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A MICROPLANE CONSTITUTIVE MODEL FOR SOILS

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Abstract

A microplane model for the inelastic behavior of soils is presented. The microscopic constitutive equations are defined independentely for a set of microplanes covering all possible orientations, with a kinematic micro–macro constraint. The stresses on each microplane are defined as explicit functions of the volumetric and deviatoric normal and shear components of the macroscopic strain tensor. The model can reproduce drained as well as undrained behavior, with an uncoupled formulation between the stress–strain and the pore water pressure terms. The model is calibrated and verified by comparisons with several test data both drained and undrained, and good agreement is attained for most of the basic features of the material behavior. The model involves nine material parameters but four can be fixed constant and five (or four in some cases) have to be determined by data fitting. The fact that the stress is given as an explicit function of strain makes the model suitable for finite elements applications.

1. Introduction and Basic Assumptions

During the last decade, the multilaminar and microplane models have been developed for several types of materials, and its use proved adequate to predict the basic features of the materials under most usual loading conditions (Bažant and Oh, 1983, 1985; Bažant, 1984; Bažant and Gambarova, 1984; Bažant and Kim, 1986; Bažant and Prat, 1987, 1988; Carol et al., 1990; Pande and Sharma, 1980, 1983; Zienkiewicz and Pande, 1977). In this paper we present a generalization of a previously presented microplane model for drained behavior of soils (Prat and Bažant, 1989], to include undrained response as well. The original idea of the method is due to Taylor (1938) who proposed that the stress–strain relation be specified independently on planes of various orientations in the material, assuming that either the stresses on that plane (now called the *microplane*) are the resolved components of the macroscopic stress tensor (static constraint), or

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the strains on the plane are the resolved components of the macroscopic strain tensor (kinematic constraint). The responses on the planes of various orientations are then related to the macroscopic response simply by superposition or, as it has been done in recent works (Bažant, 1984; Carol et al, 1990) by means of the principle of virtual work. In the initial application to metals (Batdorf and Budianski, 1949) only the static constraint was considered, and so it was in the early applications to soils (Pande and Sharma, 1980, 1983; Zienkiewicz and Pande, 1977) which successfully described some of the basic aspects of soil behavior. It appeared, however, that the microplane system under a static constraint becomes unstable when strain-softening takes place (Bažant and Oh, 1983, 1985; Bažant and Gambarova, 1984). For this reason, as well as others, it is necessary to use the kinematic constraint, which will be adopted here.

To model the undrained behavior of clays, we make the following basic hypotheses and assumptions: (a) The strains on a microplane are the resolved components of the macroscopic strain tensor ε_{ij}, which represents a kinematic constraint; (b) The response on each microplane depends explicitly on the volumetric strain ($\varepsilon_V = \frac{1}{3}\varepsilon_{kk}$), in the sense that the microplane equations must include a separate treatment of the volumetric and deviatoric components. The equations governing this decomposition have been developed elsewhere (Bažant and Prat, 1988; Carol et al, 1990) and will not be repeated here; (c) Overall we consider three strain components on a microplane: volumetric ε_V, deviatoric normal ε_D and shear $\underline{\varepsilon}_T$, with the respective responses mutually independent (decoupled); (d) The vector of shear stress $\underline{\sigma}_T$ and the vector of shear strain $\underline{\varepsilon}_T$ acting on a microplane are parallel, i.e. $\sigma_{T_i} \sim \varepsilon_{T_i}$; and (e) The microplane stress–strain relations for monotonic loading are path-independent (note that this does not imply path-independence at the macroscopic level).

It is well known in geomechanics that soils undergo deformations only when a change in the "effective stresses" is produced. The effective stress tensor $\sigma'_{ij} = \sigma_{ij} - p_w\delta_{ij}$ (where σ_{ij} is the total stress tensor and p_w is the pore water pressure) characterizes the stresses transmitted by the solid skeleton. It can be shown that the deviatoric effective and deviatoric total stress tensors are identical ($s'_{ij} = s_{ij}$) and therefore we may conclude that the effective pore water pressure needs to be introduced int the equations governing the volumetric behavior only. Therefore the *effective* stress–strain equations will be formulated using the microplane theory, while volumetric stress–strain–pore pressure equations can be formulated macroscopically without the use of microplanes

2. Microplane Material Functions

According to the hypotheses formulated before, the microplane equations are defined independently for volumetric, deviatoric and shear components, assuming a functional relation between effective stresses and strains: $\sigma'_V = \mathcal{F}_V(\varepsilon_V)$, $\sigma'_D = \mathcal{F}_D(\varepsilon_D)$, and $\sigma'_T = \mathcal{F}_T(\varepsilon_T)$.

