Disc. 81-38/From the September-October 1984 ACI JOURNAL, p. 456

Size Effect in Shear Failure of Longitudinally Reinforced Beams. Paper by Zdeněk P. Bažant and Jin-Keun Kim

Discussion by K. D. Chiu, J. C. Walraven, and Authors

By K. D. CHIU

Associate Consulting Engineer, Ebasco Services, Inc., New York, N.Y.

The authors presented a very interesting and long sought-after subject. The correlation of the test results with the formula presented was impressive. However, in deriving Eq. (11), the authors overlooked the term $(x/a)^r$ in Eq. (3). Eq. (6) should be

$$V_1 = k_1 \rho^{1/2-m} f_c'^{\,q}\, bd \left(\frac{x}{a}\right)^r \quad (6)$$

when $x = d$

$$V_1 = k_1 \frac{\rho^{1/2-m}}{\left(\frac{a}{d}\right)^r} f_c'^{\,q}\, bd$$

Therefore Eq. (11) becomes

$$v = k_1 \frac{\rho^p}{\left(\frac{a}{d}\right)^r}\left(f_c'^{\,q} + k_2\sqrt{\rho}\right)\left(1 + \frac{d}{\lambda_0 d_a}\right)^{-1/2} \quad (11)$$

Accordingly, Eq. (14) and (19) should be revised and the values of the constants must also be reevaluated.

In this new form, the first part (ignore the dimensional factor) of Eq. (11) is similar to Zsutty's formula as presented in Eq. (18), except for the additional term $k_2\sqrt{\rho}$ which corresponds to the arch action.

One minor but important correction is $\alpha = M_u/V_u d$ instead of $V_u d/M_u$ as presented in Eq. (19). Since the presentation is based on simple beam formulation, the proposed relationship of Eq. (19) is only good for a one-way beam or slab. In the case of two-way concrete panels such as flat plates or curved shells, Eq. (19) is no longer applicable due to the more complex constitutive relationship and energy-releasing mechanism. Further studies in this area should be carried out to establish a criterion for the two-way panels.

by J. C. WALRAVEN

Professor, Institut für Massivbau, Technische Hoschschule Darmstadt, Germany

The authors are complemented for their very interesting article, which shows that a highly accurate formulation based on physical arguments can be obtained for the shear capacity of members without shear reinforcement. An important aspect is that the expression concerns the ultimate failure load and not the inclined cracking load, which was always used up to now for its supposed higher reliability. However, it is believed that the high accuracy is obtained in spite of the scale factor, which contains the ratio of beam depth to maximum particle diameter (d/d_a). Only a limited number of tests have been carried out in which this factor was systematically varied, but these all show tendencies contrary to what the theoretical scale factor suggests.

Fig. A shows a comparison between the experimental results of Taylor[45] and the theoretical results obtained with Eq. (14) of the article. The ratio $v_{u,th}/v_{u,\exp}$ should be independent of d/d_a, but the diagram shows a clear decrease. Fig. B shows test results of Chana[46] for experiments with perfectly scaled concrete for constant d/d_a. According to the theory, v should be independent of d, which is not the case. Finally, Fig. C shows a comparison between comparable normal aggregate and lightweight concrete beams with varying depths.[47] Since the cracks in lightweight concrete intersect the lightweight aggregate particles instead of going around them, a different behavior regarding the variation of depth could be expected; however, in spite of the fundamentally different concrete composition, the behavior is similar.

