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Size Effect on Compressive Strength of Sandwich Panels with Fracture of Woven Laminate Facesheet

Prismatic sandwich specimens of various sizes, geometrically scaled in the ratio 1:2:4:8, are subjected to eccentric axial compression and tested to failure. The sandwich core consists of a closed-cell polyvinyl chloride foam, and the facesheets are woven glass-epoxy laminates, scaled by increasing the number of plies. The test results reveal a size effect on the mean nominal strength, which is strong enough to require consideration in design. The size effect observed is fitted with the size effect law of the energetic (deterministic) size effect theory. However, because of inevitable scatter and limited testing range, the precise form of the energetic size effect law to describe the test results is not unambiguous. The Weibull-type statistical size effect on the mean strength is ruled out because the specimens had small notches which caused the failure to occur in only one place in the specimen, and also because the observed failure mode was kink band propagation, previously shown to cause energetic size effect. Various fallacies in previous applications of Weibull theory to composites are also pointed out.
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Introduction

Quasibrittle materials are brittle materials characterized by a fracture process zone (FPZ) that is not negligible compared to the characteristic size D (or cross-sectional dimension) of the structure. Typically, the size of the FPZ, taken as the material characteristic length l_{ch}, is about 5 to 50 times the maximum inhomogeneity size, and quasibrittle behavior is observed only for $D/l_{ch} \approx 1$ to 1000. For larger D/l_{ch}, the FPZ can be regarded as a point and then the behavior is brittle, while for $D/l_{ch} < 1$, and approximately for up to about 5, the behavior can be regarded as quasiplastic. Small structural parts made of fiber-reinforced composites, rigid foams, and sandwich plates (as well as not too large concrete structures) can be treated as quasiplastic. In that case, fracture mechanics need not be used and size effect on the mean structural strength is negligible. Neglect of size effect has been the norm in most of the practice and virtually all the textbooks, e.g., [1–7].

However, larger composite and sandwich structures, sandwich skins, and foam cores, display pronounced quasibrittle behavior, as demonstrated by recent theoretical studies and experiments at Northwestern University, (Evanston, IL) [8–13], supplemented by analysis of experiments done elsewhere [14–18]. This is of concern for load-bearing panels in aircraft fuselage, and especially for structures of large ships made of composites. Quasibrittle behavior automatically implies a significant deterministic size effect of the energetic (deterministic) type, generally dwarfing the Weibull-type statistical size effect on the mean structural strength [10,19–22]. Such size effect has already been experimentally demonstrated for tensile fracture of notched laminates with various plies and layups [8], tensile fracture of polyvinyl chloride (PVC) closed-cell foam [11], flexural strength of various laminates [23–25], and kink band failure of compressed unidirectional composites [9]. From the fracture mechanics viewpoint, the cause of deterministic size effect is the energy release associated with stress redistribution prior to maximum load, engendered either by a large FPZ (type 1 size effect) or by a large crack (type 2 size effect) [10,20,21].

In quasibrittle materials, the energetic size effect also occurs in compression [26], although it is pushed to larger specimen sizes because of a larger FPZ. The size effect in compression was experimentally demonstrated and analytically modeled for kink band compressive failure of unidirectional carbon composites [9] and this model was also validated against tests using similar materials carried out by Soutis et al. [27].

As for sandwich structures, the size effect has already been experimentally demonstrated for shear failure of four-point-bend sandwich beams without and with interface notches between laminate skins and foam core [13].

An important remaining question is whether the deterministic size effect also operates for axially compressed laminate-foam sandwich facesheets. An affirmative answer is to be expected on the basis of the previous studies at Northwestern University and experimental demonstration is the objective of this study.

Choice of Test Specimens and Separation of Weibull Statistical Size Effect

Test specimens in the form of prismatic sandwich columns of rectangular cross section were selected; see Fig. 1. They were designed so that skin wrinkling instability as well as global buckling could not take place, thus forcing a facesheet compressive failure mode. The core consisted of PVC (Divinycell 250, density 250 kg/m^3). The skins were glass-epoxy laminates (7781 style satin weave glass and Bryte 250 epoxy resin), consisting of several plies. The prepreg material was manufactured by hot-melt film coating and was oven cured under vacuum consolidation. Two types of skins were used in the tests: porous and nonporous.

