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PROBABILISTIC ANALYSIS OF CREEP EFFECTS IN CONCRETE STRUCTURES

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## ABSTRACT

The paper has a two-fold purpose: 1) to give a brief overview of recent works at Northwestern University, and 2) to present a new application of the latin hypercube sampling to Bayesian prediction of creep and shrinkage effects. First a probabilistic prediction model in which the creep and shrinkage structural effects are considered as functions of a certain number of random parameters is reviewed, and determination of the statistics of these effects by latin hypercube sampling of the random parameters is outlined. Subsequently it is shown how this sampling approach can be combined with the short-time data for a given particular concrete to predict long time effects. The Bayesian updating of the prior statistical information is based on comparing the measured short time data with samples of these data predicted on the basis of the prior information and theoretical model. Finally, the modeling of the effect of random environmental humidity or temperature in the context of diffusion theory and spectral analysis is reviewed. Results of a few typical numerical examples accompany the presentation.

## KEYWORDS

Probabilistic methods, concrete, concrete structures, creep, shrinkage, environmental effects, random material properties, random environment, Bayesian prediction, sampling, extrapolation, spectral analysis, aging.

## 1. INTRODUCTION

Creep and shrinkage is the most uncertain mechanical property of concrete, exhibiting a much greater random variability than strength or elastic modulus. Even with the most sophisticated and comprehensive prediction model [1,2], the prediction errors (confidence limits) exceeded with a 10% probability are about  $\pm 31\%$  of the mean prediction. For the ACI Committee 209 model [3], which is much simpler, this increases to about  $\pm 63\%$ , and for the 1978 CEB-FIP model code [4] to about  $\pm 76\%$ . This situation is obviously unsatisfactory, and is

no doubt the cause that many concrete structures suffer damages due to excessive deflections, cracking and inherent corrosion, etc., long before the end of their design lifetime.

It is, therefore, imperative to develop probabilistic design procedures which take this great random variability into account.

A research program dealing with probabilistic design for creep and shrinkage was recently initiated at Northwestern University, and the intent of this paper is to review some of the recent findings as well as to present one new development consisting in the application of a sampling method to Bayesian prediction of creep and shrinkage in situations where an analytical solution appears impossible, due to nonlinear aspects of the problem as well as the presence of too many random variables.

## 2. RANDOM PARAMETER APPROACH

The simplest way to obtain the probability distributions of long-time creep and shrinkage and their structural effects is to regard these effects as deterministic functions of certain random parameters. Random parameters are variables such as the cement content, mix ratios, design strength, factors for the inherent uncertainty of creep and shrinkage prediction formulas and of the principle of superposition used in calculations, environmental humidity and temperature, etc. A model of this type was formulated by Madsen and Bazant [5]. Due to nonlinearity of the functional dependence of the creep and shrinkage effects on these random parameters, and the fact that this functional dependence is expressed as a computational algorithm rather than a closed form expression, the mean and variance of the structural effects had to be calculated numerically. The method of two-point estimates was applied for this purpose, and solution of a number of typical structural design problems was carried out [5]. It was found that in certain situations the coefficient of variation of the effect can be much smaller than that of the random parameters involved, while in other situations it can be far larger. Therefore, the mean predictions with constant safety factors would not represent an acceptable design approach, and the standard deviation of the structural effect must be predicted so that confidence limits with a certain prescribed probability cutoff (say, 5%) have to be determined.

The method of point estimates, even if only two-point estimates are used, is, however, numerically inefficient when all of the known random parameters affecting creep and shrinkage in structures are taken into account. In a recent work [6], the original formulation of Madsen and Bazant has been reworked using a sampling approach which can bring about a very large saving in computational work. The particular sampling approach adopted was the relatively new method of latin hypercube sampling [7-9], which has already been applied for a stochastic finite element analysis of concrete creep by Anderson [10]. This sampling approach has the merit that the number of required deterministic structural creep analyses is reduced from  $2^N$ , as required for the two-point estimates [5], to approximately  $2N$  where  $N$  is the number of random parameters. Numerical examples for various typical structural creep and shrinkage problems have been worked out, and it has been demonstrated that the distribution of the creep effects can be approximately considered as normal if the random parameters are normally distributed.

