Bažant, Z.P. (1965). "Electric analogues for creep of concrete structures." Proc., *Conf. on Exper. Methods of Investigating Stress and Strain in Structures*, held at Techn. Univ. Prague, Building Res. Inst., 207–218.

Proceedings

of the

Conference on Experimental Methods of Investigating Stress and Strain in Structures

Praha, October 5 - 8, 1965

International Hotel

Reprint

Edited by the

Building Research Institute of the Technical University in Prague,

Praha 6 Dejvice, Šolínova 7, Czechoslovakia

August 1965

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Electric Analogues for Creep of Concrete Structures

/Elektrické analogie pro dotvarování betonových konstrukcí/

Jsou možné dva typy analogie, v nichž k napětím a rychlostem deformace odpovídají elektrická napětí a proud /A/ nebo obráceně /B/. Model různých zákonů dotvarování betonu. Model pro řešení n staticky neurčitých veličin nehomogenní konstrukce je tvořen obvodem /A/ o n navzájem propojených smyčkách a n zdrojích napětí, resp. /B/ o n uzlových párech a n zdrojích proudu, přičemž jednotlivé prvky obvodu jsou tvořeny elementárními obvody pro dotvarování. Obvod se skládá ze samoindukcí a odporů /A/, resp. z kapacit a odporů /B/, jež jsou obecně časově proměnné.

Modern concrete structures, such as concrete bridges cast by the cantilever method, concrete suspension beams, etc., are for the most part clearly nonhomogeneous because of cooperation between concrete and steel, concrete parts of different ages , of various thicknesses, etc. An analysis of the effects of creep leads in such cases to relatively involved systems of ordinary differential equations or, more generally yet, to Volterra´s integral equations which must be solved on digital or analogue computers. Although their electric modelling is essentially known /see[2] -by conversion to one differential equation of higher order/, one can arrive at an analogue circuit far more simply in a direct way, namely by finding an analogy for the law of creep of the material and an electric model corresponding to the given statically indeterminate structure. The present paper will first indicate that Dischinger-Whitney s or more accurately, Arutyunyan-Maslov's laws of concrete creep can be described by simple rheologic models consisting of springs and dashpots with time variable constants,i.e. by Maxwell's or Boltzmann's models. Thereafter it will determine the electric circuit analogous to the mechanical model and hence also to the law of creep, which can be built up of resistances, self-inductions and capacities. Finally it will show how to set up an analogue circuit for solving the statically indeterminate quantities of the entire structure by coupling together such elementary circuits.

1. Analogue for creep of time invariable material. The linear creep of materials with time invariable parameters, e.g. viscoelasticity of plastics at constant temperature, can be represented by mechanical rheologic models consisting of springs and dashpots. As it is well known [4] ,[5] ,their electric analogue can be realized by the following four types:

/A/ $U_i = a\, d\varepsilon_i/dt$, $I_i = b\sigma_i$, $R = a/b\eta$, $L = a/bE$

/B/ $U_i = b\sigma_i$, $I_i = a\, d\varepsilon_i/dt$, $R = b\eta/a$, $C = a/bE$

/C/ $U_i = a\varepsilon_i$, $I_i = b\sigma_i$, $R = a/bE$, $C = b\eta/a$

/C/ $U_i = b\sigma_i$, $I_i = a\varepsilon_i$, $R = bE/a$, $L = b\eta/a$

where σ_i or ε_i denote the stress and deformation in a spring or dashpot, E - the spring constant / $\sigma_i = E\varepsilon_i$ /, η -the viscosity of the dashpot/ $\sigma_i = \eta\, d\varepsilon_i/dt$ /, U_i-the voltage and I_i-the current in the corresponding electric element, R-the ohmic resistance / $U_i = R I_i$ /, C-the capacity / $I_i = C\, dU_i/dt$ /, L-the self-induction / $U_i = L\, dI_i/dt$ /, a, b-the selectable parameters of the analogue, t-the time.

