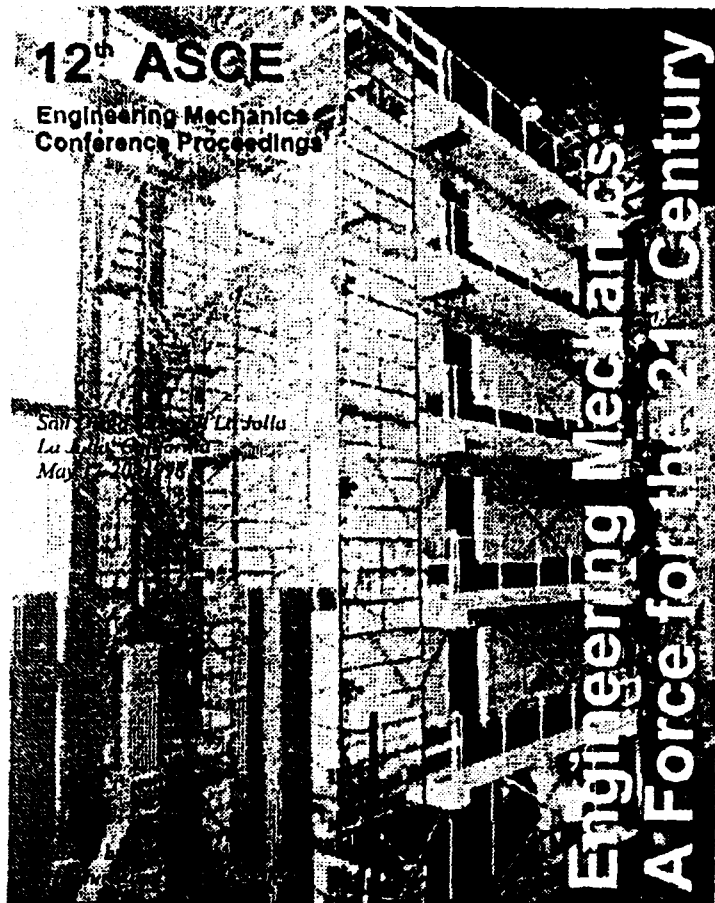
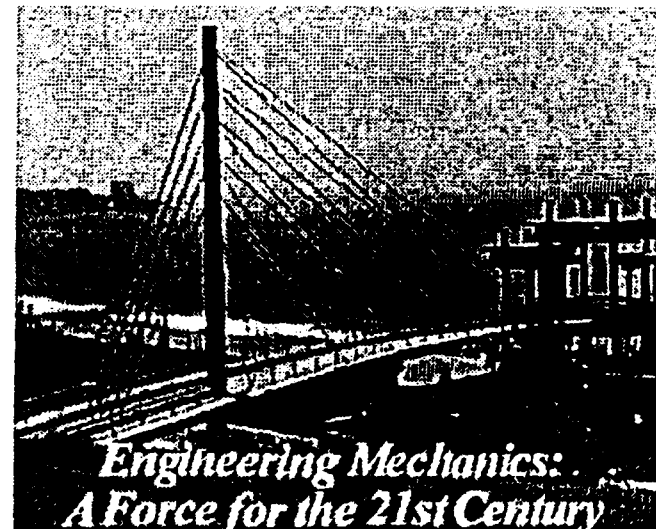


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pp. 48-51

# Micromechanical Fracture Analysis of ASR-Induced Damage in Concrete with Waste Glass

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## *Abstract*

Recent experiments of C. Meyer and W. Jin at Columbia University show that a decrease of waste glass particle in concrete causes increase of damage only if the particle size is larger than a certain pessimum size. For smaller particle size the damage diminishes and vanishes for a sufficiently small particle size. The papers outlines a fracture mechanics theory that can explain these recent experimental findings.

## *Introduction*

Using waste glass as an aggregate in concrete is an attractive means of waste disposal but can cause severe damage because of the alkali-silica reaction (ASR) between the alkali in the cement paste and the silica in the glass. Recent experiments of C. Meyer and W. Jin at Columbia University reveal that the damage to concrete caused by expansion of the ASR gel, which is manifested by strength reduction and volume expansion, depends strongly on the size of the glass particles.