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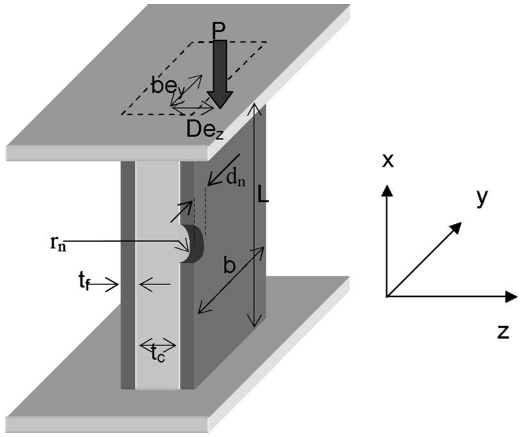

Fig. 1 Test specimen

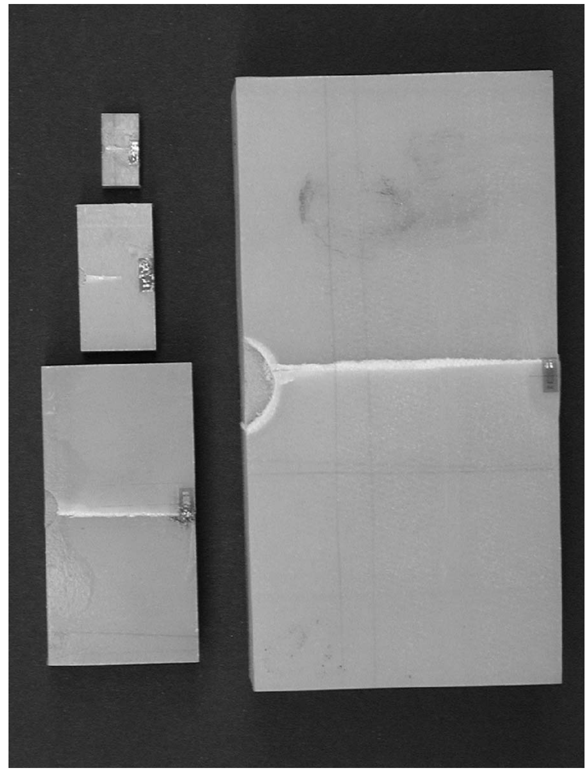

Fig. 2 Specimens showing size variation

The porosity was 3% to 5%, which is representative of manufacturing defects. The extent to which the porosity degrades the strength of the skins needs to be known for defining maximum allowable areas of excess porosity [28].

The elastic modulus of the core was $E_c=400$ MPa and the axial modulus of the orthotropic skins (nonporous) was $E_s=24{,}300$ MPa. The compression strength values were about 390 MPa (nonporous skin) and 320 MPa (porous skin) [29], and 5.7 MPa for the foam core [3]. The fracture energy and the material characteristic length (or FPZ length) were not measured but they were doubtless of the same order or magnitude as measured previously for similar laminates and foam [8,11–13].

The axial load is chosen to be doubly eccentric (Fig. 1), in order to ensure that only one FPZ develops within the cross section. Centric loading might be more difficult to interpret because several interacting FPZs could be developing simultaneously in the cross section prior to the maximum load. The axial load is applied through rigid end plates clamped to the ends of the specimen [29].

To determine the size effect, the specimens are geometrically scaled in three dimensions to four different sizes, which are in the ratio 1:2:4:8, and the ratios e_z and e_y of load eccentricities to D and b (the specimen width) are kept constant (see the dimensions in Table 1, and photograph in Fig. 2). To scale the laminate skins, the number of plies n is increased progressively, and is $n=2$, 4, 8, and 16 for the respective four specimen sizes. The sandwich thicknesses shown in Table 1 are in the range of planned load-bearing fuselage panels for aircraft, while for marine sandwich structures (such as ship hull, bulkhead, deck, mast, or antenna cover), considerably greater thicknesses are contemplated.

If the material was deterministic, the energetic size effect would be the only one to expect. However, composites always exhibit large statistical fluctuations of local material strength. Such fluctuations not only cause statistical scatter in σ_N but may also introduce a structural size effect on the mean value of σ_N, the theory of which was developed by Weibull [30,31] (see, for example, Bažant and Planas [22]). To make the size effect measurements unambiguous, it is necessary to choose specimens that exhibit only one kind of size effect.