2.1 Volumetric Stress–Strain relation

We need to distinguish between hydrostatic compression and tension. For volumetric compression we assume a relationship similar to the known experimental curves obtained from oedometric tests, e.g. a bilinear relation between ε_V and $\log \sigma'_V$. For volumetric tension we use a curve with a peak and a softening branch (Fig. 1). If the soil is overconsolidated, we will use for virgin (initial) loading:

$$\sigma'_V = \sigma^\circ_V e^{\varepsilon_V / C_s^*} \tag{1}$$

where σ°_V is the initial effective volumetric stress *in situ* and C_s^* and empirical material parameter. If the initial vertical stress is not less than the preconsolidation pressure, then the virgin loading branch can be described as

$$\sigma'_V = \hat{\sigma}_V e^{\varepsilon_V / C_c^*}, \quad \text{with} \quad \hat{\sigma}_V = \sigma_m e^{-\varepsilon_m / C_c^*} \tag{2}$$

where σ_m and ε_m are the maximum effective volumetric stress and strain ever reached. These two state variables have the initial values $\sigma^\circ_m = \sigma^\circ_V$, $\varepsilon^\circ_m = C_s^* \log \sigma^\circ_m / \sigma^\circ_V$. Note that if the soil is normally consolidated then $\varepsilon^\circ_m = 0$.

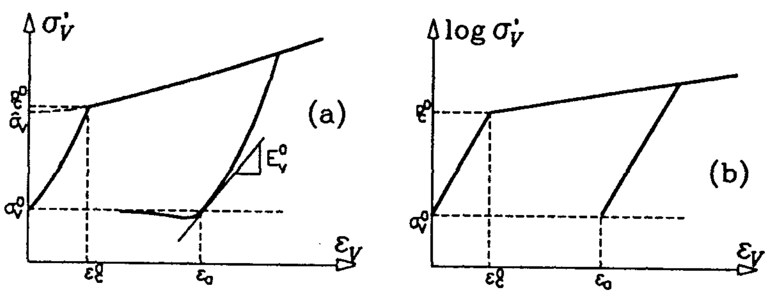

Fig. 1 — *Microplane volumetric stress–strain relation.*

The unloading branches in compression are defined so that in the (ε_V–$\log \sigma'_V$) space they are straight lines of slope $1/C_s^*$ (see Fig. 2):

$$\sigma'_V = \sigma^\circ_V e^{\frac{\varepsilon_V - \varepsilon_a}{C_s^*}} \tag{3}$$

where ε_a is the value of ε_V corresponding to the point on the unloading branch at which $\sigma_V = \sigma^\circ_V$:

$$\varepsilon_a = \left[1 - \frac{C_s^*}{C_c^*}\right]\varepsilon_m + C_s^* \log\left[\frac{\sigma^\circ_V}{\hat{\sigma}_V}\right] \tag{4}$$

and C_c^* is an empirical material parameter.

For "tension," i.e. when the current $\sigma'_V < \sigma^\circ_V$, we assume a stress–strain curve with a peak and a descending branch asymptotically approaching zero. The curve is shifted by a distance equal to the latest value of ε_a, so that continuity is maintained in the transition from compression to tension;

$$\sigma'_V = \sigma^\circ_V + E^\circ_V(\varepsilon_V - \varepsilon_a) e^{-\frac{1}{p}\left|\frac{\varepsilon_V - \varepsilon_a}{\varepsilon_p}\right|^p} \tag{5}$$

where $E^\circ_V = \sigma^\circ_V / C_s^*$, and p and ε_p are material parameters.

Finally, for "tension" unloading, we assume a linear branch with slope E°_V such that

$$\Delta\sigma'_V = E^\circ_V \Delta\varepsilon_V \tag{6}$$

2.2 Deviatoric Stress–Strain Relation

The equations for compression and tension are as follows (Fig. 2):