2. Analogue for creep of time variable material.The creep of concrete is complicated by its dependence on concrete age. Its representation can also be effected - as will be shown later on - by rheologic models assembled of springs and dashpots, the parameters E_i and η_i of which are, however, time variable. For a spring of such a model it no longer holds that $\sigma_i = E(t)\,\varepsilon_i$ but rather that $d\sigma_i/dt = E(t)\,d\varepsilon_i/dt$, and similarly for a dashpot $\sigma_i = \eta(t)\, d\varepsilon_i/dt$. Since it holds for a time variable resistance that $U_i = R(t)\, I_i$ rather than $dU_i/dt = R(t)\frac{dI_i}{dt}$, analogues /C/ and /D/ can evidently be not applied to creep of concrete. On the other hand, comparing the equations for variable self-induction $U_i = L(t)\, dI_i/dt$ and capacity $I_i = C(t)\, dU_i/dt$ with those for a spring, we note that analogues /A/ and /B/ are applicable to materials with time variable parameters and can represent the creep of concrete as an aging material.

Since in a mechanical model stresses are additive in the parallel and deformations in the series coupling, whereas in an electric circuit currents are additive in the parallel and voltages in the series coupling, series coupling corresponds to series, and parallel coupling to parallel in analogue /A/ /and /C//, and parallel coupling to series coupling and vice versa in analogue /B/ /and /D//.

For variable parameters analogues A and B can be formulated yet more generally. Let us require that real time t corresponds to time ϑ in the electric model, the correspondance $\vartheta = \vartheta(t)$ being any arbitrary continuous increasing function with finite limit for $t \to \infty$; thus infinite age of concrete is associated with finite time in the model. Furthermore, in analogues A and B any arbitrary continuous function $a(t)$ may be chosen in lieu of constant a ; in this way one of the parameters $R(t)$, $L(t)$, or $C(t)$ canbecomes constant. It can be proved that the analogies are then given by relations

/A'/ $U_i(\vartheta) = a(t)\frac{d\varepsilon_i(t)}{dt}$, $I_i(\vartheta) = b\,\sigma_i(t)$, $R(\vartheta) = \frac{a(t)}{b\,\eta(t)}$, $L(\vartheta) = \frac{d\vartheta(t)}{dt}\,\frac{a(t)}{b\,E(t)}$.

/B'/ $U_i(\vartheta) = b\,\sigma_i(t)$, $I_i(\vartheta) = a(t)\frac{d\varepsilon_i(t)}{dt}$, $R(\vartheta) = \frac{b\,\eta(t)}{a(t)}$, $C(\vartheta) = \frac{d\vartheta(t)}{dt}\,\frac{a(t)}{b\,E(t)}$.

3. Creep law of concrete and its model. To obtain general relations for stress σ and strain ε in uniaxial state of stress at linear creep of concrete /which we can assume for stresses lower than about 0.5 of strength, except for the case of a large decrease in stress/, we can use the principle of superposition, and by integration by parts derive Volterra's integral equation[1] , [6] , [9] as follows:

/1/
$$\varepsilon(t) = \frac{\sigma(t)}{E(t)} - \int_{t_0}^{t} \sigma(\tau)\,\frac{\partial}{\partial\tau}\left[\frac{1}{E(\tau)} + C(t,\tau)\right] d\tau$$

where $E(t)$ is the time variable modulus of elasticity of concrete, t – the age of concrete, $C(t,\tau)$ – the creep function expressing creep strain at time t produced by the action of constant unit stress applied in time τ , t_o – the instant of the application of the first stress.