As the particle size decreases, the tensile strength first also decreases, which is expected because the surface-volume ratio of the particles, and thus their chemical reactivity, increases. However, experiments show that there exists a certain worst (pessimum) size (approximately 0.05 inch or 1.25 mm) below which a further decrease of particle size improves the strength and diminishes the volume expansion. The damage becomes virtually nonexistent if the particles are sufficiently small. This experimental observation is contrary to intuition. The present paper briefly outlines a micromechanical fracture theory that can explain the reversal.

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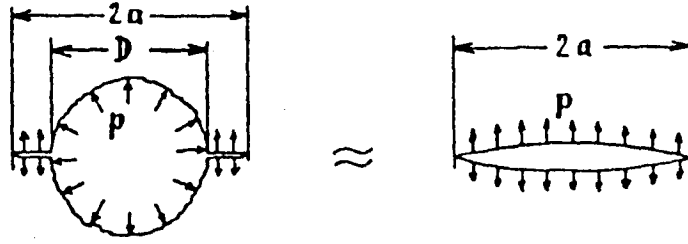


Figure 1: Glass particle with flaws (mortar cracks) at its surface, and approximation by a penny-shaped crack.

### Proposed Theory

The ASR occurs at the surface of the particle. The expanding layer of the solid ASR gel applies radial pressure on the surrounding cement mortar. Around each particle, there are likely to be pre-existing flaws. Two diametrically opposite flaws, as shown in Fig. 1, are the most critical ones. Considering the expanding particle with two opposite flaws to be embedded in an infinite elastic space, neglecting the interaction with the adjacent particles, and assuming the length  $c_f$  of the flaws (cracks) not to be much shorter than the particle diameter, the radial pressure applied onto the spherical surfaces is approximately equivalent to a radial pressure applied on the surfaces of a penny-shaped crack of radius  $a = a_0 + c_f$  where  $a_0 = D/2 =$  initial radius of the glass particle and  $c_f$  is the radial length of the pre-existing flaws on the spherical surface (Fig. 1).

The penny-shaped crack with pressure  $p$  applied to its faces has the same stress intensity factor as a penny-shaped embedded in an infinite solid subjected at infinities to a uniform normal stress  $p$  in the direction normal to the crack. This can be easily shown on the basis of the principle of superposition. The mode I stress intensity factor,  $K_I$ , of the penny-shaped crack in an infinite space stressed uniformly at the infinities is well-known (e.g., Bažant and Planas 1998). The energy release rate  $\mathcal{G}$  of the crack can be obtained as  $K_I^2/E$ , where  $E =$  Young's modulus;

$$\mathcal{G} = \left[ \frac{\partial}{\partial a} \left( \frac{p^2}{2E} \frac{1}{3} \pi a^2 \cdot ka \right) \right]_{p=\text{const}}, \quad k = \frac{8}{\pi} \quad (1)$$

According to the energy balance condition (fracture equilibrium condition) of fracture mechanics,  $\mathcal{G}$  must be equal to the rate of the energy required to form the new crack surfaces, which is  $\pi a^2 G_f$ , i.e.  $\mathcal{G} = \frac{\partial}{\partial a} (\pi a^2 G_f)$ , in which  $G_f$  is the fracture energy of concrete (typically 50N/m). Substituting this into (1), we have the condition  $2\pi a G_f = (p^2/2E)(\pi k/3)3a^2$  from which one can solve:

$$p = p_{cr} = 2\sqrt{\frac{EG_f}{ka}}, \quad a = \frac{D}{2} + c_f \quad (2)$$

This represents the approximate critical pressure of ASR gel that will cause fracturing (disintegration) of concrete. (The present result is analogous to that indicated by fracture analysis of the effect of rock aggregate size on the strength of concrete; Bažant and Kim, 1984.)

The critical pressure required for breaking up concrete increases in inverse proportion to  $\sqrt{a}$ . In other words, the critical pressure increases with a decreasing diameter of the particles. Therefore, it is logical to expect that, for sufficiently small particles, the ASR may become incapable of breaking up concrete. But the amount of ASR gel matters, too.