This study is focused on demonstrating the energetic (deterministic) size effect in the compressive failure of sandwich structures, and so the specimen should be designed so as to eliminate the statistical size effect on the mean of σ_N (even though that size effect is expected to be very mild). This can be easily ensured by cutting notches, as shown in Fig. 1 (the notches are semicircular, cut by a horizontal mill into the most highly stressed edge of the laminate skins, with the circle radius r_n and notch depth d_n scaled in proportion to D).

A statistical size effect arises when structural failure can be initiated at many different points of the structure. This idea was conceived already in the mid 1600s by Mariotte [32], with the justification that the larger the structure, the smaller is the minimum local material strength likely to be encountered in the structure (the mathematical description of this idea, which leads to extreme value statistics of the weakest-link model, had to await the work of Fisher and Tippett [33]). In a specimen without any high local stress concentration, failure can initiate at many different points (or, more precisely, in many different representative volume elements). Therefore, at least some statistical size effect must generally be expected, not only in tension but also in compression. This would likely be the case for the present specimens if no notches were introduced.

The effect of a notch, inducing high local stress concentration, is that failure can initiate from only one place—the notch. Hence, the statistical size effect is obviously excluded. Consequently, if

Table 1 Dimensions of test specimens and axial load eccentricities (b, D, t_c, t_f, l, r_n, d_n are in millimeters, N, e_y, e_z are dimensionless)

n	b	D	t_c	t_f	l	r_n	d_n	e_y	e_z
2	10	7.5	6.25	0.55	20	1.59	1	0.075	0.135
4	20	15	12.5	1.1	40	3.18	2	0.075	0.135
8	40	30	25	2.2	80	6.35	4	0.075	0.135
16	80	60	50	4.4	160	12.7	8	0.075	0.135

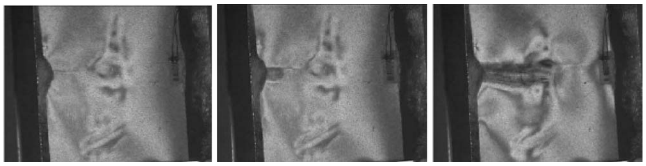

Fig. 3 Development of fracturing compressive kink band at the notch in the laminate skin

any structural size effect on the mean of σ_N is found in the present specimens, it can be explained only by the deterministic (energetic) size effect.

The compression failure consists of horizontal propagation of a softening fracturing kink band. This failure mode can be detected in the photoelastic coating fringe pattern captured during loading; see Fig. 3. Kink band propagation with microbuckling causes a reduction of the normal stress transmitted across the band, which is properly regarded as a phenomenon of cohesive fracture, characterized by a certain kink band fracture energy, implying a certain characteristic length of the kink band FPZ, and a finite residual stress on the softening stress-displacement relation of the kink band. A detailed analytical study and numerical simulation of kink band failure was given in Bažant et al. [9] and Zi and Bažant [34], along with experiments on polyetheretherketone (PEEK)-carbon specimens. The results showed that this mode of failure generally produces a significant energetic size effect, and so it is no surprise that a pronounced size effect is exhibited by the present tests.

Size Effect Laws

The size effect is understood as the effect of structure size on the nominal strength of structure, which is a parameter of the maximum load P defined as

$$\sigma_N = cP/bD \tag{1}$$

where c is a dimensionless constant introduced for convenience, often taken as $c=1$ but here defined so that σ_N would represent the maximum stress calculated from the elastic theory of bending (because b/D=constant for the present tests, one could, of course, alternatively use the definition $\sigma_N=cP/D^2$). In the theory of plasticity, as well as elasticity with an allowable strength limit, σ_N is independent of structure size D, i.e., there is no size effect. Linear elastic fracture mechanics (LEFM) exhibits the strongest possible size effect, in which σ_N decreases as $D^{-1/2}$ if the cracks for different sizes are geometrically similar (but when the critical crack or flaw is microscopic and independent of D, there is, of course, no size effect in LEFM) [20,21].