Fig. 1 shows the cumulative histogram of the effects from 8, 16, and 32 latin hypercube samples, drawn in the normal probability plot [6]. The ordinate is the deflection of an unreinforced (but possibly prestressed) beam, simply supported at the ends. The figure demonstrates that the effects are indeed almost normally distributed, and that increasing the number of latin hypercube samples affects the result only slightly. The slope of this plot indicates the standard deviation. For details, see Ref.6. The curves of deflection versus the age of the beam, representing the mean  $\pm$  standard deviation are shown in Fig. 2 (bottom). The diagram on top represents the results for the same beam, but with reinforcement, and the strong suppressing effect of reinforcement on the random variability should be noted (this is not true of prestressing). The solid curves in these plots correspond to 16 latin hypercube samples, the dashed-dot lines to 8 samples, and the dashed lines show for comparison the results obtained previously for the same problem by the method of two-point estimates [5]. The results for another example, representing the horizontal reaction in a portal frame caused by shrinkage, are shown in Fig. 3 [6], which confirms that the uncertainty in the structural effects can be enormous.

## 3. BAYESIAN ANALYSIS OF NONLINEARIZABLE PROBLEMS VIA SAMPLING

### 3.1 Previous Work on Linearizable Problems

The uncertainty in the prediction of creep and shrinkage effects can be tremendously reduced if even a few short-time measurements for the given concrete to be used are made [2]. The proper framework to do this is the Bayesian framework, in which the prior information consists in the statistics of the creep and shrinkage prediction model used, determined from comparisons with measurements on all kinds of concretes and under many different situations. The most realistic (and also the most laborious and sophisticated) creep prediction model, which was calibrated against a data bank involving over 10,000 data points and over 80 different concretes from different laboratories [1, 11], has been used for this purpose [12]. This prior information was combined, according to Bayes's theorem, with the probability of the measured creep values for given concrete to yield the posterior probability distribution of the long-time creep values. The calculation could be carried out analytically since the creep model was linearized through the use of a certain reduced time, and since the number of random parameters was reduced to only two. It is, however, more realistic to consider all known random parameters, the number of which is at least eight, and to consider a more realistic form of the creep law in which the dependence on the random parameters is not linearizable. The inadvisability of linearization is especially true for shrinkage. Therefore, a sampling approach will now be proposed and described in detail.

### 3.2 Random Variables and Latin Hypercube Sampling

Let  $Y_i$  ( $i = 1, 2, \dots, I$ ) be the long-time effects of creep or shrinkage (at times  $t_i$ ) which we want to predict, and let  $X_m$  ( $m = 1, 2, \dots, M$ ) be the short-time effects of creep or shrinkage (at times  $t_m$ ) which have been measured and which we want to use for improving the predictions of  $Y_i$ . For example,  $Y_i$  may be either a long-time creep or shrinkage strain, or long-time structural effects, such as the maximum deflections of the structure, maximum bending moments or

