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INTERACTION OF SIZE EFFECT AND RELIABILITY OF DESIGN: CASE STUDY OF FLEXURAL STRENGTH OF CONCRETE

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Abstract. *In spite of the fact that size effect phenomena have been extensively studied and their importance has been amply demonstrated, the size effect is still not considered in most design specifications. The aim of the paper is to demonstrate the influence of size effect on the reliability of design. This is done by comparing the effect of the dead and live load factors as specified in the design codes. The so-called hidden size effect implied by the currently used dead load factor is reviewed. With the help of a simple example of flexural strength of a concrete beam failing at crack initiation, it is shown that the structure size may have a drastic influence on the failure probability, and particularly that even large load factors cannot play the role of a "safety bell". Further it is explained that the size effect at fracture initiation must have been a significant contributing factor in many catastrophic structural failures, e.g., the collapse of Schoharie Creek Bridge caused by fracture of an unreinforced foundation plinth. Because of its simplicity, the example of flexure of an unreinforced beam is helpful for showing the general methodology. The method of realistic prediction of the interaction of size effect and reliability using computational modeling based on nonlinear fracture mechanics combined with structural reliability approaches is briefly discussed. Finally, critical comments are made on the AASHTO LRFD Bridge Design Specifications*[1]*, particularly on the limit state Strength IV which characterizes the case of very high dead-to-live load effect ratios, which is typical for large-span bridges.*

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1 INTRODUCTION

The reliability of design is, according to the current specifications, achieved by load factors for the dead and live loads (aside from material capacity reduction factors). For example, according to ACI Standard 318[2], the designer must consider the dead load factor 1.4 and live load factor 1.7. The internal forces caused by the live load L and the dead load D are magnified by these load factors to get the internal force caused by ultimate load U, i.e., $U = 1.4D + 1.7L$. Noting that the dead load factor 1.4 is excessive by far in view of true uncertainties in design, Bažant and Frangopol (2001)[3] showed that this implies a hidden size effect which partially compensates for the absence of the actual size effect in the code. They emphasized that a reduction in the dead load factor, which has recently been contemplated, would be dangerous because of the existence of size effect. While these previous investigators showed the consequences of the excessive dead load factor and the hidden size effect, they did not utilize any specific size effect formula. Nevertheless, some important questions arise: What is the influence of the size effect on reliability in a quantitative form? What would be the precise consequences of a reduction of the load factors on reliability? Can such consequences be determined using deterministic approaches or must the answer by sought in? What is the change in failure probability due to the the real size effect for different types of failures and structure geometries, especially if the most economical, i.e. "fully stressed", design is considered? To answer these questions for the simple case of a concrete beam subjected to bending, is the aim of this study.

The bending of plane concrete beams is selected as a fundamental yet simple case in which the size effect can be expressed easily using a one-line simple energetic-statistical size effect formula[4]. This formula, giving the modulus of rupture f_r, can be regarded as asymptotic matching between the deterministic-energetic formula, which is approached for small sizes, and the power law size effect of the classical Weibull statistical theory, which is approached for large sizes. The formula, which has been extensively verified by experimental data, reads as follows:

$$f_r = f_r^0 \left[\left(\frac{D_b}{D} \right)^{rn/m} + \frac{rD_b}{D} \right]^{1/r} \tag{1}$$

where f_r^0, D_b, r and m are positive constants, representing unknown empirical parameters; n is the number of dimensions in geometric similarity, n = 2 or 3; and D_b is a constant that has approximately the meaning of the thickness of a boundary layer of cracking.

2 CONSEQUENCE OF LOAD FACTORS ON RELIABILITY

2.1 Consequence in deterministic sense

Consider geometrically similar beams of different sizes D, where D represents the depth of cross section (characteristic size). Assume also that the dead load consists of the own weight only. The modulus of rupture, f_r, is defined as the maximum normal stress in the beam, calculated elastically from the maximum (ultimate) bending moment M_u, $f_r = 6\,M_u/bD^2$, where D, b = beam depth and width. The value calculated from the real ultimate bending moment

M_u^{real} will be denoted as f_r^{real}. The size dependence of f_r^{real} is defined by the size effect formula (1). Then the ultimate internal force caused by the ultimate loads is simply:

$$M_u^{real} = f_r^{real} \frac{D^2 b}{6} \tag{2}$$

The part of bending moment, M_{dead}, caused by the dead load q_{dead} considered as uniformly distributed is $q_{dead} l^2/8$, where l is the span of beam; q_{dead} naturally depends on size D. Then maximum real value of the bending moment due to live load is:

