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INELASTIC ANALYSIS OF STRUCTURES

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Inelastic Analysis of Structures

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Preface

Our main objective in writing this book has been to provide a textbook for courses on plasticity, with some ramifications to time-dependent inelastic behaviour. In our selection of the topics and the sequence of their exposition, we put emphasis on structural engineering applications. There is nevertheless plenty of material for using the book in postgraduate courses in geotechnical, mechanical, aerospace, naval, petroleum and nuclear engineering. We assume the background level of a B.S. degree in civil or mechanical engineering.

Plasticity has already been the subject of many books. So why another? We hope to provide a book that is unique in many respects. It has been our intention to fill many needs that are not quite met by other books. Being considerably larger than a textbook for a single course, our book provides both a systematic exposition of the fundamentals of plasticity, and an up-to-date introduction to most of the advanced subjects. The courses with the coverage specified below could not be taught from some other existing book. We proceed from simple to complex, and in each chapter examples before generalizing. We try to be systematic and mathematically rigorous while striving, above all, for clarity. We avoid an artificially formalistic presentation that hardly achieves more than impressing by mathematical sophistication. The book features complete and rigorous mathematical derivations of all the results. Some derivations are more simple and others more rigorous than those in the previous textbooks. Despite being mainly a textbook, in the advanced chapters our book also covers most of the 'hot' topics of current research, and contains new research results. A set of problems for the student is included at the end of most chapters. Both simple and hard problems are suggested, the hard ones marked by an asterisk. It is planned to make the solutions available on the internet at <http://www.wiley.co.uk/inelastic>, which will also contain some additional information, such as a set of links to sites providing software for the solution of the programming problems.

We also include a set of six appendices, four of which review, for the student's convenience, the fundamentals of linear elastic analysis and the mathematical background of linear programming, and two give information on specialized models: the code-type prediction model for creep of concrete and the size effect model for softening in plastic hinges.

A special feature, which is not encountered in the basic texts on plasticity, is a thorough exposition of the plasticity of concrete and reinforced concrete, including the basic principles of limit state design. Concrete, of course, is not a plastic material *per se*, but plasticity concepts and the yield surfaces and plastic potentials form a necessary part of models that

plasticity to damage. Besides, the theory of plasticity is well suited to reinforced concrete structures that fail by the yielding of steel reinforcement. Similar comments can be made about our inclusion of plasticity models for soils.

To keep with the nature of most civil engineering applications, as well as to make the student's entry into the subject easier, the first two among five parts of our book are restricted to beam structures whose stress state may be simplified as uniaxial. Considerable attention is devoted to shakedown, another classical subject particularly important for structural engineering, but rarely treated consistently in textbooks. The classical topics at the margins of plasticity theory, such as the optimum design and linear programming, are included in our coverage. After digesting the basic concepts in the context of uniaxial stress, the students will find it easier to follow, in the third part, the extension of limit analysis to structures under multiaxial stress.

For the benefit of advanced doctoral students and postdoctoral researchers, we include in the last two parts of the book a number of advanced subjects normally not seen in basic textbooks – numerical algorithms, thermodynamic aspects, plasticity in finite strain, multisurface plasticity, anisotropic plasticity, nonlocal and gradient models for plasticity with strain softening and size effects, viscoplasticity and rate effects, microplane constitutive models, and vertex effects. We also provide a brief survey of polycrystal plasticity. With this scope, we hope to have covered a major part of what today constitutes the modern theory of plasticity. Expositions of the dislocation theory as the micromechanical basis of plasticity, dynamic plasticity, plastic buckling and bifurcations, plasticity of shells and constitutive properties of plastic composites could not be accommodated within the scope of this book.

Another special feature of our book is the inclusion of two chapters (among 29) on the creep of concrete and its effects in structures. Although this kind of inelastic behavior is very important for the durability of civil engineering infrastructure and sometimes affects the safety as well, most structural engineering curricula unfortunately do not have room for a full course devoted to this subject and, deplorably, graduate students leave the university without acquiring any knowledge of concrete creep. Our coverage of this subject provides a feasible compromise – an exposition brief enough not to lose the emphasis on plasticity yet sufficient to acquaint the student with the basic results needed for structural design. Due to space limitations, the treatment of creep is nevertheless much less systematic than that of plasticity, and most intricacies of this vast subject are inevitably skipped.