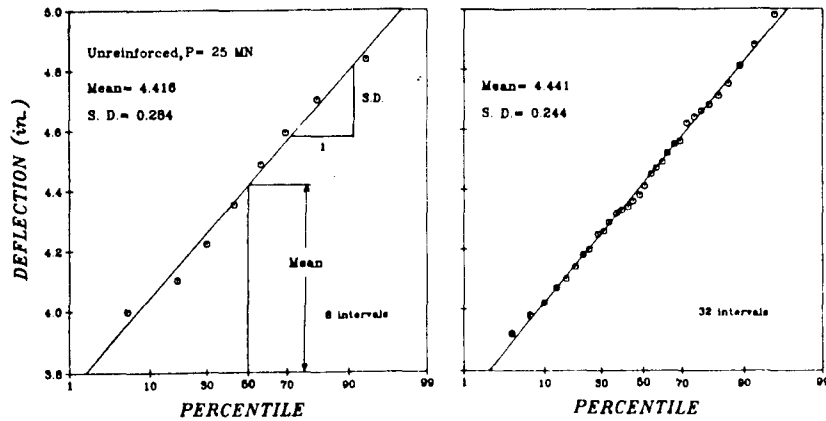


Fig. 1 - Histogram in Normal Probability Plot

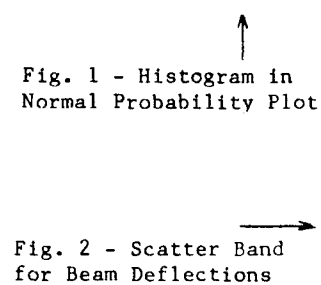


Fig. 2 - Scatter Band for Beam Deflections

Fig. 3 - Scatter Band for Shrinkage Force

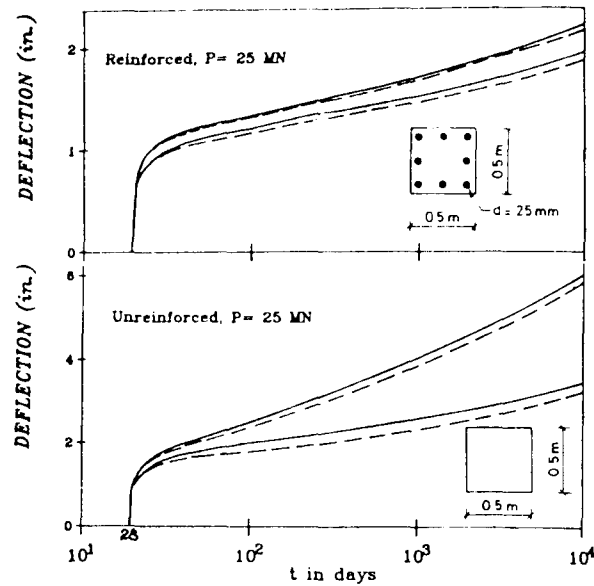
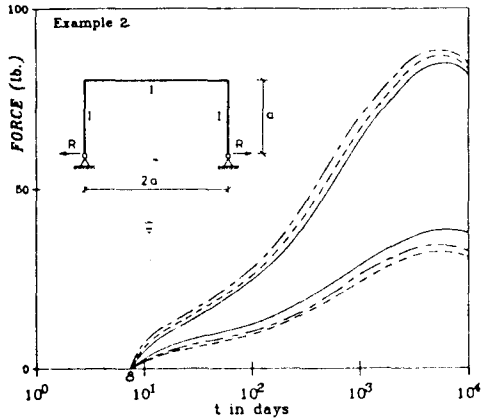


Fig. 4 - Determination of Equal Probability Intervals

maximum stress at 5, 10, 20 and 40 years, and  $X_m$  may be the strains of the control cylinder or structure deflections at the times 1 day, 7 days, and 28 days. The values of  $Y_i$  and  $X_m$  which may be predicted on the basis of the available information (without taking the measurements  $X_m$  into account) will be denoted as  $Y'_i$  and  $X'_m$ . The values of  $Y_i$  may be predicted as certain known functions  $y_i$  of certain random parameters  $\theta_1, \theta_2, \dots, \theta_N$ ; i.e.,  $Y'_i = y_i(\theta)$  where  $\theta = (\theta_1, \theta_2, \dots, \theta_N) =$  vector of random parameters. These functions need not be explicit; they may be defined by an algorithm such as the step-by-step time integration algorithm for creep analysis. Predictions can be also obtained for the short-time effects represented by  $X_m$ , and the predicted values, which generally differ from the measured values, may be regarded as functions of  $\theta$ ;  $X'_m = x_m(\theta)$ . Parameters  $\theta_1, \dots, \theta_N$  may represent, for example, the cement content, water-cement ratio, aggregate-cement ratio, concrete strength, environmental humidity and temperature, permeability, and uncertainty factors for the laws of creep and shrinkage and for the principle of superposition (for illustrations, see Refs. 5 and 6).

Statistical evaluation of various test data from the literature has led to a certain, albeit quite limited, information on the statistical properties of the random parameters  $\theta_1, \theta_2, \dots, \theta_N$  [5, 1, 2, 11]. This may be regarded as the prior statistical information, characterized by the prior probability density distributions  $f'_n(\theta_n)$  with means  $\bar{\theta}'_n$  and standard deviations  $s'_n$  ( $n = 1, 2, \dots, N$ ). With this knowledge it is possible to predict the statistical distributions of  $X'_m = x_m(\theta)$  and compare them to the available measurements  $X_m$ . The objective is to use this comparison to improve, or update the statistical information on random parameters  $\theta_n$ . This improved statistical information may be characterized by updated, or posterior, distributions  $f''_n(\theta_n)$ , from which these updated, or posterior, distributions  $f''_i(Y''_i)$  of long-time effects  $Y_i$  may be obtained according to  $y_i(\theta)$ . The double primes are used to distinguish the posterior characteristics from the prior characteristics, which are labeled by single primes.