$$M_{live}^{real} = M_u^{real} - M_{dead} \tag{3}$$

Up to now, we have worked with the "real" values which express the reality approximately in the "mean sense". But what about the design according to the building code? The design should consider the load factors and the design values of material strength. In our case, the strength is represented by the modulus of rupture, f_r^{design}. Suppose that this value is determined from standard tests using one size only, that is, from specimens of the size $D = 100$ mm. This value is in practice taken as constant for the designed structure of any size, i.e., no size effect is considered. But the value is decreased, in order to ensure safe design, by safety factor γ which reflects the statistical variability of the modulus of rupture and represents the design value prescribed by the code. Then the design ultimate moment caused by ultimate load U is

$$M_u^{design} = f_r^{design} \gamma \frac{D^2 b}{6} \tag{4}$$

The aim is to express again the maximum value of the bending moment due to live load, but now the maximum value according to the design, $M_u^{design} = 1.4 M_{dead} + 1.7 M_{live}^{design}$. The design bending moment due to live load expresses the design ultimate capacity:

$$M_{live}^{design} = (M_u^{design} - 1.4 M_{dead})/1.7 \tag{5}$$

This value of bending moment corresponds to the ultimate live load according to design. All moments not exceeding M_{live}^{design} are allowed. But in reality this maximum value is M_{live}^{real}. Comparison of these two values can give an answer to the question of design safety. In the case that $M_{live}^{real} \geq M_{live}^{design}$, the design is safe i.e., there is still some reserve. The ratio

$$M_{live}^{real} / M_{live}^{design} \tag{6}$$

can therefore be considered as a deterministic measure of design safety. If this ratio drops bellow 1, it means that a safe design is not achieved. It should be particularly noted that a change of the dead load factor from 1.4 to another value will influence the ratio (6).

2.2 Consequence in probabilistic sense

The uncertainties due to the material, structural geometry, loading, environmental effects and modeling are inevitably always involved. That is why a deterministic treatment of ratio (6) should be considered as only the beginning of a proper reliability treatment. In other words, all the uncertainties involved should be considered as random variables and the theoretical failure probability can then be defined as:

$$P_f = P(\frac{M_{live}^{real}}{M_{live}^{design}} \leq 1.) \tag{7}$$

The design according to the building code should result in a very small failure probability. In our parametric study, it is expected that this failure probability is influenced by the load factors as well as the size of beam D, i.e. the size effect.

2.3 Numerical example

To demonstrate the foregoing procedure and to show the influence of load factors, the particular size effect law in (1), relevant to the test data[7], will be considered. The parameters of formula (1) are $f_r^0 = 4.61$ MPa, $D_b = 17.89$ mm, $r = 1.14$, $n = 2$ and $m = 24$ [4]. We consider a rather broad size range of geometrically similar beams, ranging from size $D = 0.1$ m to $D = 6$ m. The plot of the modulus of rupture versus the size is shown in Fig. 1. Consider that $D = b$ and that the density of concrete is 24 kN/m^3. Suppose that f_r^{design} has been obtained by tests of size $D = 0.1$ m, and $\gamma = 0.8$. The ratio (6) is calculated for the whole range of sizes and for the dead load factors of 1.4, 1.2 and 1. (Fig. 2a).

In the classical reliability analysis of the problem, the parameters of formulae (1) to (6) are considered as random variables and the failure probability (7) is calculated by various efficient reliability techniques. For illustrative purposes, we simplify here the problem significantly by considering that the final bending moment ratio (6) is a normally distributed random variable, with the mean value taken from the deterministic analysis (Fig. 2a), and with coefficient of variation 0.1 . Under these crude assumptions, the failure probability (7) can be easily calculated. The results are shown in Fig. 2b. Significant changes of the failure probability due to a decrease of the dead load factor may be noticed. But a really drastic increase of failure probability is caused by the increase of the size. The influence of size effect clearly dominates the reliability.

As emphasized by Bažant and Frangopol[3], a decrease of the dead load factor, which has recently been contemplated, cannot be accepted without incorporating the size effect into the design codes. As can be seen, not even the present excessive value 1.4 can save the reliability of design because, e.g., the failure probability drops from 10^{-6} for small sizes to 10^{-2} for large sizes. With regard to the failure probability, the conclusion is that the size effect is dominant and decreases the reliability (i.e., increases the failure probability) very significantly. What is the influence of the live load factor on the design reliability? The same procedure can be used, keeping the value of the dead load factor as 1.4 and decreasing code value of the live load factor from 1.7, e.g., to 1.6 and 1.5 . The results are shown in Figs. 2c,d. The influence of this hypothetical change is quite pronounced, even more than in the case of the dead load factor. It shows that a hidden size effect can also be considered in the context of the live load factor. Its decrease would cause significant increase of failure probability. The failure probability for

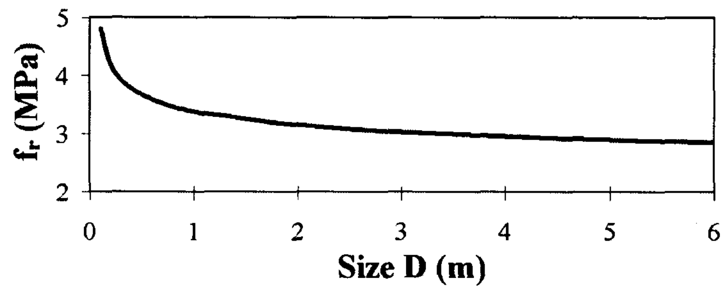

Figure 1: Modulus of rupture vs. size prediction - dependence of f_r on the beam size (depth) D according to the energetic-statistical formula.

