{\epsilon}^{cr}\Delta t$. These values are then introduced into Eqs. 2, from which the stresses on each microplane are calculated.

5. MACROSCOPIC CONSTITUTIVE EQUATIONS

The constitutive law is written in terms of the current microplane stresses and strains. The macroscopic effective stress tensor is obtained by means of the principle of virtual work (Bažant, 1984; Carol et al., 1990) which leads to the following equation:

$$\sigma'_{ij} = \sigma'_V\delta_{ij} + \int_\Omega n_i n_j \sigma'_D \Psi_N d\Omega + \int_\Omega \frac{(n_i\delta_{rj} + n_j\delta_{ri} - 2n_i n_j n_r)}{2}\sigma'_{T_r}\Psi_N d\Omega \tag{8}$$

where Ψ_N is a weight function of the orientations $\mathbf{n}$ which in general can introduce anisotropy of the material in its initial state. The macroscopic total stress tensor, σ_{ij}, then results from the principle of effective stresses of soil mechanics ($\sigma_{ij} = \sigma'_{ij} + p_w\delta_{ij}$) and is obtained solving the system of equations $\sigma_{ij} - \alpha\sigma_{kk}\delta_{ij} = B_{ij}$ where $B_{ij} = \sigma'_{ij} + 9K\alpha\epsilon''\delta_{ij}$ and $\alpha = C_w/3(nK + C_w)$.

6. NUMERICAL IMPLEMENTATION AND RESULTS

The model presented has been developed for constitutive equations in finite element computations. The computer code can be used without modification as the constitutive equation in a finite element code, or as a stand-alone program for model verification (see Carol et al., 1990). The computer subroutine procedure shows a single loop over the number of microplanes, 28 in our case. Since Eq. 8 is written in terms of the total stresses (not of their increments), no numerical integration is necessary within the microplane loop. This feature is particularly important when using finite element analysis because it reduces the necessary computation time by more than one order of magnitude (the microplane model involves numerical integration on the surface of a hemisphere at each Gauss point of each finite element), making the algorithm suitable for this kind of applications.

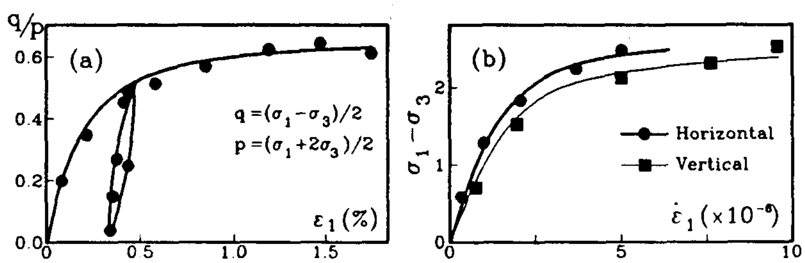

Fig. 1 *(a) Quasi-instantaneous data measured by Wood (1975). (b) Data for anisotropically consolidated specimens 1000 min. after loading (Bažant et al., 1975)*

Figs. 1 and 2 show comparisons of the model results with test data taken from the literature. Fig. 1a shows the results from Wood (1975) on cubic tests of kaolin clay (no creep), with one unloading-reloading cycle which is well reproduced by the microplane model. Fig. 1b exhibits data for anisotropic clay obtained from Bažant et al. (1975). The two data sets correspond to specimens which were horizontally and vertically trimmed. The difference on the microplane results is obtained by means of function Ψ_N in Eq. 8, with good qualitative agreement to the experimental data.

Fig. 2a represents data for isotropically consolidated specimens obtained by Campanella and Vaid (1974). Fig. 2b exhibits data from oedometer tests reported by Leroueil et al. (1985). The present model can provide good qualitative agreement with these creep test results, including the variation of the curves with the strain rate.

7. SUMMARY AND CONCLUSIONS

The microplane model for inelastic behavior of cohesive soils has been extended to take into account time-dependent (creep) behavior. The formulation has been found to

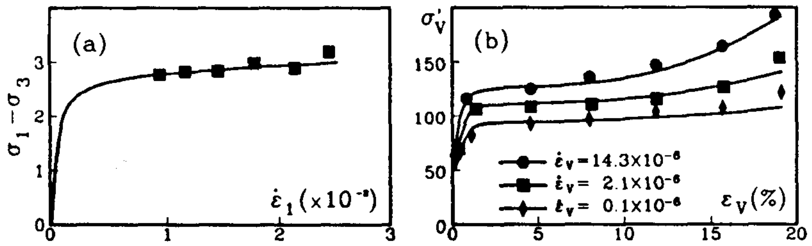

Fig. 2 *(a) Data for isotropically consolidated specimens 1 min. after loading (Campanella & Vaid, 1974). (b) Oedometer tests data (Leroueil et al., 1985) with different strain rates.*

describe qualitatively well both anisotropically and isotropically consolidated samples, as well as the strain rate effect on creep. Since the model is explicit (no iterations required within the microplane integration loop), its use in finite element codes should be easy and efficient despite the need to integrate over a unit hemisphere at each integration point of each finite element in each load step.

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REFERENCES

Ansal, A.M., Bažant, Z.P. & Krizek, R.J. (1979). Viscoplasticity of Normally Consolidated Clays. *J. Geotech. Engrg.* ASCE, **105**(4):519–537.

Bažant, Z.P. & Krizek, R.J. (1975). Saturated sand as an inelastic two-phase medium. *J. Engrg. Mech.* ASCE, **101**(4):317–332.

Bažant, Z.P., Ozaydin, K. & Krizek, R.J. (1975). Micromechanics Model for Creep of Anisotropic Clay. *J. Engrg. Mech.* ASCE, **101**(1):57–78.

Bažant, Z.P. (1984). Microplane model for strain-controled inelastic behavior. In C.S. Desai & R.H. Gallagher (eds.), *Mech. of Engineering Materials*, p. 45–59. New York:Wiley.

Bažant, Z.P. & Prat, P.C. (1987). Creep of Anisotropic Clay: New Microplane Model. *J. Engrg. Mech.* ASCE, **113**(7):1050–1064.

Campanella, R.G. & Vaid, Y.P. (1974). Triaxial and Plane Strain Creep Rupture of an Undisturbed Clay. *Canadian Geotechnical Journal*, **11**(1):1–10.

Carol, I., Bažant, Z.P. & Prat, P.C. (1990). *New Explicit Microplane Model for Concrete: Theoretical Aspects and Unified Implementation for Constitutive Verification and F.E. Analysis.* Report, E.T.S.E.C.C.P.B.-U.P.C.; C/ Gran Capità, S/N; E-08034 Barcelona, Spain.

Glasstone, S., Laidler, K.J. & Eyring, H. (1941). *The Theory of Rate Processes.* New York:McGraw-Hill.

Leroueil, S., Kabbaj, M., Tavenas, F. & Bouchard, R. (1985). Stress–strain–strain rate relation for the compressibility of sensitive natural clays. *Geotechnique*, **35**(2):159-180.

Prat, P.C. & Bažant, Z.P. (1990). *Microplane model for triaxial deformation of saturated cohesive soils.* Report, E.T.S.E.C.C.P.B.-U.P.C.; C/ Gran Capità, S/N; E-08034 Barcelona, Spain. Also *J. Geotech. Engrg.* ASCE, in press.

Wood, D.M. (1975). Explorations of Principal Stress Space with Kaolin in a True Triaxial Apparatus. *Geotechnique*, 25(4):783–797.