Having eliminated the statistical size effect, we can expect, according to the previously developed theory [9,10,19–22,24], that one of the following two types of the energetic (deterministic) size effect law should be followed:

$$\sigma_N = \sigma_\infty(1 + rD_b/D)^{1/r} \quad \text{(Type 1)} \tag{2}$$

$$\sigma_N = \sigma_0(1 + D/D_0)^{-1/2} + \sigma_r \quad \text{(Type 2)} \tag{3}$$

where σ_∞, r, D_b, D_0, σ_0, and σ_r are constants (related to the geometry and properties of the material). The type 1 size effect law applies to failures at fracture initiation from a smooth surface, and type 2 to failures when a large notch or a large crack is present at maximum load (there exists also type 3, but it is too similar to type 2 to be distinguished experimentally). Parameter σ_r represents the residual nominal strength of the specimen, due to frictional-plastic resistance after the fracture is fully formed. Usually $\sigma_r=0$ for tensile failures, but for compression failure σ_r can be nonzero [26] (and is definitely nonzero for compression kink bands [9,34]).

Table 2 Measured values of maximum load *P* and nominal strength σ_N for all individual sandwich specimens (*n*=number of prepreg layers)

Porous		Nonporous	
n	P/n^2	n	P/n^2
2	333	2	402
2	350	2	411
2	527	2	485
4	346	4	363
4	348	4	380
4	349	4	397
4	353	4	433
4	388	8	334
8	296	8	338
8	312	8	353
8	322	8	381
8	323	8	433
8	331	16	311
8	344	16	342
16	275		
16	295		

Because the present specimens have a sizeable notch, but not a very deep notch, the size effect must represent a transition from type 1 to type 2, in which the energetic size effect formula unfortunately becomes considerably more complex [35], with a greater number of coefficients. In view of the inevitable high scatter in laminate testing and the limited size range of the present test data, it is impossible to identify from the tests more than two coefficients of the size effect law. So we must choose one of the foregoing two formulas. Because a sizeable notch is present, the type 2 size effect, Eq. (3), is probably closer to reality.

Note: Initially it was planned also to carry out tests with notches in the form of a deep and wide enough slit, in which case the size effect would have to follow closely type 2, and further tests with very small notches. These tests would lie in the realms of pure type 2 and pure type 1 size effects, rather than in the more complex transitional range. However, these tests were not carried out; they are planned for a possible future project.

Experimental Results and Their Implication for Size Effect

The sandwich specimens were compressed at a constant displacement rate, for each size equal to 0.01 in./min. (which is the minimum possible on the testing machine used, Instron 8500). The viscoelastic effects on σ_N are expected to be unimportant. The maximum loads measured for all individual specimens of each size are given in Table 2.

For small sandwich specimens, with a size (thickness) denoted as D_1, the elastic analysis with a compressive strength limit f_c is known to give good results. Thus it is suitable to define the convenience parameter c in Eq. (1) in such a way that the nominal strength σ_N would coincide with f_c. To this end, we ignore the small notch, neglect the shear deformation of the foam core (which is small for nearly centric axial loading), and use the elastic theory of bending (in which the cross sections are assumed to remain planar), to calculate the maximum stress σ_{max} in the cross section, which occurs at the edge of skin, with coordinates $y=b/2$ and $z=D/2=(t_c+2t_f)/2$. Then, setting $\sigma_{max}=\sigma_N=f_c$, one obtains from the theory of bending the following expression (in which e_y and e_z are dimensionless ratios of the eccentricities to b and D):

$$c = \frac{1}{2t_f/D + E_c t_c/E_f D} + \frac{6e_y}{2t_f/D + E_c t_c/E_f D} + \frac{6e_z}{(1 - t_c^3/D^3) + E_c t_c^3/E_f D^3} \tag{4}$$

For the present sandwich geometry, the numerical value is $c=10.92$.