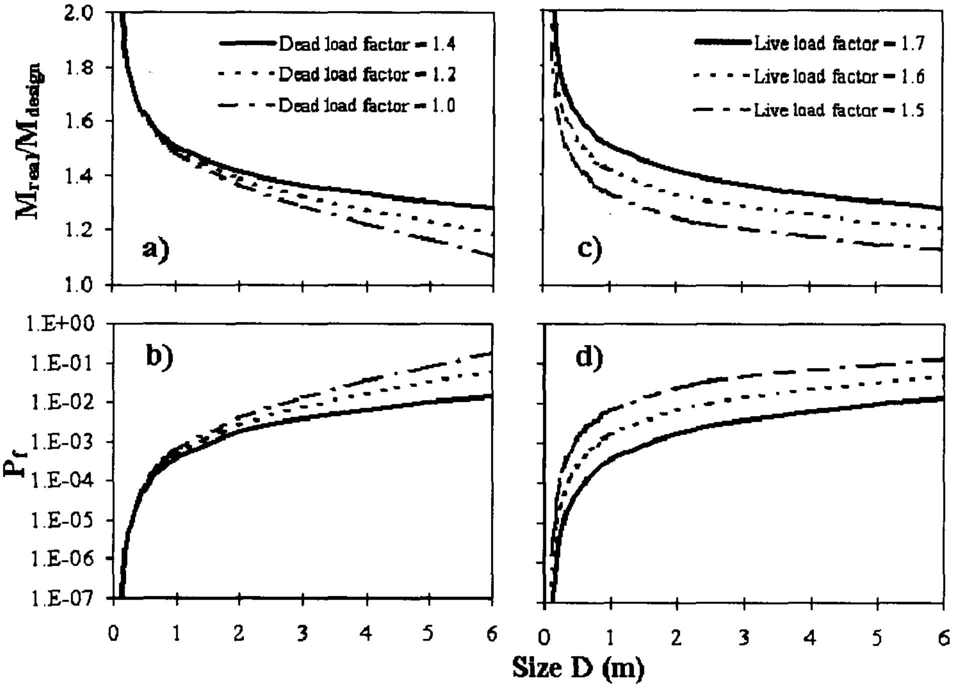

Figure 2: Dead load factor influence a) Deterministic; b) Probabilistic; Live load factor influence c) Deterministic; d) Probabilistic.

large sizes is unacceptable even for the load factors 1.4 and 1.7 presently considered. This is due to influence of size effect and its neglect in the current code specifications.

3 COMMENTS ON AASHTO LRFD BRIDGE DESIGN SPECIFICATIONS

The design of bridges is based on the Load Resistance Factor Design (LRFD)[1]. These specifications represent a statistical enhancement of the plastic limit design and thus do not capture the fracture mechanics aspects. The deterministic size effect in quasibrittle failures due to energy release and stress redistribution is not covered.

There are five strength limit states to be checked by a designer. Strength IV represents the load combination characterizing the case of a very high ratio of the dead load effect to the live load effect (i.e., the case where the live load effect is negligible compared to the dead load effect). This is typical for large-span bridges (spans over 60 m). But it is indicated in the commentary (AASHTO 1994), that the calibration of the load and resistance factors has been carried out for bridges of short and medium spans not exceeding 60 m[9]. Combinations of safety factors that yield a reliability index close to target value of 3.5 were selected. As one could intuitively predict, this reliability cannot be met for large bridges if the foregoing deductions are followed, in spite of the fact that the size effect for real bridges, with their complexity, has a different form than the simple size effect of crack initiation. Note that the load combination Strength IV will govern when the ratio of dead load to live load force effects exceeds about 7 ($1.5DC + 1.5DW$, DC is the load due to the structural components and nonstructural attachments, DW is the dead load due to wearing surfaces and utilities). Again, the excessive value of the dead load factor in the ultimate load requirements of the code specifications implies a hidden size effect[3]. It is debatable whether the dead load factor, obviously too high, can "cover" the influence of size effect, especially in the case of large bridges. A detailed reliability analysis of such an influence is desirable, and collaboration of the experts in reliability and fracture mechanics is a prerequisite of progress.