3.1 The most simple and frequently used, yet only roughly descriptive expression for $C(t,\tau)$ is Dischinger-Whitney's law [3] , [8] , [9]

/2/ $$C(t,\tau) = \frac{\varphi(t) - \varphi(\tau)}{E_o}$$

where $\varphi(t)$ is the creep factor /an increasing continuous function of one variable with finite limit $\varphi(\infty)$ /. Introducing /2/ in /1/ and differentiating with respect to φ , we obtain differential equation

/3/ $$\frac{\partial \varepsilon}{\partial \varphi} = \frac{1}{E(t)} \frac{\partial \sigma}{\partial \varphi} + \frac{\sigma}{E_o}$$

with the initial condition of $\sigma = E\varepsilon$ for $t = t_o$. The equation of creep of so-called Maxwell's model /Fig.1a/ is as follows:

/3a/ $$\frac{d\varepsilon}{dt} = \frac{1}{E(t)} \frac{d\sigma}{dt} + \frac{\sigma}{\eta}$$

On comparing /3/ and /3a/ it becomes evident that the creep of concrete according to law /2/ can be represented by Maxwell's model /Fig.1a/ in which the spring constant is equal to $E(t)$ and dashpot viscosity $\eta(t) = \frac{E_o}{d\varphi/dt}$. On introducing these expressions in /A'/ and /B'/ we obtain an electric analogue of this creep law. In analogue /A'/ it is circuit Aa according to Fig.2 for which it holds that

/4a/ $$L(\vartheta) = \frac{d\vartheta(t)}{dt} \frac{a(t)}{b\,E(t)}, \qquad R(\vartheta) = \frac{d\varphi(t)}{dt} \frac{a(t)}{b\,E_o}$$

In analogue /B'/ it is circuit Ba according to Fig.2 where

/4b/ $$C(\vartheta) = \frac{d\vartheta(t)}{dt} \frac{a(t)}{b\,E(t)}, \qquad R(\vartheta) = \frac{b\,E_o}{a(t)\,d\varphi/dt}$$

If we select parameter a in the form of $a(t) = \frac{a_o}{d\varphi/dt}$, we conveniently obtain resistance R constant in both analogues. Since for concrete we can approximately consider $E = const.$, we arrive at constant self-induction and capacity if we choose $\vartheta(t) = \vartheta_o \varphi(t) + \vartheta_1$. Hence time invariable models are approximately satisfactory for this creep law. /Constant self-induction or capacity would

be attained for variable E by choosing $\vartheta = \int E\,d\varphi$. This cannot be done, however, if the structure is nonhomogeneous because ϑ would be different for different parts of the structure/.

3.2 A more exact law of concrete creep is given by Arutyunyan-Maslov s function [1] , [6] , [9]

/5/ $$C(t,\tau) = \psi(\tau)\left(1 - e^{-\gamma(t-\tau)}\right)$$

where it is to advantage to choose $\psi(\tau)$ in the form of $\psi = A_o + A_1 e^{-\gamma\tau}$ [9]. On introducing eq. /5/ to /1/ we can prove that the same integral appears in the first and the second derivatives of eq./1/ with respect to t . Eliminating it from the two equations we get the following differential equation

/6/ $$E\frac{d^2\varepsilon}{dt^2} + E\gamma\varepsilon = \frac{d^2\sigma}{dt^2} + \gamma\left(1 + E\psi - \frac{1}{E\gamma}\frac{dE}{dt}\right)\frac{d\sigma}{dt}$$

with the initial conditions $E\varepsilon = \sigma$ and

$E\frac{d\varepsilon}{dt} = \frac{d\sigma}{dt} + \gamma E\psi\sigma$ for $t = t_o$

implied by eq./1/ and its first derivative.

A differential equation of so-called Boltzmann s /or standard/ model according to Fig.1b with time variable parameters E , E_γ , η is obtained from the deformation equations of the model elements $\frac{d\sigma}{dt} = E\left(\frac{d\varepsilon}{dt} - \frac{d\varepsilon_\gamma}{dt}\right)$, $\frac{d\sigma_\gamma}{dt} = E_\gamma \frac{d\varepsilon_\gamma}{dt}$, $\sigma - \sigma_\gamma = \eta\frac{d\varepsilon_\gamma}{dt}$ by eliminating ε_γ and σ_γ , in the form of [9] :