In previous works [12,6],  $f''_n(\theta_n)$  and  $f''_i(Y''_i)$  for long-time creep strains, have been obtained analytically for the case when the functions in Eq. 1 are linearized in certain transformed variables, and when there are only two random parameters,  $\theta_1$  and  $\theta_2$ . These simplifications may be acceptable for approximate prediction of long-time creep strains but are insufficient for the prediction of long-time shrinkage or structural effects of creep and shrinkage, for which generally the functions  $y_i(\theta)$  are highly nonlinear and more than two random parameters are involved. In such a case an analytical approach, such as that in Refs. 12, presents considerable difficulties, and a sampling approach is preferable.

One such sampling approach, consisting in the use of latin hypercube sampling will be shown here. In this method, the known distribution  $f''_n(\theta_n)$  of each input parameter  $\theta_n$  is partitioned into  $K$  intervals  $\Delta\theta_n^{(k)}$  (strata) of

equal probability  $1/K$  ( $k = 1, \dots, K$ ). The subdivision may be obtained according to the cumulative probability distribution as illustrated in Fig. 4. The number of random parameter samples is chosen in latin hypercube sampling to be the same as the number of computer runs to be made, and from each interval the random parameter value is sampled exactly once, i.e., it is used in one and only one computer run. The sampled value need not be taken randomly within the interval, but may be taken at the centroid of the interval (Fig. 4). The random selection of the intervals  $\Delta\theta_n^{(k)}$  to be sampled for a particular computer run can be carried out as follows. With each random parameter one may associate a sequence of integers representing a random permutation of the integers  $1, 2, \dots, K$ . For eight intervals ( $K = 8$ ), such random permutations are illustrated in the columns of Table 1. A different random permutation is used for each column (each  $\theta_n$ ). Thus, each interval number appears in each column of Table 1 once and only once. The individual latin hypercube samples are then represented by the lines in Table 1. For each such sample  $\theta_n^{(k)}$  ( $n = 1, \dots, N$ ) one can then calculate according to the creep or shrinkage prediction model the effects  $X_m^{(k)}$  which correspond to the measured short-time effects  $X_m$ , and also the corresponding long-time effects  $Y_i^{(k)}$ . From these results, one can then obtain the mean prior prediction  $\bar{X}_m'$  for the measured effects  $X_m$ , and also the mean prediction  $\bar{Y}_i'$  of the long-time effects, as well as the corresponding standard deviations;

$$\bar{X}_m' = \frac{1}{K} \sum_{k=1}^K X_m^{(k)}, \quad s_{X_m'} = \left[ \frac{1}{K} \sum_{k=1}^K \left( X_m^{(k)} - \bar{X}_m' \right)^2 \right]^{\frac{1}{2}} \quad (m = 1, 2, \dots, M) \quad (1)$$

$$\bar{Y}_i' = \frac{1}{K} \sum_{k=1}^K Y_i^{(k)}, \quad s_{Y_i'} = \left[ \frac{1}{K} \sum_{k=1}^K \left( Y_i^{(k)} - \bar{Y}_i' \right)^2 \right]^{\frac{1}{2}} \quad (i = 1, 2, \dots, I) \quad (2)$$

### 3.3 Determination of Posterior Probabilities by Sampling

We now try to determine the posterior (updated) probability of values  $Y_i^{(k)}$ . Since one calculated value  $Y_i^{(k)}$  is obtained for each random sample  $\theta^{(k)} = (\theta_1^{(k)}, \dots, \theta_N^{(k)})$ , i.e.,  $Y_i^{(k)} = y_i(\theta^{(k)})$ , the posterior probability of  $Y_i^{(k)}$  is the same as the posterior probability of  $\theta_1^{(k)}$ , i.e.,  $P''(Y_i^{(k)}) = P''(\theta^{(k)})$ . While the prior probabilities of all latin hypercube samples  $\theta^{(k)}$  and of the corresponding  $X_m^{(k)}$  and  $Y_i^{(k)}$  are equal the posterior ones are not. The posterior probability is a conditional probability, the condition being that the values  $X_1, \dots, X_M$  have been observed; i.e.,  $P''(X_m^{(k)}) = P''(Y_i^{(k)}) = P''(\theta^{(k)}) = P(\theta^{(k)} | X)$  where  $X = (X_1, X_2, \dots, X_M)$  = vector of the measured creep or or shrinkage effects on which the updating is based. According to Bayes' theorem [5-12],

$$P''(X_m^{(k)}) = P''(Y_i^{(k)}) = P(\theta^{(k)} | X) = c_1 L(X | \theta^{(k)}) P'(\theta^{(k)}) \quad (3)$$