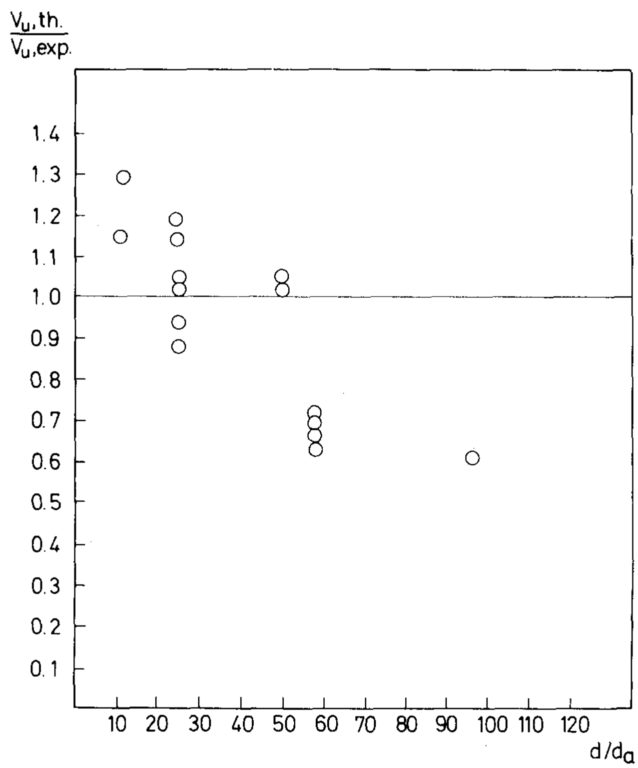

Fig. A—Comparison of test results of Taylor[45] with theoretical values obtained with Eq. (14)

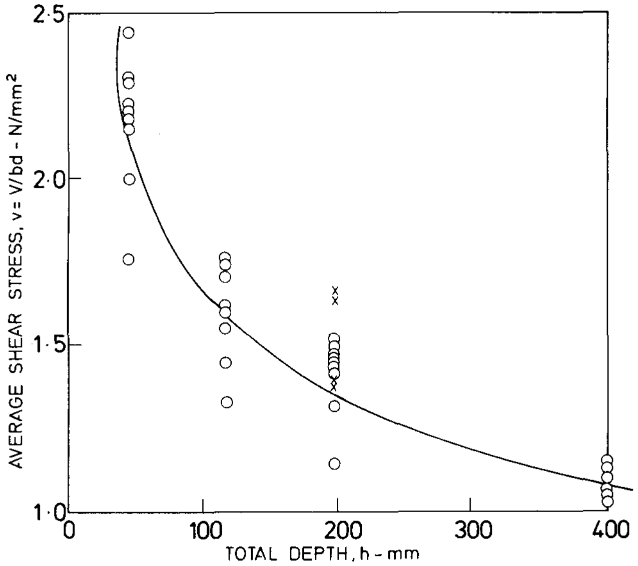

Fig. B—Relationship between ultimate shear strength and depth for constant value of d/d$_a$ *according to Chana*[46]

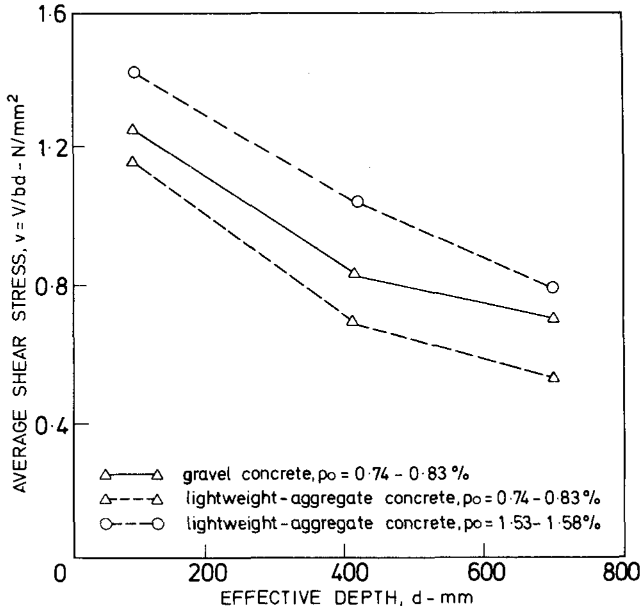

Fig. C—Ultimate nominal shear stresses as a function of the effective beam depth for normal weight and lightweight beams according to Walraven[47]

Considering these experimental data it seems obvious that the presence of d_a in the equation is unjustified. Similar accuracy may be obtained without it.

REFERENCES

45. Taylor, Howard P. J., "The Shear Strength of Large Beams," *Proceedings,* ASCE, V. 98, ST11, Nov. 1972, pp. 2473-2490.

46. Chana, P. S., "Some Aspects of Modelling the Behaviour of Reinforced Concrete Under Shear Loading," *Technical Report* No. 543, Cement and Concrete Association, Wexham Springs, July 1981, 22 pp.

47. Walraven, J. C., "The Influence of Depth on the Shear Strength of Lightweight Concrete Beams Without Shear Reinforcement," *Stevin Laboratory Report* No. 5-78-4, Delft University of Technology, 1978, 36 pp.

