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COMMENT ON HILLERBORG'S COMPARISON OF SIZE EFFECT LAW WITH FICTITIOUS CRACK MODEL (*)

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In the preceding paper in this volume (1), Arne Hillerborg presented a very interesting and valuable comparison of his nonlinear fracture model with the size effect law. However, omission of certain aspects of the comparison might be misleading and invites further comment.

Hillerborg properly points out that a comparison with his model does not necessarily tell the truth, and that a comparison with real materials would be more relevant. However, it may be also pointed out that his model has not yet been validated by comparison with the bulk of the concrete fracture data available in the literature. On the other hand, for the crack band model (2), which appears to yield rather similar but not identical results, such a comparison has been made and a satisfactory agreement was demonstrated for the practical size range of interest. Thus the crack band model might have been a less objectionable yardstick than Hillerborg's fictitious crack model.

Neither model is, however, unobjectionable for the purpose of verifying the extrapolation to infinite size, which is the essence of Hillerborg's argument. There exist no test data to verify such an extrapolation, and they are not urgently needed for practical applications. For extrapolations from laboratory fracture tests to real concrete structures, the relative structure sizes need to cover roughly the range of 1:50 , in which the size effect law approximates the test data, as well as both fracture models, adequately, espe-

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cially in comparison with the typical random scatter of test results. There is no practical need for the theoretical value of fracture energy for an infinite structure size. The fracture energy, G_f , should be regarded as a material parameter to characterize concrete fracture within the size range of the usual test specimens and the usual structures (i.e., about 1:50) ; G_f represents the fracture energy value for the infinite size extrapolation of this practical size range, and not the actual fracture energy value for a structure of actually infinite size. In this sense it is a fictitious value, just like Hillerborg's crack model is fictitious. If the size effect law is used to obtain the fracture energy for the crack band model (as in Hillerborg's Ref. 6) or for Hillerborg's model, fracture analysis will then yield good results for structures up to about 10-times larger than the largest fracture specimens, which is sufficient for most practical applications.

Sufficient though the original size effect law may be, we should nevertheless realize that a more general and more accurate size effect law is possible. As derived in (1), the most general size effect law for blunt fracture may be expressed in terms of the following asymptotic expansion:

$$\sigma_N = B\, f_t^* \left[1 + \frac{d}{\lambda_0 d_a} + \frac{\lambda_1 d_a}{d} + \left(\frac{\lambda_2 d_a}{d} \right)^2 + \left(\frac{\lambda_3 d_a}{d} \right)^3 + \cdots \right]^{-\frac{1}{2}} \tag{1}$$

σ_N is the nominal stress at failure (f_{net} in Hillerborg's notation), d is the characteristic size of structure, d_a is the maximum aggregate size, B, λ_0, λ_1, λ_2, ... are constants characterizing the shape of the structure, and $f_t^* = f_t$ = concrete strength if d_a is constant. If d_a is varied, then $f_t^* = f_t'[1 + (c_0/d_a)^{1/2}]$ where c_0 = positive constant (see Bazant and Kim's discussion reply in ACI Journal, July-August, 1985). Furthermore, if there is plastic dissipation outside the fracture process zone, a positive constant σ_0 must be added on the right-hand side of Eq. 1. For the present discussion we may use $\sigma_0 = 0$ and $f_t^* = f_t'$.

The linear elastic fracture mechanics represents the zero-th order asymptotic approximation ($\lambda_0 = \lambda_1 = \lambda_2 = \ldots = 0$) and the original size effect law (Hillerborg's Ref. 5) represents the first-order approximation ($\lambda_0 > 0$, $\lambda_1 = \lambda_2 = \lambda_3 = \ldots 0$) . If enough terms of the expansion are included, Eq. 1 would be able to fit the solid curve in Hillerborg's Fig. 5 as closely as desired.

Another, although not the most general possible, refinement of the size effect law (with r = positive constant) is

$$\sigma_N = B\, f'_t \left[1 + \left(\frac{d}{\lambda_0 d_a}\right)^r\right]^{-\frac{1}{2r}} \tag{2}$$

It, too, has the plastic limit analysis and the linear elastic fracture mechanics as its limits, and is consistent with the argument that originally led to the size effect law (Hillerborg's Ref. 5) and later, in a more rigorous manner, to Eq. 1 (2). Eq. 2 may be regarded as the first order approximation based on the generalized asymptotic series expansion

$$\sigma_N = B\, f^*_t \left[1 + \left(\frac{d}{\lambda_0 d_a}\right)^r + \left(\frac{\lambda_1 d_a}{d}\right)^r + \left(\frac{\lambda_2 d_a}{d}\right)^{2r} + \left(\frac{\lambda_3 d_a}{d}\right)^{3r} + \cdots\right]^{-\frac{1}{2r}} \tag{3}$$

With regard to Hillerborg's argument it is now interesting that Eq. 2 can fit the solid curve from Hillerborg's Fig. 5 so closely that the fit is virtually indistinguishable. This fit, which is obtained for $B = 3.061$, $\lambda_0 =$ $= 0.6079$ and $r = 0.4402$ is, therefore, compared to Hillerborg's curve numerically; see Table 1.

TABLE 1 - Comparison of Generalized Size Effect Law (Eq. 2) to Hillerborg's Curve ($r = 0.44$)

$df_t'^2/EG_f$	Hillerborg	σ_N/f'_t (Eq. 2)
0.02	2.43	2.44
0.05	2.22	2.21
0.1	2.01	2.00
0.2	1.77	1.78
0.5	1.46	1.46
1	1.222	1.222
2	0.992	0.995
5	0.725	0.739

Obviously, if the generalized size effect law with $r = 0.44$ (Eq. 2) is used to extract the fracture energy value from size effect tests in the same manner as proposed before (Hillerborg's Ref. 6), close agreement with Hillerborg's value of G_f is obtained. However, as already explained, this would be more of academic than practical interest until a fracture test series of an extremely broad range, capable of unambigous determination of r, is carried out. For a limited range of sizes needed for practice, the gain from using in Eq. 2 values of r other than 1 is probably not very significant.

The fact that Eq. 2 should fit Hillerborg's results so closely is not surprising. It seems there has been a misunderstanding that the size effect law were applicable only to failures due to a crack band of finite width at the front. However, in the last section of Hillerborg's Ref. 5 it was shown that the same size effect law applies to line crack models of Barenblatt-Dugdale type (including Hillerborg's model), in which the crack front blunting stems from a finite length of the fracture process zone in the crack direction. A similar conclusion has recently been reached, strictly on the basis of dimensional analysis, by Elices and Planas (3). These authors hare also independently derived for the blunt crack band theory a generalization of the size effect law different from Eq. 1 and 2.

In conclusion, there is no fundamental disagreement between the size effect law and Hillerborg's model. With a simple refinement of the original size effect law, a perfect agreement can be achieved.

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