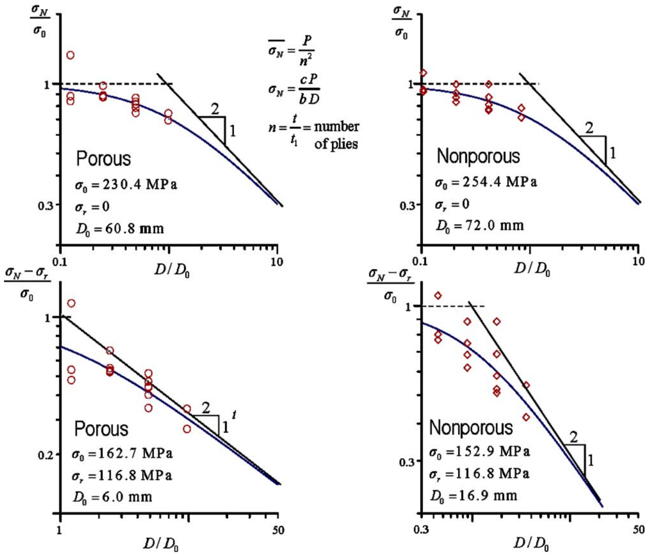

Fig. 4 Measured values (data points) of nominal strength σ_N of eccentrically compressed sandwich prisms of various sizes (thicknesses) D, plotted as $\log(\sigma_N-\sigma_r)$ versus $\log D$, and the their fit by type 2 size effect law, Eq. (3). Left: Porous laminate skins from standard manufacture. Right: Nonporous laminate skins.

The values of measured nominal strength σ_N, calculated from the maximum load P with this c value, are shown by the data points in Fig. 4, in which the optimum fits by the type 2 size effect law in Eq. (3) are shown by continuous lines and the asymptotes of this law are also marked. In the plots on top, it is assumed that there is no residual strength (σ_r=0), while in the plots at the bottom, the residual strength σ_r is finite. While Fig. 4 shows the data in logarithmic scales, Fig. 5 shows the same data in the plots of $1/\sigma_N^2$ or $1/(\sigma_N-\sigma_r)^2$ versus D/D_0, which are useful because Eq. (3) gets transformed, in such coordinates, to a linear regression plot. In such a plot, the optimum (least-squares) fit of the data

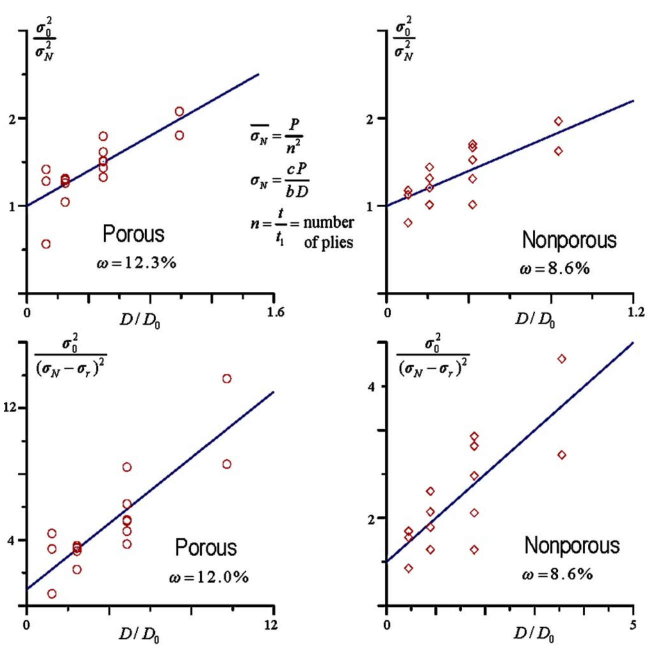

Fig. 5 The same data and fits as in Fig. 4, but replotted as $(\sigma_N-\sigma_r)^{-2}$ versus D, to obtain a linear regression plot.

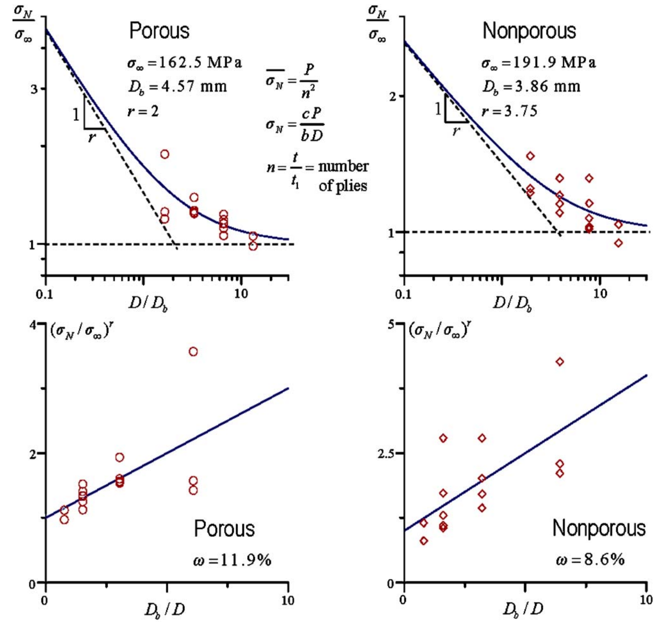

Fig. 6 Top: The same data as on top of Fig. 4, but fitted with the type 1 size effect law, Eq. (2). Bottom: The same as on top but replotted as σ_N–$1/r$ versus 1/D, to obtain a linear regression plot.

by Eq. (3) is easily obtained, along with the coefficient of variation ω of the data deviations from the regression line.