AUTHORS' CLOSURE*

K. D. Chiu has identified an interesting question that was not addressed in the paper. This question, which was considered before writing the paper, cannot be answered solely by theoretical analysis because the answer depends on the empirical approximations made in the underlying assumptions.

The ultimate shear force is assumed to be a sum of two parts: (1) the force V_1 due to the composite beam action [Eq. (6)], and (2) the force V_2 due to the arch action. Now the question is where to locate the critical cross section for each type of action. Chiu proposes to take it at distance $x = d$ not only for the arch action but also for the composite beam action. This would be correct if we assumed that both types of action interact but then it might be questionable to assume that the total shear force V is simply a sum of the composite beam contribution V_1 and the arch action contribution V_2. With respect to the assumption that $V = V_1 + V_2$, it seems more reasonable to consider the composite beam action and the arch action as independent, as if the total shear force V was the sum of the shear force of one beam capable only of the composite beam action and another capable only of the arch action. Then each location of the critical cross section should be considered independently for each beam.

For the arch action, the critical cross section is as close to the support as allowed by the assumptions of beam theory, which is roughly at distance $x = d$ from the support. For the composite beam action, however, the critical cross section occurs right at the load, which is at $x = a$ (provided we consider the arch action as independent). According to this argument, Fig. 6 based on $x = a$ is better justified than Chiu's equation for V_1 based on $x = d$.

The final answer depends, of course, on the test results. Unfortunately, the data presently available are too scattered to decide this fine point. Both versions of the regression for V_1 based on $x = a$ and $x = d$ were, in fact, tried in the computer analysis of data, and for neither was the fit significantly better than for the other. In the end, the regression for V_1 that corresponds to $x = a$ was selected because of the aforementioned argument and also because the resulting formula is more similar to that used in the current ACI Code. When, however, test data with better control and a broader range become available, the question raised by Chiu should be reexamined.

In his perspicacious discussion, Walraven expands the scope of inquiry. Although the objective of the paper was a study of the effect of the beam's depth d for one given concrete and not of the maximum aggregate size d_a (d was divided by d_a mainly for the purpose of having a nondimensional variable), the effect of d_a is of

*Dr. S. Sener, Visiting Scholar, Northwestern University, on leave from Istanbul Technical University, Faculty of Civil Engineering, Istanbul, Turkey, assisted in the preparation of this closure.

interest. Walraven identified an important limitation of the proposed new formula in regard to d_a. He showed that some of the existing test data do not indicate any significant dependence of the nominal shear stress at failure ν_u on the ratio d/d_a. However, Walraven's suggestion that an effect of the relative structure size d/d_a is unjustified is questionable for the following reasons:

1. The existence of the size effect is required by the current theoretical knowledge about strain-softening failures in general. If the size effect were absent, the failure would have to be ductile rather than brittle, and then the load-deflection diagram for diagonal shear failure would have to exhibit a horizontal plateau rather than a gradual decrease of the load after the peak, which is not the case. The width of crack-band front, in which strain softening (progressive distributed microcracking) takes place, is proportional to d_a (see Reference 25 of the paper). The similitude argument given in the Appendix of the paper (and more rigorously in Reference 48) inevitably leads to d/d_a rather than d as the independent variable for the size effect. Moreoever, even if the existence of strain softening is denied and the fracture-process zone ahead of the crack tip is treated as a line (assuming the width of the fracture-process zone to be zero), a certain characteristic length in the crack direction must be introduced for the fracture-process zone. Similitude arguments will once again require a dependence on the relative structure size rather than d (see p. 532 of Reference 37 in the paper).

2. More important than the theoretical argument, the effect of size d is clearly evident from a majority of existing test data, including Walraven's data in his Fig. C. Some data, e.g., those considered by Reinhardt (Reference 32 of the paper), even show a stronger size effect than the proposed formula. Again, a dependence on d alone is not quite correct physically because parameter d is not nondimensional. Every physically correct formula must be reducible to a nondimensional form, even though the current codes for concrete structures do not quite adhere to this principle. Therefore, the structure size parameter d must be divided by some material constant that has a dimension of length, and the only question becomes which material constant should be used for the nondimensionalization. The discusser suggests no material length constant for this purpose. The maximum aggregate size d_a is the only nondimensionalizing constant which is easily defined. As an alternative one could use the characteristic fracture length of the material, such as the effective width w_c of the crack-band front, but this width is again found to be proportional to d_a (see Reference 35 of the paper). Thus Walraven's data in his Fig. C, also used in the paper in Fig. 4(a), really imply a dependence on d/d_a if one recognizes that any physically sound formula should be reducible to a nondimensional form. Since d_a was the same for all of Walraven's tests, the plots of ν versus d/d_a look the same as the plots of ν versus d in Walraven's Fig. C.