Our book can serve as a textbook for courses of several types:

- *A Quarter-Length First-Year Graduate Course* with a slight civil engineering emphasis may consist of the following chapters and sections: **1, 2, 4–6, 7.5, 8.2, 9**, essentials of **10** (without proofs), **11, 12.1, 12.2, 13.1–13.3, 14.1.1, 15.1, 15.2.1–15.2.3, 16.1, 17.1–17.3, 18, 19.1, 19.2.1–19.2.2**, only essential ideas and graphs from **19.5, 28.1–28.3, 28.4** without proof, and selections from **29.2.2**.
- *A Quarter-Length First-Year Graduate Course* with a slight mechanical engineering emphasis may consist of the following chapters and sections: **1–7, 13, 15, 16.1, 16.3, 17.1–17.3, 18.1–18.3, 19.1, 19.2.1, 19.5, 20.1, 25.4, 27.1**.
- *A Semester-Length First-Year Graduate Course* in structural engineering may fully cover chapters **1–19** and **28** and sections **29.1** and **29.2**. In mechanical engineering, one may omit **8–11, 12.3** and **14** and add **20.1–20.3, 22.1, 22.2.1, 22.3.1, 22.3.2, 25.4** and **27.1**.

- *A Second Course on Plasticity for Doctoral Students* in structural engineering cover chapters and sections **20, 21, 22.1, 22.2.1, 22.2.2, 22.3.1, 22.3.2, 25.2** and **26.1–26.3**. In mechanical engineering one may omit **21, 25.2, 25.2.5** and add **25.3, 25.4** and **27.1**. In a computationally oriented course entire chapter **22** can be covered.
- *A Short Course for Post-Doctoral Researchers and Advanced Doctoral Students* may start with the three-dimensional formulation of plasticity in chapters **20** and include the advanced topics in chapters **22–24**. A course with emphasis on structural engineering may also cover chapters and sections **21, 25.2, 26.1, 28.1, 28.2** and **29.3**, while a course with emphasis on mechanical engineering instead cover chapters **26, 27** and **25** without sections **25.2.4–25.2.5** and

The first course outline listed above has been used by the second author course on Inelastic Structural Analysis, which he has been regularly teaching at Northwestern University since 1970. The lecture notes that he had prepared for this course during the 1970s served as the point of departure for writing this book – an arduous effort that began in earnest in 1993, right after the first author completed his doctoral study at Northwestern University. The presentation of various advanced subjects in this book have been tried in a number of courses or advanced graduate courses taught at various institutions¹. The book was completed during the first author's Visiting Scholar appointment at Northwestern University in the summer of 2000, and the second author's Visiting Professor appointment at the Swiss Federal Institute of Technology at Lausanne (EPFL) in March 2001.

We would like to express our thanks for valuable comments and discussions on drafts of various chapters to Giulio Maier, professor at Politecnico di Milano; Z. Cohn, Professor Emeritus at the University of Waterloo; Zuzana Dimitrova, researcher at the Technical University of Lisbon; Andrzej Truty, associate professor at the Cracow University of Technology; Cino Viggiani, professor at Université Joseph Fourier, Grenoble; and Bořek Patzák and Simon Rolshoven, colleagues of the first author at EPFL.

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M.J. and Z.P.B.
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Introduction

The roots of some elementary ideas of the theory of plasticity can be traced over three and half centuries. In Galileo's (1638) calculation of the collapse load of a cantilever, one may discern the assumption of a uniform distribution of tensile stress over the cross section, even though the assumption of a concentrated compressive stress at the compression face was far from realistic. About a century later, Giovanni Galvani discussed the safety of Michelangelo's dome of Saint Peter's cathedral in a manner which one could detect the ideas of the static approach to limit analysis (Benvenuto, 1991). In the debates of the stability of masonry arches, vaults and domes in the nineteenth century, one could also perceive various elementary ideas of plasticity (Benvenuto, 1991). The first realistic and almost complete static analysis of a structure along with the concept of plastic slip and yield condition, is found in Coulomb's study of earth-retaining walls of military fortifications.

Various elementary ideas of plastic deformation and failure, and the reduction of buckling loads gradually emerged throughout the nineteenth century in the work of pioneers such as Lüders (1854), Tresca (1868), de St. Venant (1870), Lévy (1876), Rankine (1876), Bauschinger (1881), Considère (1891), Engesser (1895), Haigh (1896) and Mohr (1900). The static theorem of limit analysis was anticipated in the work of Rankine in 1859 and Kötter in 1891 and its intuitive enunciations can be found in the work of Kazinczy (1914) in his inaugural lecture of Kist (1917). During the first quarter of the twentieth century, basic concepts, such as the yield surfaces, flow rules, slip lines, and plasticity, appeared, principally in the works of von Kármán (1909), von Mises (1913), Prandtl (1924) and Reuß (1930). An important milestone was the resolution of the torsion problem (Nádai, 1923) and indentation problem (Hencky, 1923; Prandtl, 1923). The materials science foundation of metal plasticity in the dislocation theory was laid by Taylor (1934) and others.

The static and kinematic theorems of limit analysis were in general first introduced in a Russian conference proceedings article by Gvozdev (1938), long unknown in the West. At about the same time, the static shakedown theorem was first proposed by Melan (1936), being anticipated a few years earlier by himself and Bleich (1933). The fact that Melan's theorem implies the static theorem of limit analysis was recognized much later.