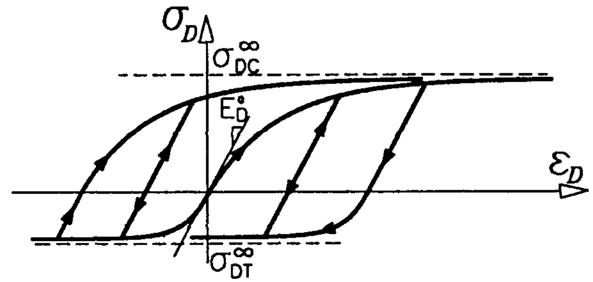

Fig. 2 — *Microplane deviatoric stress–strain relation.*

$$\begin{aligned} \sigma_D &= \mathcal{F}_D(\varepsilon_D) = \sigma^\infty_{DC}[1 - e^{-k_{DC}|\varepsilon_D|}] \quad \text{if} \quad \sigma_D \geq 0 \\ \sigma_D &= \mathcal{F}_D(\varepsilon_D) = \sigma^\infty_{DT}[1 - e^{-k_{DT}|\varepsilon_D|}] \quad \text{if} \quad \sigma_D < 0 \end{aligned} \tag{7}$$

where σ^∞_{DC}, σ^∞_{DT}, k_{DC}, and k_{DT} are empirical material constants, not entirely independent if we enforce continuity of slopes at the origin. In that case, the following relation must hold: $|\sigma^\infty_{DC} k_{DC}| = |\sigma^\infty_{DT} k_{DT}| = E^\circ_D$, where E°_D is the initial elastic modulus. Eqs. 7 apply only for loading on the microplane; for unloading, we assume on each microplane linear elastic behavior with elastic modulus E°_D. It must be noted that the relationships defined by Eqs. 7 act as the envelopes for future loading–unloading–reloading cycles (Fig. 2).

2.3 Shear stress–strain relation

The shear stress–strain relation must shows a dependency on the overconsolidation ratio r_{OCR}(Fig. 3):

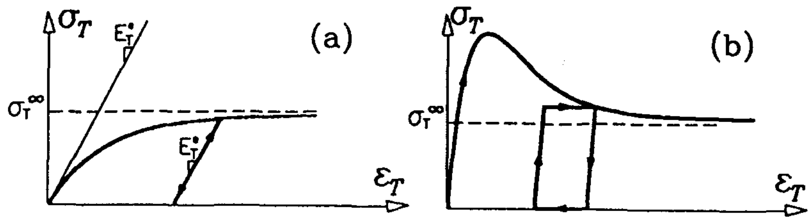

Fig. 3 — *Microplane shear stress–strain relation.*

$$\sigma_T = \mathcal{F}_T(\varepsilon_T) = \sigma_T^{\infty}[1 + (a\varepsilon_T - 1)e^{-k_T\varepsilon_T}] \tag{8}$$

where $a = a_0(r_{OCR} - 1)$ and σ_T^{∞}, k_T, and a_0 are empirical parameters. The unloading rules are shown on Figs. 3a and 3b. Note that if the initial shear modulus E_T° is known, then the exponent k_T in Eq. 8 can be computed as $k_T = E_T^{\circ}/\sigma_T^{\infty} - a$.

Eq. 8 represents a relation between the norms of the shear stress and shear strain vectors. However, since we have assumed that these vectors are parallel, we can easily obtain the components of the stress vector as

$$\sigma_{T_i} = \sigma_T \frac{\varepsilon_{T_i}}{\varepsilon_T} \tag{9}$$

where $\sigma_T = \|\underset{\sim}{\sigma}_T\| = \mathcal{F}_T(\varepsilon_T)$, and $\varepsilon_T = \|\underset{\sim}{\varepsilon}_T\|$.

3. Pore Water Pressure

To model the pore water pressure under undrained conditions, we will use the following set of equations first proposed by Bažant and Krizek (1975) and later developed by Ansal et al. (1979). We assume that compressibilities of free and bound water are the same, that both behave ellastically and that both can carry only volumetric stress.

The pore water pressure can be calculated as

$$p_w = \frac{C_w K}{nK + C_w}\Big[\frac{\sigma_V}{K} + 3\varepsilon''\Big] \tag{10}$$

where ε'' is the accumulated inelastic volumetric strain, n the porosity, C_w the water compressibility, $K = 2G(1+\nu)/3(1-2\nu)$ and G the shear modulus which is non-constant along the stress path. The accumulated inelastic strain ε'' is obtained by integrating the inelastic volumetric strain increment $d\varepsilon''$ which is a measure of the time-independent densification-dilatancy and can be written as

$$d\varepsilon'' = \frac{C(1 + 2500\varepsilon_V)}{(1 + 1000J_2^{\varepsilon})(1 + \frac{\sigma_V}{4p_a})(1 + 9000\varepsilon'')} d\xi \tag{11}$$

where C is a material parameter, $J_2^{\varepsilon} = \frac{1}{2}\varepsilon_{ij}\varepsilon_{ij}$, σ_V is the total volumetric stress, p_a is the atmospheric pressure, ε'' is the accumulated inelastic volumetric strain (densification-dilatancy), and $d\xi = \sqrt{\frac{1}{2}d\varepsilon_{ij}d\varepsilon_{ij}} \sim$ path length increment.