Even the data by Taylor and Chana when considered alone do not necessarily imply that there is no effect of d/d_a. In Taylor's and Chana's tests, it may be that the effect of the relative beam's size d/d_a was overridden by the effect of some other variable. Studies subsequent to the paper showed that this indeed appears to be the case — the additional variable is the maximum aggregate size itself.

By applying fracture mechanics to the spread of mortar cracks in the contact regions between large aggregate pieces, Bažant recently found* that the direct tensile strength of concrete f_t' should vary as

$$f_t' = f_t^0\left(1 + \sqrt{\frac{c_0}{d_a}}\right) \quad (20)$$

with other influencing factors (such as the mortar component of concrete, curing, etc.) being the same; c_0 and f_t^0 are material constants, with c_0 depending on the characteristic length (or the sand grain size) of the mortar in concrete (and also on the differences in elastic moduli and strengths between the mortar component and the large aggregate pieces); and f_t^0 is the ideal tensile strength for the extrapolation to infinitely large aggregate. Eq. (20) happens to be of the same form as the Petch's formula for the tensile strength of ductile polycrystalline metals, which was derived from the dislocation theory and verified by many tests of metals.[49-51]

In light of Eq. (20), the formula proposed in the paper [Eqs. (11), (14), and (19)] is strictly applicable only when d varies and the aggregate size d_a is fixed, and approximately applicable when d_a varies within a narrow range. This is sufficient for most concrete construction since the aggregate size is usually between 1 and ¾ in. However, Taylor's and Chana's tests, invoked by the discusser, involved rather broad ranges of d_a, spanning from fine mortars to concrete — Taylor's from 2.4 to 38 mm, and Chana's from 2.4 to 20 mm. Chana's tests were unknown to the authors and were not included in the comparisons in the paper. Taylor's tests for $d_a > 9$ mm were included, but not those for fine mortar. Nearly all data used in the paper had $d_a \geqslant 0.75$ in. and for all $d_a \geqslant 0.375$ in.

To cover a wide range of maximum aggregate sizes, Eq. (20) must be incorporated into Eq. (11), (14), and (19). Two possibilities exist: multiply by the factor $[1 + \sqrt{c_o/d_a}\,]$ either the value of $\sqrt{f_c'}$ or the value of ν (or ν_u). Presently, it is not yet clear which approach is better justified, and for the time being we choose the latter approach, i.e., we generalize Eq. (11), (14), and (19) to the form

$$\nu = k_1 C_1\left(1 + \frac{d}{\lambda_0 d_a}\right)^{-1/2}\left(1 + \sqrt{\frac{c_0}{d_a}}\right) \quad (21)$$

in which

$$C_1 = 0.07\rho^{1/3}\left(11.95\sqrt{f_c'} + 3000\sqrt{\frac{\rho}{(a/d)^5}}\right) \quad (22)$$

*Bažant, Z. P., "Effect of Aggregate Size on the Strength of Brittle Cementitious Composites," in preparation.