Here  $L(X | \theta^{(k)}) = L(X_1, X_2, \dots, X_M | \theta^{(k)})$  is called the likelihood function; it represents the relative joint conditional probability of observing the measured values  $X_1, X_2, \dots, X_M$  under the condition that the parameter vector  $\theta$  coincides with the  $k^{th}$  sample  $\theta^{(k)} = (\theta_1^{(k)}, \dots, \theta_N^{(k)})$ . The prior probability of each sample is

$$P'(\theta^{(k)}) = P'(X_m^{(k)}) = P'(Y_i^{(k)}) = 1/K = \text{constant} \quad (4)$$

and  $c_1$  is a normalizing constant to be determined from the condition that

$$\sum_{k=1}^K P''(Y_i^{(k)}) = 1 \quad (5)$$

As is usually the case in Bayesian estimation, the most difficult task is the determination of the likelihood function, representing the relative joint probability of observing the measured values  $X_1, X_2, \dots, X_M$ . They are certainly to some extent correlated, but since we have at present no information on the correlation of the successive values of creep or shrinkage, we will assume for the sake of simplicity that the values  $X_1, X_2, \dots, X_M$  are statistically independent, i.e., uncorrelated. Obviously, the larger the time interval, the weaker the correlation of creep or shrinkage values at the beginning and the end of the interval, and so the assumption of statistical independence should be acceptable if the measurements are spaced sparsely in time.

It may be noted that a better approach would be to assume statistical independence for the successive increments of creep and shrinkage rather than for their total values. This would, however, lead to the problem of Bayesian estimation for a nonstationary stochastic process with independent increments, which is a much more difficult problem to handle than the present Bayesian problem in the setting of statistical regression. That problem is difficult enough for practical application even without the Bayesian aspect [13].

According to the simplifying assumption of statistical independence, we have

$$L(X | \theta^{(k)}) = L(X_1, X_2, \dots, X_M | \theta^{(k)}) \approx \prod_{m=1}^M f_m(X_m | \theta^{(k)}) \quad (6)$$

Here we introduce the distribution  $f_m^X(X_m | \theta^{(k)})$  which represents density distribution of the conditional probability to obtain any value  $X_m$  under the condition that the random parameter values are  $\theta_1^{(k)}, \dots, \theta_N^{(k)}$ . Note that this probability density is not the prior probability of  $X_m$  and cannot be taken the same as  $f_m^{X'}(X_m^{(k)})$  for the prior, which characterizes the statistical scatter of the properties of all kinds of concretes in general. Rather,  $f_m^X(X_m | \theta^{(k)})$

should be interpreted simply as a characteristic of the statistical scatter of the properties of one particular concrete. Generally, the standard deviation  $s_m^X$  of  $f_m^X(X_m|\theta^{(k)})$  will be smaller than that of the prior,  $f_m^{X'}(X_m^{(k)})$ . The fact that the probability described by  $f_m^X$  is conditional to  $\theta^{(k)}$  means that it refers to one particular concrete, for which  $\theta^{(k)}$  are essentially fixed.

Substituting now Eqs. 6 and 4 into Eqs. 3 and 2, we obtain the result

$$P''(Y_i^{(k)}) = P''(\theta^{(k)}) = c_0 P_k, \quad P_k = \prod_{m=1}^M f_m^X(X_m|\theta^{(k)}) \quad (7)$$

in which  $c_0 = c_1/K = \text{constant}$ . This constant may be determined from the normalizing condition (Eq. 7);

$$c_0 = \left( \sum_{k=1}^K P_k \right)^{-1} \quad (8)$$

The foregoing relations hold in general for any probability distributions. To obtain the prior probability density distribution  $f_m^{X'}(X_m^{(k)})$ , one may use many samples  $\theta^{(k)}$  to generate many values  $X_m^{(k)}$  ( $k = 1, \dots, K$ ), then construct the density histogram of these values and fit to it a suitably chosen formula. To obtain the posterior probability density distributions  $f_m^{X''}(X_m'')$  and  $f_i^{Y''}(Y_i'')$ , one needs to construct the weighted density histograms of values  $p_k X_m^{(k)}$  and  $p_k Y_i^{(k)}$  calculated for all samples  $\theta^{(k)}$ , and then again fit to these histograms a suitably chosen formula. The posterior (updated) means and standard deviations of the predictions of  $X_m''$  and  $Y_i''$  are obtained as