4. Macroscopic Constitutive Law

The basic structure we will use for the effective stress–strain law has been developed recently by Carol et al. (1990). The constitutive law is written in terms of the current effective stresses and strains (and not in terms of their increments), which allows the model to be explicit. The macroscopic effective stress tensor can be expressed as:

$$\sigma'_{ij} = \sigma'_V\delta_{ij} + \int_{\Omega} n_i n_j \sigma'_D \Psi_N d\Omega + \int_{\Omega} \frac{1}{2}(n_i\delta_{rj} + n_j\delta_{ri} - 2n_i n_j n_r)\sigma'_{T_r}\Psi_N d\Omega \tag{12}$$

where σ'_V, σ'_D, and σ'_{T_r} are the microplane effective stresses and Ψ_N is a weighing function of the orientations $\underset{\sim}{n}$ which in general can introduce anisotropy of the material in its initial state (Prat and Bažant, 1990). If such a function is unknown, we can take approximately Ψ_N = constant. The macroscopic total stress tensor, σ_{ij}, then results from the principle of effective stresses of soil mechanics: $\sigma_{ij} = \sigma'_{ij} + p_w\delta_{ij}$ where σ'_{ij} is obtained from Eq. 12 and the pore water pressure p_w from Eq. 10. The latter equation can be rewritten as

$$p_w = \alpha\sigma_{kk} + \beta\varepsilon'' \tag{13}$$

with $\alpha = C_w/3(nK + C_w)$ and $\beta = 9K\alpha$. Thus,

$$\sigma_{ij} = \sigma'_{ij} + \alpha\sigma_{kk}\delta_{ij} + \beta\varepsilon''\delta_{ij} \tag{14}$$

Calling $B_{ij} = \sigma'_{ij} + \beta\varepsilon''\delta_{ij}$ (which is a known tensor), we obtain the following system of equations in the unknowns σ_{ij}:

$$\sigma_{ij} - \alpha\sigma_{kk}\delta_{ij} = B_{ij} \tag{15}$$

The solution of this system of equations gives the values of the macroscopic total stress tensor and, therefore, the value of the pore water pressure as well.

5. Verification with Experimental Data

The model has been verified with several typical test data from the literature as exhibited in Figs. 4–6. Fig. 4 shows a comparison between the present microplane model and the results obtained by Pande and Sharma (1983) using the critical state model (Schofield and Wroth, 1968). These results correspond to a normally consolidated soil under triaxial compression and extension, in drained and undrained conditions.