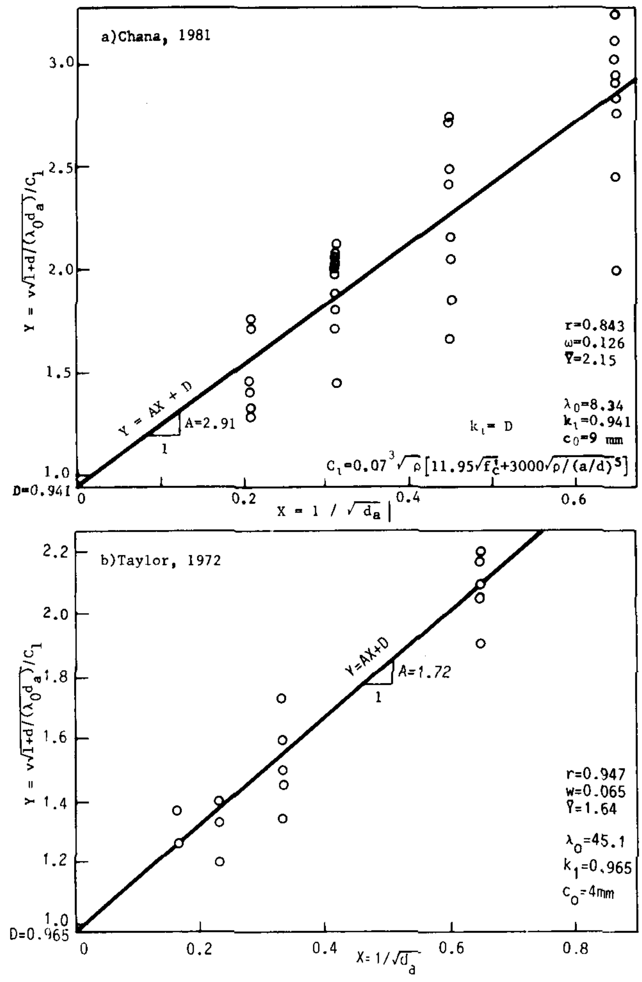

Fig. D — Aggregate size regression plots for Chana's and Taylor's data

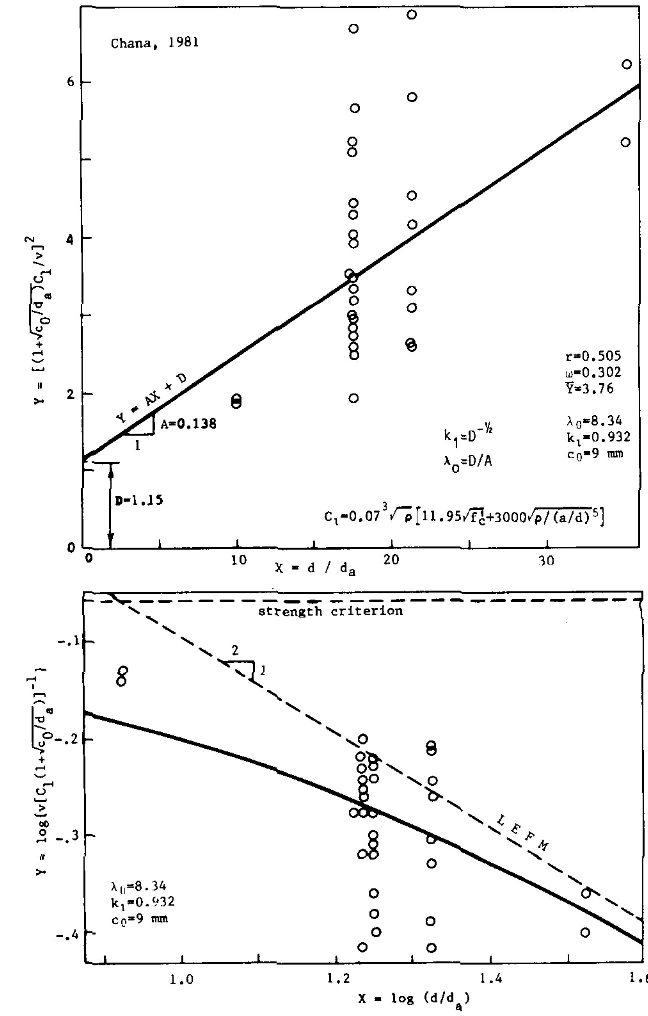

Fig. E — Size effect regression plots for Chana's data

Eq. (21) can be algebraically rearranged so as to allow two types of linear regression $Y = AX + D$ in which A and D are expressed only in terms of nondimensional material constants:

Type I

$$X = \frac{d}{d_a}, \quad Y = \left[\frac{C_1}{v}\left(1 + \sqrt{\frac{c_0}{d_a}}\right)\right]^2, \quad D = \frac{1}{k_1^2}, \quad A = \frac{D}{\lambda_0} \tag{23}$$

Type II

$$X = \frac{1}{\sqrt{d_a}}, \quad Y = \frac{v}{C_1}\left(1 + \frac{d}{\lambda_0 d_a}\right)^{1/2}, \quad D = k_1, \quad A = k_1\sqrt{c_o} \tag{24}$$

Parameter c_0 for Type I and parameter λ_0 for Type II must be found by nonlinear optimization.