$$\bar{X}_m'' = \frac{1}{\sum_k P_k} \sum_k P_k X_m^{(k)}, \quad s_m^{X''} = \left[ \frac{1}{\sum_k P_k} \sum_k P_k (X_m^{(k)} - \bar{X}_m'')^2 \right]^{\frac{1}{2}} \quad (9)$$

$$\bar{Y}_i'' = \frac{1}{\sum_k P_k} \sum_k P_k Y_i^{(k)}, \quad s_i^{Y''} = \left[ \frac{1}{\sum_k P_k} \sum_k P_k (Y_i^{(k)} - \bar{Y}_i'')^2 \right]^{\frac{1}{2}} \quad (10)$$

From now on we assume that all distributions are normal (Gaussian), which is likely sufficient for practice. Then the prior probability densities are

$$f_m^{X'}(X_m') = \frac{1}{s_m^{X'} \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{X_m' - \bar{X}_m'}{s_m^{X'}} \right)^2 \right] \quad (11)$$

$$f_m^X(X_m|\theta^{(k)}) = \frac{1}{s_m^X \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{X_m - X_m^{(k)}}{s_m^X} \right)^2 \right] \quad (12)$$

where  $X_m^{(k)}$  is the value calculated for the k-th parameter sample,  $\theta^{(k)}$ .

According to Eq. 7

$$P_k = \exp \left[ -\sum_{m=1}^M \frac{1}{2} \left( \frac{X_m - X_m^{(k)}}{s_m^X} \right)^2 \right] \quad (13)$$

in which we dropped the multiplicative constant since only the relative values of  $P_k$  affect the result. The posterior probability density distributions are

$$f_m^{X''}(X_m'') = \frac{1}{s_m^{X''} \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{X_m'' - \bar{X}_m''}{s_m^{X''}} \right)^2 \right] \quad (14)$$

$$f_i^{Y''}(Y_i'') = \frac{1}{s_i^{Y''} \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{Y_i'' - \bar{Y}_i''}{s_i^{Y''}} \right)^2 \right] \quad (15)$$

where  $\bar{X}_m''$ ,  $s_m^{X''}$ ,  $\bar{Y}_i''$ , and  $s_i^{Y''}$  are obtained from Eqs. 9-10.

### 3.4 Simplified Alternative for Sharp Likelihood

The foregoing procedure works, however, only if the likelihood distribution  $f_m^X$  is not much sharper (narrower) than the prior distribution  $f_m^{X'}$ . This is the case if the measured data are very scant (only a few values) in which case the standard deviation of the measured values (likelihood)  $X_m$  must be guessed by analogy with similar concretes used before at the same locality, or if the measurements are carried out in haste, in an unsophisticated laboratory, with make-shift instrumentation or inexperienced personnel. More frequently, however, the measured data are good and their standard deviation  $s_m^X$  is small, i.e., the likelihood is sharp compared to the prior. This creates problems for the sampling approach. In the space of random parameters  $\theta_1, \theta_2, \dots, \theta_N$ , the region for which the likelihood distribution  $f_m^X$  does not have negligible values represents only a small island compared to the domain in which the prior probability distribution values  $f_m^{X'}$  are nonnegligible. If, for example,  $s_m^X < s_m^{X''}$ , and if there are eight random parameters, then the fraction of the parameter samples  $\theta^{(k)}$  that falls within the aforementioned island is roughly  $3^{-8}$  or 0.00015 of the number of all samples. In such a case, of course, no meaningful information can be extracted from the sampling results since the

$p_k$  values would be also almost 0 for 99.985% of all samples. The nature of the difficulty is actually similar to that encountered with the numerical integration of the probability distributions, as described in [12]. We now describe an alternative simplified method which avoids this difficulty.