/6a/ $$E\frac{d^2\varepsilon}{dt^2} + \frac{E}{\eta}\left(E_\gamma + \frac{d\eta}{dt}\right)\frac{d\varepsilon}{dt} = \frac{d^2\sigma}{dt^2} + \frac{1}{\eta}\left(E + E_\gamma + \frac{d\eta}{dt} - \frac{\eta}{E}\frac{dE}{dt}\right)\frac{d\sigma}{dt}$$

with the initial conditions $E\frac{d\varepsilon}{dt} = \frac{d\sigma}{dt} + \frac{E}{\eta}\sigma$ and $E\varepsilon = \sigma$ for $t = t_o$.

If $\eta(t) = \frac{1}{\gamma\psi(t)}$, $E_\gamma(t) = \frac{1}{\psi(t)} - \frac{d\eta(t)}{dt}$, equations /6/ and /6a/ with their initial conditions are identical. Hence Boltzmann's model represents the creep law according to eq./5/. On introducing E , E_γ and η to /A'/ or /B'/ we obtain as a model of this creep law in analogues A' or B' circuits Ab and Bb, respectively, according to Fig.2 for which it holds that

/7a/ $$L(\vartheta) = \frac{d\vartheta(t)}{dt}\frac{a(t)}{b\,E(t)}, \quad L_\gamma(\vartheta) = \frac{d\vartheta(t)}{dt}\frac{a(t)}{b}\frac{\gamma\psi^2(t)}{\gamma\psi(t) + d\psi(t)/dt}, \quad R(\vartheta) = \frac{a(t)}{b}\gamma\psi(t)$$

or

/7b/ $$C(\vartheta) = \frac{d\vartheta(t)}{dt}\frac{a(t)}{b\,E(t)}, \quad C_\gamma(\vartheta) = \frac{d\vartheta(t)}{dt}\frac{a(t)}{b}\frac{\gamma\psi^2(t)}{\gamma\psi(t) + d\psi(t)/dt}, \quad R(\vartheta) = \frac{b}{a(t)\gamma\psi(t)}$$

A choice of analogue parameter $a(t) = a_o/\psi(t)$ results in constant resistance R. Since we can approximately consider E=const., we arrive at constant L or C by choosing $\vartheta(t) = \vartheta_0^{*} E\int\psi(t)\,dt$. /For variable E we could also arrive at a constant L or C by choosing $\vartheta(t) = \vartheta_0^{*}\int E\psi\,dt$. This is, however, not possible in the case of nonhomogeneous structures because ϑ thus obtained would be different for different parts of the structure/. Hence constant L_V and C_V are not achievable for this law.

3.3 Boltzmann's model /Fig.1b/ and circuits Ab and Bb in Fig.2 also represent a more general creep law, namely $C(t,\tau) = g(\tau)[h(t) - h(\tau)]$ [9]. Burger's rheologic model according to Fig.1c modelled by circuits Ac and Bc describes the case of concrete still better. It is arrived at from the following law of concrete: $C(t,\tau) = f(t) - f(\tau) + g(\tau)[h(t) - h(\tau)]$.
The corresponding differential equation is also of the second order but compared with eqs./6/ and /6a/, it contains σ', too.

3.4 The electric analogues outlined in the foregoing could be evolved more rapidly by calculating /A'/ voltage or /B'/ current response in time t per unit step /A'/ of current or /B'/ voltage in time ϑ in circuits Aa and Ba, or alternately Ab and Bb. respectively, etc. according to Fig.2. Since the time variation of the rate of this voltage or current response is in the form of function /2/ or /5/, etc., the respective circuits represent the creep according to laws /2/ and /5/, etc.