To verify the effect of d_a [Eq. (20)], Chana's data are ideal since they include many tests with the same d/d_a but very different d_a. The Type II regression plot for these data is shown in Fig. D(a). The trend of data points agrees quite well with the regression line representing the new formula proposed here. [In Fig. D(a), the correlation coefficient $r = 0.843$ is quite acceptable, the coefficient of variation of deviations from the straight line $\omega = 0.126$ is not too large, and $\bar{Y}$ is the data centroid.] Fig. D(b), showing Type II regression for Taylor's data, also confirms the trend of the formula. Type I regressions for Chana's and Taylor's data are shown in Fig. E and F.

Generally, in contrast to the discusser's plots, these reevaluations of Chana's and Taylor's data indicate a good agreement with the size effect of d/d_a presented in the paper as well as the additional effect of d_a presented here. Nevertheless, further experimental verification of the influence of simultaneous variation of d/d_a and d_a is desirable.

To further verify the size effect, Fig. G shows how Eq. (21) and (22) fit the recent test data by Iguro et al.[52] This series included diagonal shear failures of unusually deep beams, ranging up to 3 m depth. Various aggregate sizes were also used. Satisfactory agreement is again found with Eq. (21) and (22).

According to the present formulas, the effect of d is more complicated than that of d/d_a (see the plots in Fig. H). A decrease of d_a at constant d/d_a always strengthens a structure that fails due to cracking of concrete. However, if structure size d is fixed, a decrease of d_a at constant d may sometimes strengthen the

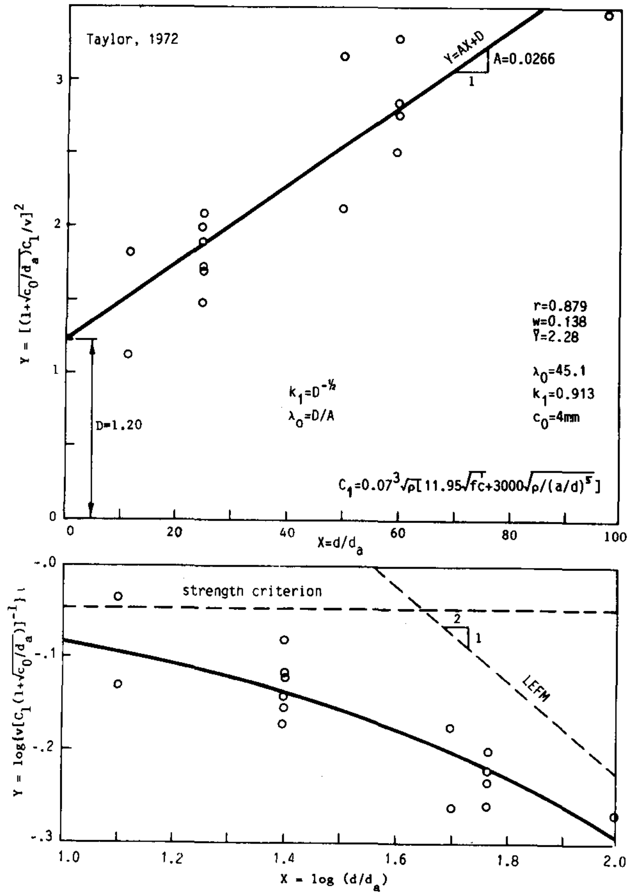

Fig. F — Size effect regression plots for Taylor's data

structure (if d is small) but another time weaken it (if d is large).

In conclusion, Taylor's and Chana's test results do not invalidate the structural size effect formulation presented in the paper. Rather, they lead to a generalized formulation which also covers the effect of variation in the aggregate size. Walraven's constructive criticism, which stimulated this development, is deeply appreciated by the authors.