The optimum values of D_0, σ_0, and σ_r obtained by regression are listed in each figure (it must be noted, though, that the sensitivity of fits to σ_r is very weak, doubtless because the data scope is too limited compared to the scatter).

Figure 6 represents the optimum fits of the same data with the type 1 size effect law in Eq. (2). The optimum values of parameters σ_∞, D_b, and r are again listed in the figures.

The data plots in Figs. 4 and 5 make it clear that the size effect exists and is quite pronounced. Therefore, the current design procedures, which are based on the concept of material strength, are not justified for larger sandwich structures under compression. It follows that cohesive (or quasibrittle) fracture mechanics, or nonlocal damage mechanics, must be used in the analysis, or at least a size effect correction must be applied to the results obtained with the classical strength theory.

Looking at all the fits shown, one has difficulty in deciding which size effect law provides the best fit. The fit in Fig. 4 by the type 2 size effect law in Eq. (3), which seems more logical, is perhaps slightly better than the others but, in view of the scatter, the differences are insignificant. To reach an unambiguous conclusion, it would be necessary either to reduce the random scatter of the test data, which however seems impossible, or to extend the data range in terms of the size and include other geometries with different brittleness.

Estimation of Nominal Strength of Arbitrary Sandwich of Any Size

Parameters D_0, σ_0, and σ_r in Eq. (3), or σ_∞, r, and D_b in Eq. (2), should in principle be calculated from the properties of the cohesive crack model, particularly the material strength and fracture energy. This has been achieved for tensile fracture of some materials (such as concrete [20,22]), but is difficult for compressive failure of complex structures such as a sandwich. Therefore, a simplified approach is needed.

It can probably be safely assumed that for a very small sandwich structure, say $D=D_1=25.4$ mm, the elastic analysis with compressive strength limit f_c gives a realistic result. Therefore, we may set $\sigma_N = f_c$ in Eq. (2) or (3), and solve the equation for σ_∞ or σ_0. This yields

$$\sigma_\infty = f_c(1 + rD_b/D_1)^{-1/r} = f_c(1 + rD_b)^{-1/r} \quad \text{(Type 1)} \tag{5}$$

$$\sigma_0 = (f_c - \sigma_r)(1 + D_1/D_0)^{1/2} = (f_c - \sigma_r)(1 + 1/D_0)^{1/2} \quad \text{(Type 2)} \tag{6}$$

However, the values of D_0/t_1, D_b/t_1, and r must still be estimated. If analysis based on fracture mechanics cannot be carried out, one may use for this purpose, as very crude estimates, the means of the values from Figs. 4–6.

Fallacies in Adaptations of Weibull Statistical Theory

The Weibull power law for size effect on the mean structural strength reads $\sigma_N = kD^{N/m}$ where k, m, N=constants; N=number of dimensions in which the structure is scaled (N=1, 2 or 3), and m=Weibull modulus (or shape parameter), which is a local material property, characterizing the cumulative tail distribution of the survival probability P_e of a small material element, or representative material volume, subjected to stress ($P_e \propto \sigma^{-m}$). Weibull modulus governs not only the size effect but also the coefficient of variation ω of the scatter of measured structure strength; $\omega = [\Gamma(1+2/m)\Gamma^{-2}(1+1/m)-1]^{1/2}$. So, ω is a function of m but is independent of structure size and shape, provided that, as usual, the strength threshold is taken as zero (although the Weibull distribution with a finite threshold would give a size dependent ω, it is known that data histograms are generally fitted equally well with finite and zero thresholds, and besides, if one should not be over-optimistic in predicting the strength for a failure probability such as 10^{-7}, realistic for design, one must choose the threshold to be very small, but then the size dependence of ω is negligible).