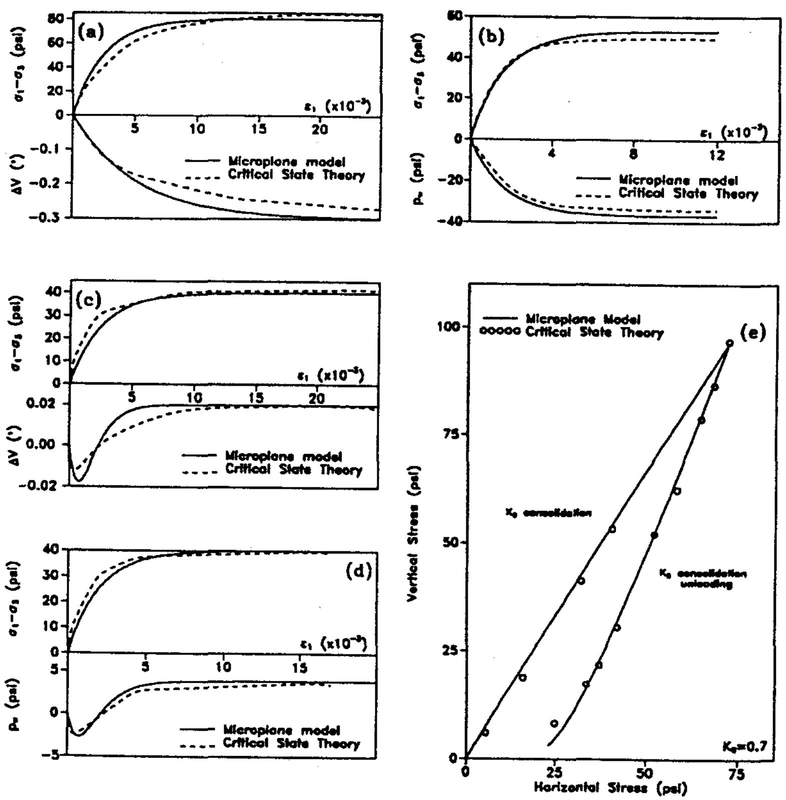

Fig. 4 — *Comparison with critical state model results: (a) drained compression; (b) undrained compression; (c) drained extension; (d) undrained extension; and (e) k_0 consolidation.*

Fig. 5 shows (a) test data on overconsolidated clays by Henkel (1956); and (b) data from standard and true triaxial tests of clays by Nakai et al. (1986).

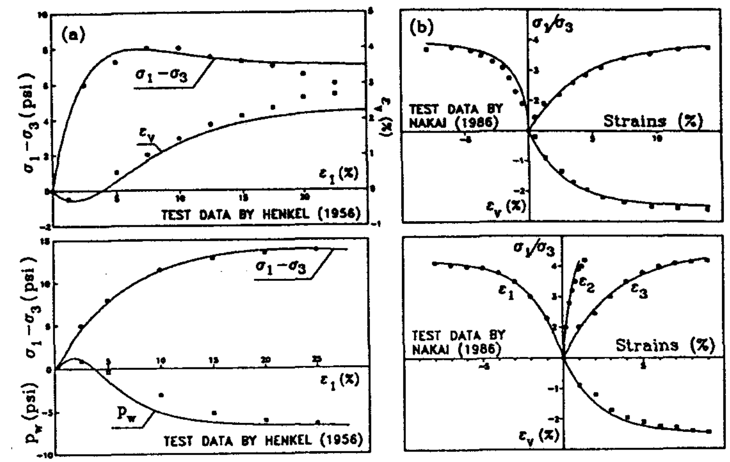

Fig. 5 — *(a) Comparison with data on overconsolidated clays by Henkel; (b) Comparison with standard and true triaxial tests by Nakai.*

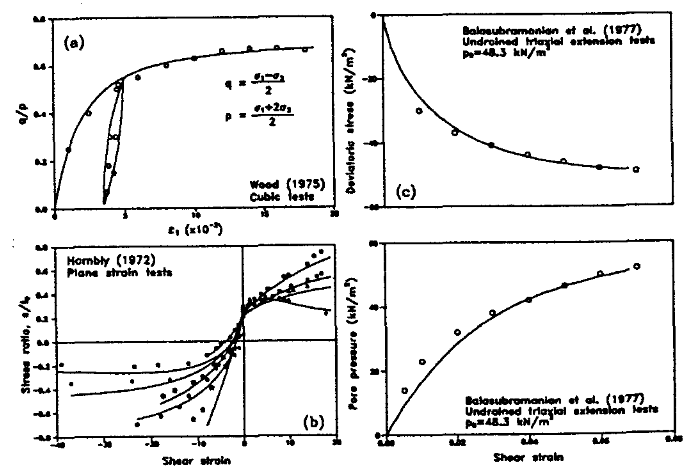

Fig. 6 — *Comparison with (a) cubic tests by Wood; (b) plane-strain tests by Hambly; and (c) triaxial extension tests by Balasubramanian and Uddin.*

Fig. 6 exhibits (a) results on cubic tests of kaolin clay by Wood (1975), with one cycle of unloading–reloading; (b) data from plane-strain tests by Hambly (1972); and (c) triaxial extension tests by Balasubramanian and Uddin (1977).

6. Conclusions

1. The work presented is an extension of a simplified model developed earlier for deviatoric creep of *drained* soils. A pore water pressure term is included in a manner that gives undrained as well as drained behavior as special cases. The model seems capable to reproduce well the main trends of both types of behavior, including the pore pressure variation (for undrained tests) and the volume change (for drained tests).

2. The number of parameters that need to be adjusted to fit complex data (five at the most, four in some cases) is small enough for practical purposes.

3. The present formulation can be used as a general constitutive equation in nonlinear finite element programs.

4. The present model is *explicit* in the sense that no iterations are required to obtain a stress state from a given initial strain state and strain increment. Therefore, its use in a finite element code is as easy and efficient as that of a similar model for concrete (despite the need to integrate over a hemisphere, typical of the microplane formulations).

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