4. Models of creep of statically indeterminate structures.

4.1 Let us first determine an electric circuit modelling the relation between load $\underline{Z}(t)$ and deformation $\delta_X(t)$ produced by it in the direction of force X applied to a statically indeterminate concrete structure. So far as concrete is concerned, one can very well accept the assumption that the functions of concrete creep in various parts of the structure are affine [9], [10], [11], i.e. that the creep function in all parts of the structure may be written in the form of $\varkappa\, C(t,\tau)$ where $C(t,\tau)$ is the chosen basic creep function for the entire structure, and $\varkappa$ is the coefficient of creep affinity of the respective part /e.g. $\varkappa = 0$ for steel, $\varkappa = 1$ for a homogeneous

structure/. Deformation $\delta_X(t,\tau)$ produced in time t by constant load $\underline{Z}(\tau)$ applied in time τ is given by the principle of virtual work in the form

$$/8/ \quad \delta_X(t,\tau) = \int_V \bar{\sigma}\sigma\left[\frac{1}{E(\tau)} + \varkappa\, C(t,\tau)\right]dV = \int_V \frac{\bar{\sigma}\sigma}{E(\tau)}\,dV + C(t,\tau)\int_V \bar{\sigma}\sigma\varkappa\, dV$$

where σ is the time invariable stress produced by constant load $\underline{Z}(\tau)$, $\bar{\sigma}$ - the stress due to virtual load $X = 1$ in the X-direction, V - the volume of the structure. As the last term of expression /8/ immediately indicates, long-time component of $\delta_X(t,\tau)$ is proportional to function $C(t,\tau)$ /assuming creep affinity/.

This is the reason why the electric analogue for relation between δ_X and $\underline{Z}(t)$ under an arbitrary load is again given by circuits Aa and Ba, and Ab and Bb, respectively, etc. in Fig.2 for the respective laws of creep. The parameters of these circuits are ascertained in a similar way from eq./8/ /interchanging functions $E(t), \varphi(t), \text{or } \psi(t)$, etc. in eqs./4a,b/ or /7a,b/ with functions corresponding to eq./8/ /.

4.2 The deformation of a statically indeterminate nonhomogeneous structure is bound by the conditions of compatibility requiring zero resultant deformation of the equilibrium basic system statically determinate in the sense of the various statically indeterminate quantities X_i / i = 1, 2, ...n/ produced by external load $\underline{Z}(t)$ and by various statically indeterminate quantities

Since in accordance with para. 4.1 these deformations are again expressed at constant stress by equations in the form of /8/ - assuming creep affinity - circuits in the form of Aa, and Ba, or alternately Ab and Bb denoted as /iZ/ or /ik/ whose parameters are determined from an equation in the form of /8/ describing the respective stress and deformation, again interpret the relation for deformation in the X-direction vs. external load $\underline{Z}(t)$ or vs. statically indeterminate quantities X_k. The electric model of a statically indeterminate structure is then obtained by interconnecting circuits /iZ/ and /ik/ /i,k = 1, ...n/ accordingly with the conditions of equilibrium and compatibility.

In circuits thus interconnected Kirchhoff's laws are valid irrespective of the internal arrangement of the circuits /i.e. whether we consider circuit Aa or Ba, etc./ similarly as in structures conditions of equilibrium and compatibility apply irrespective of the deformation law of the material. It is, therefore, sufficient, to set up a circuit modelling the solution of the statically indeterminate quantities of an elastic structure consisting only of elements of one kind /i.e. of self-inductions in analogue A' or of capacities in analogue B'/; interchanging these elements with elementary circuits / iZ / and / ik / of the form of Aa or Ab, etc. we get a circuit modelling the solution of the statically indeterminate quantities of a structure undergoing creep. Alternatively, using only resistances, we can first set up a circuit modelling the solution of an elastic structure in analogues C or D, and interchanging resistances with self-inductions or capacities, we obtain a model of the elastic structure in analogues A' or B'.

As the definition of the analogues clearly indicates, statically indeterminate quantities are modelled in analogue A' /similarly as in C/ by current, and in analogue B' /or D/ by voltage; furthermore, that the conditions of equilibrium correspond in analogue A' to the first Kirchhoff's law /zero sum of currents meeting at a node/, while in analogue B' to the second Kirchhoff's law /zero sum of voltages in a closed loop/. The conditions are a priori satisfied [7] if we solve the circuit on the basis of loop currents in analogue A' and on the basis of voltages in nodal pairs in analogue B'. The conditions of the structure compatibility in statically indeterminate connections then correspond to the second Kirchhoff's law in analogue A' /or C/, and to the first Kirchhoff's law in analogue B' /or D/.