Errata

1. In Eq. (4), replace p with ρ and raise ½ to the exponent level.
2. In Eq. (19), $\alpha = M_u/(V_u/d)$.

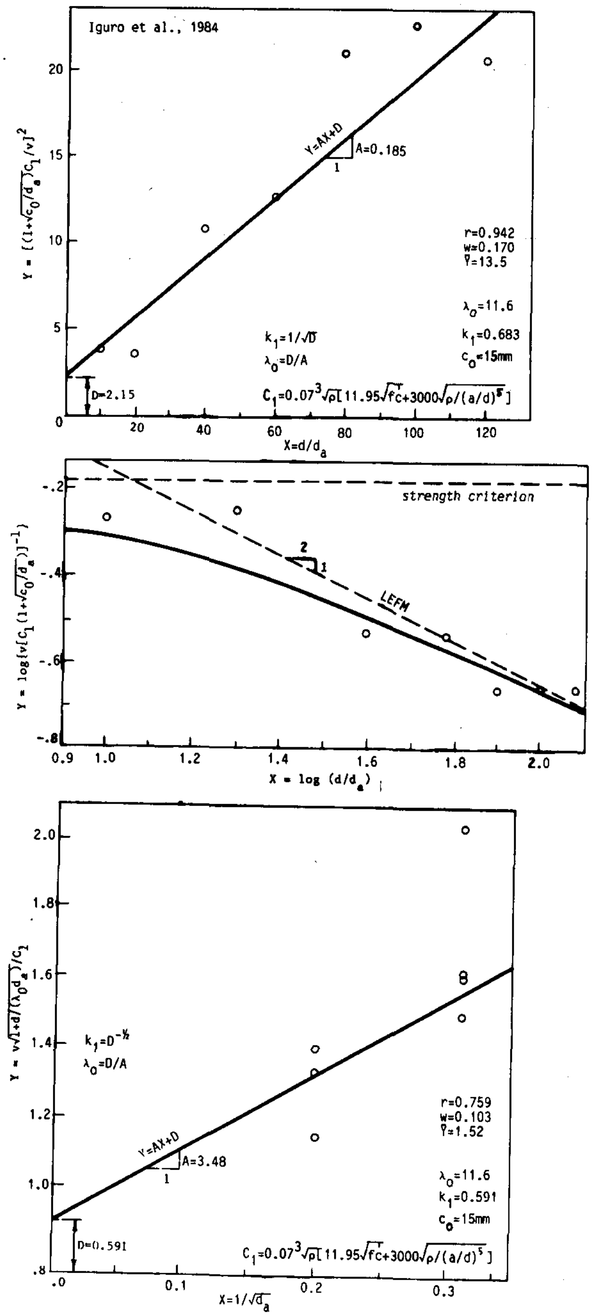

Fig. G — Fit of Iguro et al.'s data on the effects of beam depths and of aggregate size using Eq. (21) and (22)

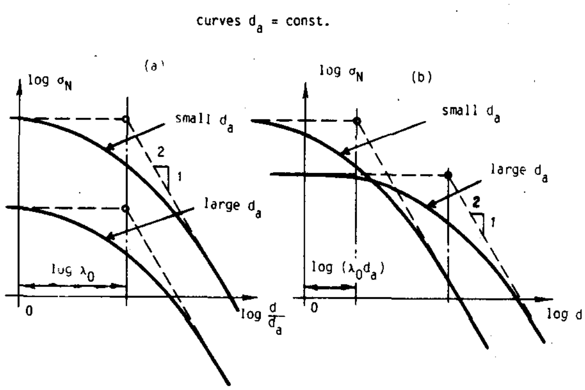

Fig. H — Size effect according to Eq. (21)

REFERENCES

48. Bažant, Z. P., "Fracture Mechanics and Strain-Softening in Concrete," *Preprints,* U.S.-Japan Workshop on Finite Element Analysis of Reinforced Concrete (Tokyo, May 1985), Japan Concrete Institute, Tokyo, Eqs. 26-38, pp. 63-67.

49. Petch, N. J., "The Cleavage Strength of Polycrystals," *Journal,* Iron and Steel Institute, V. 173, 1953, pp. 25-28.

50. Tetelman, A. S., and McEvily, A. J., Jr., *Fracture of Structural Materials,* John Wiley & Sons, New York, 1967, p. 186.

51. Cottrell, A. H., *The Mechanical Properties of Matter,* John Wiley & Sons, New York, 1967, p. 282.

52. Iguro, M.; Shioya, T.; Nojiri, Y.; and Akiyama, H., "Experimental Studies on Shear Strength of Large Reinforced Concrete Beams under Uniformly Distributed Load," *Proceedings,* Japan Society of Civil Engineers (Tokyo), No. 345/V-1, Aug. 1984. (In Japanese. English translation available from authors.)