The reason for identifying the random parameter samples with various relative likelihoods  $p_k$  has been the need to use these parameter samples to calculate subsequently the long time effects  $Y_i$ . However, the long time effects  $Y_i$  can be also predicted by regression or optimization methods, and this is what we now try to exploit. At first we forget about the prior information, and simply try to extrapolate the measured short time data into the long times  $t_i$ . For this, we need to determine first the statistics of the measured short time data, at least their mean trend and standard deviation. For this purpose we need to consider a suitable formula for the effect, for example the formula for the shrinkage curve [1, 2]

$$\epsilon_s(t) = \epsilon_\infty \left[ 1 + \left( \frac{t}{\tau_{sh}} \right)^r \right]^{-1/2r}, \quad X_m = \epsilon_s(t_m), \quad Y_i = \epsilon_s(t_i) \quad (16)$$

in which  $\epsilon_s$  is the shrinkage strain,  $t$  is the duration of drying, and  $\epsilon_\infty$ ,  $\tau_{sh}$  and  $r$  are empirical parameters;  $\epsilon_\infty = \theta_1$ ,  $\tau_{sh} = \theta_2$  and  $r = \theta_3$ . We can now obtain the optimum fit of the measured values  $X_m$  ( $m = 1, \dots, M$ ) with this law (Eq. 16), and considering the deviations  $\Delta X_m$  of the measured data  $X_m$  from the optimum fit curve (Eq. 16) we can get the distributions of the deviations  $\Delta X_m$  and their standard deviation  $s_m^X$ . Normally this statistic is calculated simultaneously for all measurement times  $t_m$ , and then  $s_m^X$  is the same for all  $t_m$ . However, if many measurements on different specimens are taken for each time  $t_m$ , then the statistics can be also calculated separately for each time  $t_m$ , in which case the  $s_m^X$ -values for different  $t_m$  are not the same. If the number  $M$  of the measured values  $X_m$  is too small, a suitable value of  $s_m^X$  must simply be guessed from experience.

If the law (e.g. Eq. 16) can be linearized in some transformed variables the task just described is better carried out by linear regression, which yields both the mean and the variation of  $s_m^X$  with time. This then directly provides the standard deviation for the long time extrapolations at times  $t_i$ . However, here we are interested in the cases where linear regression is impossible, which is the majority of cases. The extrapolation to  $t_i$  may then be approximately obtained as follows.

Within the time range of the measured values  $X_m$ , we select as many uniformly spaced representative times  $T_j$  ( $j = 1, 2, \dots, J$ ) as there are unknown

parameters in the law. E.g., for Eq. 16 there are three unknown parameters  $\epsilon_\infty$ ,  $\tau_{sh}$ , and  $r$ , and so we choose three times  $T_j$  ( $j = 1, 2, 3$ ). Times  $T_j$  may, but need not, coincide with some of the measurement times  $t_m$ . If the numbers of all  $T_j$  and of all unknown parameters are the same, the law (such as Eq. 16) can be fitted exactly to any given set of values at times  $T_j$  (except for some pathological cases where the law permits only a certain range of  $\epsilon_s$ ). If there were more times  $T_j$  than unknown coefficients, then exact fitting to any given values at times  $T_j$  would be impossible, and if there were fewer times than unknown coefficients, then the fit would be indeterminate. We should note that times  $T_j$  should be spaced uniformly over the time range of the measured data and should cover it fully. The uniform distribution need not be in the actual time, in fact for creep and shrinkage a uniform distribution in log-time is more appropriate.

For each representative  $T_j$  we know from the previous statistical analysis of the measurements the cumulative distribution  $F_j^X$  of  $X_j(T_j)$ , including the mean  $\bar{X}_j(T_j)$  and the standard deviation  $s_j^X(T_j)$ . We can then subdivide this cumulative distribution into  $R$  intervals of equal probability  $1/R$  and consider the centers of these intervals as possible sampling values  $X_j^{(r)}(T_j)$ . From among these values we may generate samples  $X_1^{(r)}(T_1) \dots X_J^{(r)}(T_J)$  ( $j = 1, \dots, J$ ) in the latin hypercube manner, and then we can solve the empirical parameters in the law (Eq. 16) so as to make the law fit exactly the sampling values. The empirical parameters for the exact fit may be found most conveniently by a nonlinear optimization subroutine such as the Marquardt-Levenberg algorithm. For each sample we thus obtain an extrapolated effect  $Y_i^{(r)}(t_i)$  at each long time  $t_i$ , one extrapolated value for each sample  $j$  and each long time  $t_i$ .