4 REINTERPRETATION OF THE COLLAPSE OF SCHOHARIE CREEK BRIDGE

The size effect at fracture initiation must have been a significant contributing factor in many catastrophic structural failures. One example is the collapse of Schoharie Creek Bridge on New York Thruway, built in 1952[6]. A flood scoured the river bed to a depth of 5.5 m (18 ft.) and bared about one half of the length of an unreinforced foundation plinth of 6.7 m in depth, forcing it to act as a cantilever. Fracture of the plinth caused the pier to sway, which in turn caused the precast prestressed beams to slip out of their bearings. The collapse represents an exception in the history of structural engineering disasters in the sense that in this case the investigating committee did actually recognize the size effect as a significant contributing factor, which was later confirmed also by finite element analysis[11]. Bažant and Novák[4] published a simplified estimation of the importance of size effect in this case; assuming $D_b = 0.05$ m, it may be concluded from (1) that the nominal strength must have been reduced to about 54% of the value calculated from the standard strength measured for $D \approx 0.15$ m.

Although the plinth was doubtless not designed for a large scour underneath, the probabilistic implications of the collapse of the plinth should be noted. The size effect must have increased real failure probability, and the increase must have been similar to that found in the present simplified study of a beam: The excessive load factors could not have saved the plinth because at a certain critical length scour the hidden size effect has been eliminated. Is a more realistic quantitative reliability assessment possible? This question is explored next.

5 STOCHASTIC FRACTURE MECHANICS COMPUTATIONAL MODELING

The foregoing numerical characterization of the influence of size effect on the reliability of design, focused particularly on the reliability load factors for design, is considerably simplified. The simplification is two-fold: (1) from the viewpoint of fracture mechanics, and (2) from the viewpoint of the theory of reliability. The former was expressed by a realistic fracture mechanics based formula, while the latter consisted mainly in reducing all the uncertainties of the problem to just one random variable, considered to be Gaussian, etc. The present state of the art of structural reliability theory would certainly allow a more sophisticated analysis. The present example is a straightforward illustration of the problem at hand in general. The real goal is how to capture the size effect probabilistically when dealing with the design of complex concrete structures. Several obstacles to achieving this goal can be identified:

Efficient methods for numerical analysis of reinforced concrete structures have been the objective of much research during the last few decades, and the main difficulty has been how to best capture material nonlinearity. The aim is to model the complete response of a structure including the crack propagation in the pre-peak, peak and post-peak states. A form of fracture mechanics that can be applied to such kind of fracture analysis has been developed during the last three decades. It tranpired that the deterministic explanation of size effect is to be found in nonlinear fracture behavior. The deterministic size-effect represents the transition from a ductile failure of relatively small specimens to a brittle failure of large structures. Objective modeling of this size effect must be based on the tools of fracture mechanics[5]. Recently, commercial finite element programs, using the crack band approach, have become available for this purpose. These tools, however, remain at the deterministic level.

There is still a considerable gap between the research achievements and the engineering practice. The academic community deals with various sophisticated aspects such as the damage localization, crack propagation, size effect and strain softening laws. The statistical treatment of fracture mechanics problems still relies on the classical approaches conceived by Weibull, Fischer and Tippett, and Fréchet which try to explain the size effect exclusively by statistics. Interest in the fracture mechanics research community in utilizing the reliability theory for large-scale nonlinear fracture is very recent, and the progress is still mainly academic (see the proceedings of the main conferences such as FraMCoS and ICOSSAR).

On the other hand, the design practice in industry provides motivation mainly for efficient implementation of existing simple material models, solution strategies, discretization and interpretation of results. These topics will naturally remain as the priorities for commercial software developers. But exceptions are nowadays appearing—the interdisciplinary field of stochastic fracture mechanics is now finally infringed on by some advanced software developers, e.g. those of ATENA[8,10] and DIANA[12]. These new exciting efforts are advancing beyond strict boundaries of design codes and attempt to treat in a combined manner the fracture nonlinearity, size effect and reliability.

6 CONCLUSIONS

1. The simple example of flexural failure of an unreinforced concrete beam shows the methodology to realistic problems of foundation plinths, pavements, retaining walls and arch dams. The example demonstrates that the structure size can have a drastic influence on the failure probability, such that even an excessive dead load factor cannot guarantee the desired reliability.
2. The size effect at fracture initiation must have been a significant contributing factor in many catastrophic structural failures, e.g., the collapse of Schoharie Creek Bridge, precipitated by fracture of a foundation plinth in a flood. The fact that the plinth must have collapsed at fracture initiation implies that the size effect caused an increase of the real failure probability. The excessive value of the dead load factor could not have saved the plinth.
3. A more realistic prediction of the interaction of size effect and reliability necessitates computational modeling based on fracture mechanics in combination with a structural reliability approach. A promising interdisciplinary marriage of these two distinct fields is now being studied by commercial code developers[8,10,12].

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