The problem of the circuit structure has thus been reduced to that of finding a circuit modelling the solution of a system of n linear algebraic equations with a symmetric matrix. In analogue A' /or C/, the circuit must contain n loops with n elements, of which each two loops must generally have a

common element and each two nodal pairs must generally be interconnected through one element. Each loop must contain a source of voltage modelling the deformation of the basic system in the X-direction produced by external load /i.e. by the current in the respective loop if the remaining loops are under no current - open circuit/. Similarly in analogue B' this loading term is at each nodal pair modelled by the source of current /and the load corresponds to voltage if the remaining nodal pairs are without voltage - short circuit/. The deformation of the basic system produced by load $X_i = 1$ in the X_k-direction, i.e. the flexibility coefficient, corresponds in analogue A' to voltage on the terminals of loop i induced by unit current in loop k if all the remaining loops are without current /open circuit/, and in analogue B' to current in nodal pair i without voltage /short circuit/ induced by unit source voltage in nodal pair k if all the remaining nodal pairs are without voltage /short circuit/.

In this way we obtain in analogue A' as models of 2-times, 3-times, 4-times statically indeterminate structures, circuits A2, A3, A4 according to Fig.3 /they are so-called N-poles/, and in analogue B' generally for n-times statically indeterminate structure, circuit Bn according to Fig.3d. Elementary circuits Aa and Ab, and Ba and Bb, respectively, according to Fig.2 should be visualized as the various elements of these circuits. The coefficients of the elements of this circuit can be deduced from eq./8/ without the necessity of setting up a system of differential equations, changing it to a single equation of the first order and searching for its analogue. /Note: when drawing the circuit in the analogue, the groups of three self-inductions in T-connection occurring therein can be replaced equivalently by two parallel self-inductions with common self-induction, i.e. by transformers/.

To illustrate our discussion, we shall write a system of equations ensuing from the solution of a circuit for the special case of an elastic structure in which the various elements represent /A/ self-inductions or /B/ capacities /while in analogues C, D they represent resistances/. Using the first and second Kirchhoff's laws, respectively, we obtain /A/ a system of algebraic equations $U_i = -\sum_{k=1}^{n} L_{ik} \frac{dI_k}{dt}$ for $\frac{dI_k}{dt}$ where $L_{ii} = -\sum_{r=0(r\neq i)}^{n} L_{ir}$ /Fig.3, circuits A2, A3, A4/, or /B/ a system of equations $I_i = -\sum_{k=1}^{n} C_{ik} \frac{dU_k}{dt}$ for $\frac{dU_k}{dt}$ where $C_{ii} = -\sum_{r=0(r\neq i)}^{n} C_{ir}$ /Fig.3, circuit Bn/. As the equations suggest, all the non-diagonal coefficients of the system matrix are negative for certain directions of the loop currents or voltages of the nodal pairs. Consequently, the circuits can be employed directly only for solution of such structures in which the statically indeterminate quantities can be introduced in a manner that renders all the non-diagonal coefficients of the flexibility matrix negative; this can be done for the majority of structures in practice. In other cases it would be necessary to introduce electric elements which would act as negative self--inductions, negative capacities or negative resistances at any arbitrary stress variation.

5. Note. Various nonlinear effects observable in creep of concrete can also be modelled electrically. Thus e.g. irreversible creep deformations can be represented in a mechanical model with the aid of a ratchet pawl /Fig.2d/ [9] . In the corresponding electric models Ad, Bd in Fig.2 the ratchet pawl is represented by a diode. The nonlinear relation between creep and stress at high stresses can be represented by nonlinear self-inductions, capacities and resistances.