The foregoing procedure obviously involves some simplifications. It would be more realistic to consider not only fixed representative times  $T_j$ , but all possible samples of  $J$  representative times selected from the range of measured data. Furthermore, due to considering the effects for various times as statistically independent, the present statistical model permits a random decrease of  $X_j^{(r)}$  while going to a longer time; in such a case the law (such as Eq. 16) cannot fit the sampled values  $X_1^{(r)}(T_1), \dots, X_J^{(r)}(T_J)$  exactly, and merely an optimum fit (possibly under a certain restriction for the empirical parameters) must be accepted for the purpose of extrapolation to the long times  $t_i$ . However, this limitation could be avoided only if the independent random variables were the increments of creep or shrinkage rather than their total values.

The values  $Y_i^{(r)}$  ( $r = 1, 2, \dots, R$ ) extrapolated from the latin hypercube samples at representative times  $T_j$  have obviously equal probabilities  $1/R$ .

Constructing the histogram of the values  $Y_i^{(r)}$  for each  $t_i$ , and fitting it with a smooth curve, we can get the distribution  $f_i^Y(Y_i)$  for each variable  $Y_i$ . We can also get for each long time  $t_i$  the mean and standard deviation of the extrapolations;

$$\bar{Y}_i = \frac{1}{R} \sum_{r=1}^R Y_i^{(r)}, \quad s_i^Y = \frac{1}{R} \left[ \sum_{r=1}^R (Y_i^{(r)} - \bar{Y}_i)^2 \right]^{1/2} \quad (17)$$

The extrapolated statistical information at long time  $t_i$  may now be considered as the measured data and confronted in the Bayesian concept with the predictions  $Y_i'$  from the prior. For this purpose we carry out the other latin hypercube sampling for the random parameters  $\theta_n^{(k)}$ , as described before, and from the calculated effects  $Y_i'^{(k)}$  for all the samples we obtain the mean and the standard deviation of the prior prediction,  $\bar{Y}_i'$  and  $s_i^{Y'}$  for each time  $t_i$ , as described before.

Assuming normal distributions for both the extrapolated measured data and the predictions from the prior, we now have an elementary Bayesian problem, consisting in a combination of a normal prior with a normal likelihood (extrapolated measurements). As is well-known [16,p.74], the posterior distribution is then also normal and its mean and standard deviation are given as

$$\bar{Y}_i'' = \frac{(s_i^Y)^{-2} \bar{Y}_i + (s_i^{Y'})^{-2} \bar{Y}_i'}{(s_i^Y)^{-2} + (s_i^{Y'})^{-2}}, \quad (s_i^{Y''})^{-2} = (s_i^Y)^{-2} + (s_i^{Y'})^{-2} \quad (18)$$

#### 4. RANDOM ENVIRONMENT

Although random environment can be to a certain extent treated as a random parameter in a function describing the overall structural effects, this approach is too simplistic since it does not reveal the variation of the random humidity effect throughout the cross section and the dependence of the effect on the size of the cross section. A more realistic analysis must take into account diffusion theory, since shrinkage strains are governed by pore humidities which result from the diffusion theory. Due to the diffusion nature of the problem, the standard deviation of the random time variation of pore humidity and shrinkage attenuates with the depth below the exposed surface of concrete, and the front of mean environmental humidity propagates into the concrete at a rate decreasing as the square root of time. An analysis based on diffusion theory, which takes the foregoing aspects into account, has been developed recently [22, 23]. In this formulation, it is efficient to adopt the spectral approach, in which the environment is prescribed as a stationary random process in time with a known spectrum. One then seeks for each point of the structure and each age the mean and standard deviation of pore humidity, shrinkage strain, and induced stresses as exposures to all possible realizations of the environment are considered [24]. The solution can be carried out most generally by the finite

element method, in which the nodal displacement as well as element stresses and strains are considered as complex variables.

#### 5. CONCLUSION

Creep and shrinkage appear to be the most uncertain properties of concrete, and taking this uncertainty into account is important for long term reliability of structures, and in some instances even for safety. The sampling approach represents an efficient means of determining the statistics of the response of a concrete structure. The sampling approach can be also adapted to a Bayesian analysis in which statistics based on prior information for many concretes are updated on the basis of limited measurements for a given concrete. A fully realistic treatment of random environmental humidity and temperature requires taking diffusion theory into account, and the spectral approach appears to be appropriate for this purpose.

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