The general concepts of plasticity, which are expounded in Parts I-III of this book, and comprise the general multiaxial stress-strain relations, normality and convexity, maximization of plastic energy dissipation, limit state theorems, shakedown, optimal design, plastic hinges, yield line theory of plates and slip line theory, were established

shortly after World War II by Shanley (1947), Hill (1950), Drucker (1950), Greenberg and Prager (1951), Prager and Hodge (1951), Symonds and Neal (1951), Koiter (1953b), etc.; see Nádai (1950a) and Prager (1959) for additional references.

The second half of the last century was a period of rapid refinement and extensive ramification, which continue at an unrelenting pace until today and are for the most part described in Parts IV and V of this book.

Plasticity concepts began to impact structural analysis and design at the beginning of the last century, although design codes based on limit states were not instituted until the middle of that century. When subjected to the service loads, structures must generally respond in an elastic manner. A century ago, the standard design approach was to calculate the maximum stress according to the theory of elasticity, and make sure that it would not exceed a certain allowable stress, which was set sufficiently smaller than the material strength or yield limit. Later it was recognized that in most design problems (fatigue of metals excepted), this approach often leads to designs that are wasteful to varying degrees. The reason is that only some structures fail at a load at which the material strength or yield limit is exhausted at one point of the structure. Many structures redistribute stresses in such a way that the structure fails at a higher load, sometimes only a little higher but often a much higher load, which is attained only after a large part of the structure has plasticized. Simply setting the allowable stress value higher is not a solution, since the safety of some designs would become inadequate. If the theory of elasticity with allowable stress were still used as the basis of design, many efficient modern structures distinguished by slenderness could not even be built.

A realistic approach to design is to calculate the collapse load of the structure from the minimum expected value of material strength or yield limit, and then make sure that this collapse load would not be exceeded by the actual loads multiplied by a suitable safety factor (which is determined from experience and probabilistic considerations). Depending on the type of material, two different kinds of theories, the first older and more mature than the second, are needed for calculating the collapse load:

- If the material is plastic, as typical of most metals (provided the metal has not been fatigued), then the right approach is the *theory of plasticity*.
- If the material is brittle, then the right approach is either *fracture mechanics*, if the failure is caused by propagation of one or several large cracks, or *damage mechanics*, if the failure is caused by the spread of a zone of cracking or other distributed damage confined to the microscale.

This book deals only with the former.

To help understanding, the first two parts of this book are restricted to structures such as beams, trusses and frames whose stress state may be simplified as uniaxial. The advantage is that the basic concepts and results, such as the limit design theorems, normality and convexity, maximum plastic dissipation and shakedown, are understood more easily. This facilitates understanding of the behavior under multiaxial stress, which is the subject of Part III.

Plastic design of structures requires resolving problems of two basic types:

- Formulation of a realistic material model.
- Calculation of the collapse load if the material model is available.

Both are very rich problems. Most of the first three parts of the book deal with the latter problem, most of the fourth part with the former, and the fifth equates both.

Although 'brute-force' computational approaches such as the finite element method are nowadays capable of providing numerical answers to most problems of this type, much of the present exposition will dwell on analytical or semi-analytical solutions. It is, of course, these solutions that lead to an understanding of the structure behavior, are simple enough to be used for optimization or probabilistic safety studies, and provide indispensable checks on the correctness of computational approaches.

The constitutive models for plastic materials have proven to be a difficult problem, which has been tackled continuously from the emergence of plasticity to the present. Despite major advances in the past, this is still a very active area of research. Some important phenomena, e.g. the vertex effect, are still ignored in the constitutive models typically used in the current computational practice.

While the first two parts of the book will rest on a simple material model based only on uniaxial stress, the third part of the book will expound the basic constitutive models of multiaxial stress, and the fourth part will deal with advanced aspects such as the plastic hardening, anisotropic plasticity, restrictions on constitutive models stemming from thermodynamics, plasticity in large strain, microplane models based on the idea of polycrystal, and problems caused by material softening. The fifth part will extend the treatment to time-dependence of inelastic behavior.

As a special feature of this treatise that should be welcome by civil engineers, the fourth part of the book will discuss the plasticity aspects of quasibrittle materials such as concrete. The fifth part will briefly describe the time-dependent inelastic behavior of concrete and its consequences in structures, which are completely different from those of plasticity.

In the aftermath of the excessive enthusiasm of the 1970s which led to the development of plasticity models for all kinds of inelastic response of concrete, the development of plasticity models for all kinds of inelastic response of concrete in recent years has been in a sobering but fruitful period in which the limitations of plasticity have been properly recognized and modeled. Concrete, of course, is not a plastic material, but the theory of plasticity is useful for describing some aspects of its behavior, providing one pays proper attention to the inelastic strain localization engendered by post-peak strain softening, with the inherent size effects and the sensitivity of finite element solutions.