6. Note. The same models can also be used for representation of the creep of structures of plastics at variable temperature because in such cases the material parameters are also time variable. In the special instance of time invariable parameters, such models represent the behaviour of structures of plastics at constant temperature.

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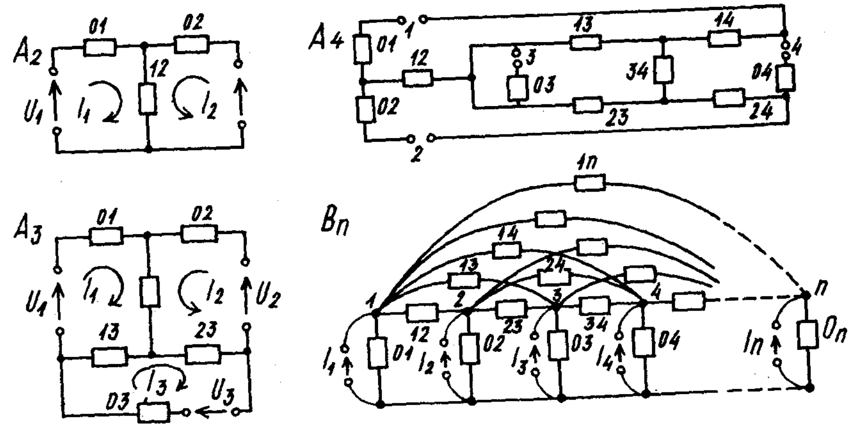

Fig. 3

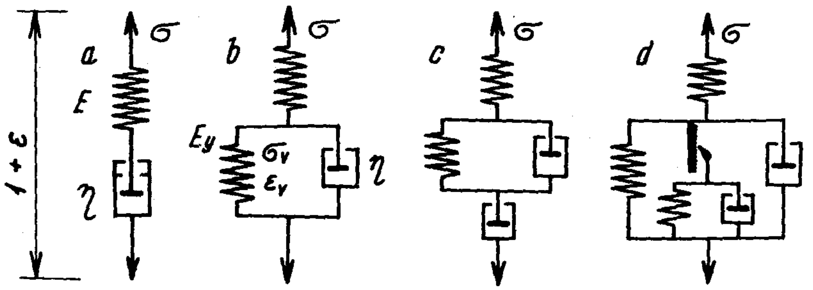

Fig. 1

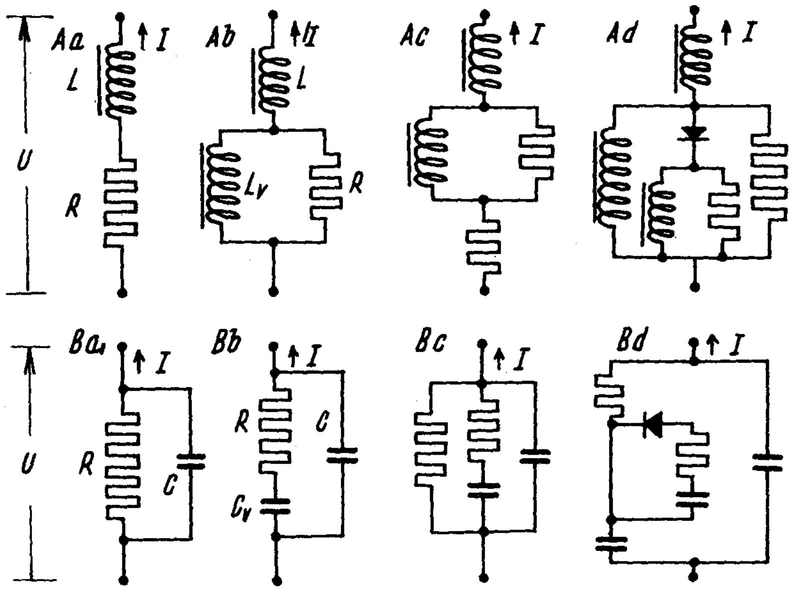

Fig. 2