The volume of  $V_a$  of glass that has undergone ASR after a long time, per unit volume of concrete, depends on a pressure  $p$  on the surface of the glass particle, temperature  $T$ , specific water content  $w$  in the pores of concrete, and time  $t$ . We are interested in the final volume  $V_a$  after a long enough time has elapsed, for which we may neglect  $w$ ,  $T$  and  $p$ . So we simply assume:

$$\begin{aligned} \text{for } D/2 > h_r : & \quad V_a = V_r \\ \text{for } D/2 \leq h_r : & \quad V_a = V_g \end{aligned} \quad (3)$$

in which

$$V_r = (\pi/6) [D^3 - (D - 2h_r)^3] \bar{N}, \quad \bar{N} = V_g/(\pi D^3/6) \quad (4)$$

Here  $h_r$  = thickness of the layer of glass into which the ASR can penetrate from the surface of the particle, assumed to be spherical,  $V_r$  = volume of these layers (spherical shells) in all the particles per unit volume of concrete, and  $\bar{N}$  = number of glass particles per unit volume of concrete. We may probably assume  $h_r$  to be approximately a constant. Substitution of (4) into (3) provides:

$$\begin{aligned} \text{for } D/2 > h_r : & \quad V_a = \{1 - [1 - (2h_r/D)]^3\} V_g \\ \text{for } D/2 \leq h_r : & \quad V_a = V_g \end{aligned} \quad (5)$$

The solid gel produced by ASR must obviously have a larger volume than the glass and silica that have reacted. This increase of volume ought to produce pressure. Thus it appears reasonable to assume that the pressure produced by ASR on the surface of the reacting glass particles is approximately given by  $p_a = \kappa V_a$  in which  $\kappa$  is a stiffness coefficient characterizing the elasticity of the materials surrounding the glass particle.

The upper limit on the tensile strength  $f'_t$  as influenced by ASR is, according to equilibrium condition,  $f'_{ta} = p_{cr} - p_a$  and  $\Delta f'_t = 2\sqrt{EG_f/ka} - \kappa V_a$ . For larger particle diameters  $D$ ,  $V_r < V_g$  and then (10) takes the form:

$$f'_{ta} = \sqrt{\frac{8EG_f}{k(D+2c_f)}} - \kappa V_g \left[ 1 - \left( 1 - \frac{2h_r}{D} \right)^3 \right] \quad (6)$$

For sufficiently large particle diameters  $D$ , the last term may be simplified, furnishing

$$f'_{ta} \approx \sqrt{\frac{8EG_f}{k(D+2c_f)}} - \kappa V_g \frac{6h_r}{D} \quad (7)$$

For sufficiently small particle diameters  $D$ , all of the glass undergoes ASR, and in that case (5) yields

$$f'_{ta} = \sqrt{\frac{8EG_f}{k(D+2c_f)}} - \kappa V_g \quad (8)$$

For large enough  $D$ , the first term in (7) increases with decreasing  $D$  slower than the magnitude of the second term. Therefore, for large enough glass particles, the strength ought to decrease with a decreasing particle diameter (which is what has been observed by C. Meyer and W. Jin at Columbia University). On the other hand, for small enough particles, Eq. (8) applies, which means that the tensile strength ought to increase with an increasing particle diameter (this trend has also been experimentally confirmed at Columbia University). Obviously, for a small enough particle size, the value of the tensile strength of concrete with no glass ought to be approached (this occurs at glass particle size about 0.2mm, according to the experiments at Columbia).

Why is the trend for insufficiently small particle sizes opposite, i.e., why does the tensile strength of material decrease with a decreasing size of particles? The proposed explanation is that the effect of the increase of the surface of particles  $S_p$  with the decreasing particle size overpowers the fracture mechanics effect.

That the opposite effect of the increasing specific internal surface  $S_p$  should disappear for sufficiently small particles is not difficult to understand. When the particles are relatively large, only a relatively thin surface layer of the glass particle gets reacted and the core of the glass particle remains shielded from ASR. In that case, the amount of ASR produced gel depends mainly on the exposed surface per unit volume of concrete, which increases with decreasing particle size. However, when the particles become smaller than a certain critical (pessimum) size, all of their volume gets reacted. This then means that the surface area of the particles become irrelevant. A further decrease of the particles cannot increase the amount of the solid gel produced, and so the volume content of glass remains the same.

### *Conclusion*

The glass particle size effect on concrete involves two opposing mechanisms: (1) the rate of chemical reaction as a function of surface area, which causes strength decrease with a decreasing particle size, and (2) the size effect of the cracks caused by expansion of the ASR gel, which causes the opposite. The pessimum size corresponds to the case where the effects of both mechanisms are balanced. For large sizes, the first mechanism prevails. For smaller sizes the second mechanism governs.

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