Many previous tests have shown that, to fit the test results, it is necessary to consider that

(a) The m value for size effect and the m value for the coefficient of variation of scatter are different;
(b) m depends on structure size, laminate layup and geometry; and
(c) ω is not constant but varies with structure size and geometry.

It must be recognized, however, that if any of these three features is observed, the quintessential principles of the Weibull statistical theory are violated. Then, the correct conclusion is not that m is variable, but that the Weibull theory itself is inapplicable or insufficient, and that at least a part of the size effect is energetic (i.e., deterministic), which is what we attempt to model here (the fact that the apparent dependence of m on the laminate layup leads to contradictions and implies inapplicability of the Weibull theory was mathematically demonstrated in the appendix of Bažant et al. [13]).

Similar fallacies are seen in various numerical simulations found in the literature. For example, it is assumed that the local stress drops to zero after the strength limit is reached, or a constitutive law of damage mechanics or plasticity with softening yield limit is used in finite element simulation of test data such as the present ones. In such practice, the computational mesh is often scaled up with the structure size and the standard local finite element code is used. In that case, the increasing mesh size forces the width of the damage localization zone to expand with the structure size rather than remain constant, and then one obtains, of course, no size effect on σ_N of the structure. If there is a suspicion of size effect, or if some test data point that way, the Weibull size effect is then simply superimposed on the finite element results. Such an approach is incorrect and misleading [36].

Properly, a nonlocal formulation for strength or damage ought to be used to achieve computational objectivity and suppress spurious mesh size sensitivity (and thus regularize the boundary value problem, which is ill-posed in the local setting). Such a formulation then automatically exhibits the energetic (deterministic) part of size effect. In type 2 failures, this is the only type of size effect on the mean σ_N. But in type 1 failures, the Weibull size effect is negligible only for small sizes. It becomes significant for large enough sizes and dominates in the limit of infinite structure size [37,38]. The transitional range between type 1 and type 2, which is the case of the present tests, is a more complex mixture of both [37,38].

The coefficient of variation ω for each group of specimens of one size shows no systematic trend (jumping from 4.4% to 22% in porous specimens and from 6.6% to 10% in nonporous specimens). Although a much greater scope of testing would be required to judge meaningfully the trend of ω, one can nevertheless detect, on the left of Fig. 4, a tendency of ω (or the scatter bandwidth) to decrease with increasing size D. Such a trend is typical of the random scatter of the energetic size effect. On the other hand, for the statistical size effect on mean σ_N, ω does not depend on D.

Conclusions

1. The present experiments prove that compressive failure of laminate-foam sandwich plates exhibits a significant size effect. This size effect is strong enough to require consideration in design.
2. The thickness range of the present tests corresponds to the thicknesses of load-bearing fuselage panels of small aircraft, while application to large ship structures will require extrapolation of the measured size effect.
3. A possible statistical part of size effect due to material strength randomness is suppressed by introducing small notches in the laminate skins. This makes it possible to conclude that the size effect observed in the mean nominal strength of sandwich specimens with small notches cannot be statistical. The size effect observed can be explained only energetically, as a consequence of stress redistribution prior to the maximum load.
4. Due to the inevitable large scatter in the testing of fiber composites, and to limited size range of tests, the mathematical modeling of test results is not unambiguous. The present test data can be fitted equally well by several different laws, particularly the energetic size effect laws of either type 1 or type 2 (and even by the Weibull statistical size effect formula). Therefore, one can safely claim only that these laws do not disagree with the present experimental data, but not that these data validate the applicability of any of these laws. Their applicability rests on the general energetic theory of size effect, which has been amply validated by many experiments on fiber composites and other quasibrittle materials, as well as extensive mesh-objective numerical simulations.
5. Even though the Weibull power law of size effect could provide an equally good fit of the present data, this law is inapplicable, as a matter of principle. One reason is that the notch, deliberately introduced into the present test specimens, prevents the place of fracture initiation from sampling various points in the random field of local material strength. In other words, fracture can initiate at only one place, the notch, for which only the size-independent local distribution of material failure probability matters. Another reason is that the observed failure mode is kink band propagation in the laminate skin, for which an energetic size effect was shown to be a theoretical necessity.
6. The typical porosity of facesheets is a manufacturing defect that makes no significant difference for the size effect.

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