

MICROPRESTRESS-SOLIDIFICATION THEORY FOR CONCRETE CREEP. I: AGING AND DRYING EFFECTS

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ABSTRACT: A new physical theory and constitutive model for the effects of long-term aging and drying on concrete creep are proposed. The previously proposed solidification theory, in which the aging is explained and modeled by the volume growth (into the pores of hardened portland cement paste) of a nonaging viscoelastic constituent (cement gel), cannot explain long-term aging because the volume growth of the hydration products is too short-lived. The paper presents an improvement of the solidification theory in which the viscosity of the flow term of the compliance function is a function of a tensile microprestress carried by the bonds and bridges crossing the micropores (gel pores) in the hardened cement gel. The microprestress is generated by the disjoining pressure of the hindered adsorbed water in the micropores and by very large and highly localized volume changes caused by hydration or drying. The long-term creep, deviatoric as well as volumetric, is assumed to originate from viscous shear slips between the opposite walls of the micropores in which the bonds or bridges that cross the micropores (and transmit the microprestress) break and reform. The long-term aging exhibited by the flow term in the creep model is caused by relaxation of the tensile microprestress transverse to the slip plane. The Pickett effect (drying creep) is caused by changes of the microprestress balancing the changes in the disjoining pressure, which in turn are engendered by changes of the relative humidity in the capillary pores. Numerical implementation, application, and comparison with test data is relegated to a companion paper that follows in this issue.

INTRODUCTION

The creep of concrete, and particularly its dependence on the age at loading and water content, has proven to be a formidable problem. Attainment of a satisfactory understanding of the physical mechanism has defied seven decades of determined research efforts. Initially, the aging and the humidity effect were simply described by empirical functions. However, the test data are highly scattered and cover a limited range of conditions. Therefore, a good model with prediction capability can be formulated and identified from the data only if it is based on a good, physically justified theory. Without a good theory, a formulation mispredicting the creep outside the range of available test data and violating some basic physical principles can easily result (for example, one widely used empirically obtained creep model exhibited conflicts with the second law of thermodynamics).

The physical justification for aging must be sought in the processes in the microstructure. Recently, one theory, called the solidification theory (Bažant and Prasanna 1989a,b), apparently explained the aging satisfactorily. The basic premise was that the age dependence of the creep compliance ought to result from changes in the composition of the system, i.e., changes in the concentrations of the constituents whose properties do not change with age.

The process of hydration of portland cement involves gradual dissolution of anhydrous cement grains in the pore water

and precipitation of the hydration products, especially the calcium silicate hydrates, from the solution. Thus the decrease of creep compliance of concrete is in the solidification theory modeled as the consequence of gradual deposition of nonaging layers of new hydration products on the walls of the capillary pores in the hardened cement paste. This process is obviously characterized by a volume growth of the hydration products. The result is an increase of the macroscopic stiffness of the material (or a decrease of the compliance). Another result is an increase of the macroscopic viscosity of the material.

A particular form of the creep constitutive equation is dictated by the fact that the newly deposited layers must be stress free at the moment they get attached to the existing load-bearing solid framework of the material (Bažant 1977). This constitutive equation agreed with the existing creep data better than the previous ones. At the same time, it eliminated certain objectionable properties that plagued the previous empirical models (e.g., the divergence of creep curves for different ages at loading, along with a nonmonotonic evolution of the creep recovery curve obtained by the principle of superposition; or the negativeness within limited time periods of some viscosities of the Maxwell chain model; or the continuation of stress relaxation curves into stress of opposite sign).

The solidification theory, however, cannot be the final solution of the aging problem because the long-term volume growth history that gives a good description of the age effect on creep does not agree with the known volume growth history of the hydration products of portland cement. In particular, the creep for a fixed load duration decreases significantly with an increasing age at loading even after many years while the volume growth of the hydration products essentially stops after one year of age. It was observed that the long-term aging ought to be explained by some kind of stiffening of the microstructure of the existing hydration products without any further increase of their volume (Bažant and Prasanna 1989). But except for attributing such stiffening to an increase in the number of bonds (as in polymerization), no specific model has been formulated. It will be offered here.

Another formidable problem has been the additional creep caused by drying, called the Pickett effect (Pickett 1942). Wittmann and Roelfstra (1980) demonstrated that a mechanism that can cause this effect is microcracking. Bažant and Wu's

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(1974) study implied that a contribution to this effect can come from tensile strain softening, which is a similar explanation because the strain softening is caused by microcracking. The explanation suggested by Pickett (1942) was also similar—he attributed the phenomenon to the nonlinearity of the stress-strain relationship. The reason that Pickett's explanation could not match a wider range of later observations was that he considered only the hardening nonlinearity (the phenomenon of strain softening was unknown at that time).

From a recent detailed analysis (Bažant and Xi 1994), however, it appears that only a part, and not even the major part, of the Pickett effect can be caused by microcracking or strain softening. Besides, the explanation by microcracking runs into trouble for thick cross sections. The reason is that microcracking localizes into continuous cracks having the property that the thicker the cross section, the wider the spacing of the cracks (Bažant and Raftshol 1982). This means that the explanation of the Pickett effect by cracking is not, in general, a problem tractable merely by a constitutive equation but represents a boundary value problem of fracture mechanics.

The remaining, major part of the Pickett effect that is not explicable by micro- or macrocracking must be explained by some processes associated with drying on the microstructural level, which somehow get manifested by a decrease of the viscosity in the creep model such as the Kelvin chain (Bažant and Xi 1994). As a possible mechanism of such a phenomenon, it was proposed that the local microdiffusion flux J_m of water molecules between the hindered adsorbed layers and the capillary pores in the hardened cement paste somehow accelerated the process of breakage of atomic bonds in the solidified calcium silicate hydrate, which is the cause of creep (Bažant 1970, 1972; Bažant and Moschovidis 1973). Because the microdiffusion must be almost infinitely faster compared to the long-term creep (Bažant and Chern 1985), J_m must be essentially proportional to the rate \dot{h} of the relative humidity h in the capillary pore, which leads to the conclusion that the creep viscosity should change proportionally to \dot{h} (Bažant and Chern 1985, 1987).

Although some kind of the aforementioned mechanism remains a possibility, the notion of acceleration of bond breakage due to the microdiffusion flux of hindered adsorbed water still lacks a precise physical explanation. More seriously, it does not lead to a unified theory of the physical mechanism that would explain both the Pickett effect and the aging. Formulation of such a unified theory, recently broadly outlined in an internal research note (Bažant 1995), is the main objective of this paper. Numerical implementation, application, and experimental verification is the objective of a companion paper that follows in this issue (Bažant et al. 1997).

PORE STRUCTURE AND WATER ADSORPTION IN HARDENED CEMENT GEL

First we need to review some relevant characteristics of the pore structure and pore water. The hardened portland cement paste is a strongly hydrophilic porous material whose pores have an enormous internal surface (about 500 m^2 per cm^3). This causes the capillary and adsorption forces to dominate the stress levels in the microstructure, far exceeding any stresses that can be produced by applied loads (Powers and Brownard 1946, 1947; Powers 1968). The paste consists of the hardened cement gel [xerogel, Wittmann (1974)] and the capillary pores [Fig. 1(a)], defined as pores larger than $1 \mu\text{m}$. The pores in the cement gel are subcapillary and range from $0.000263 \mu\text{m}$ (the effective diameter of one water molecule) to about $1 \mu\text{m}$.

The capillary water is in tension, i.e., negative pressure p_c [which determines the curvature of capillary menisci; Fig. 1(a)]. The magnitude of p_c is very large; when the relative

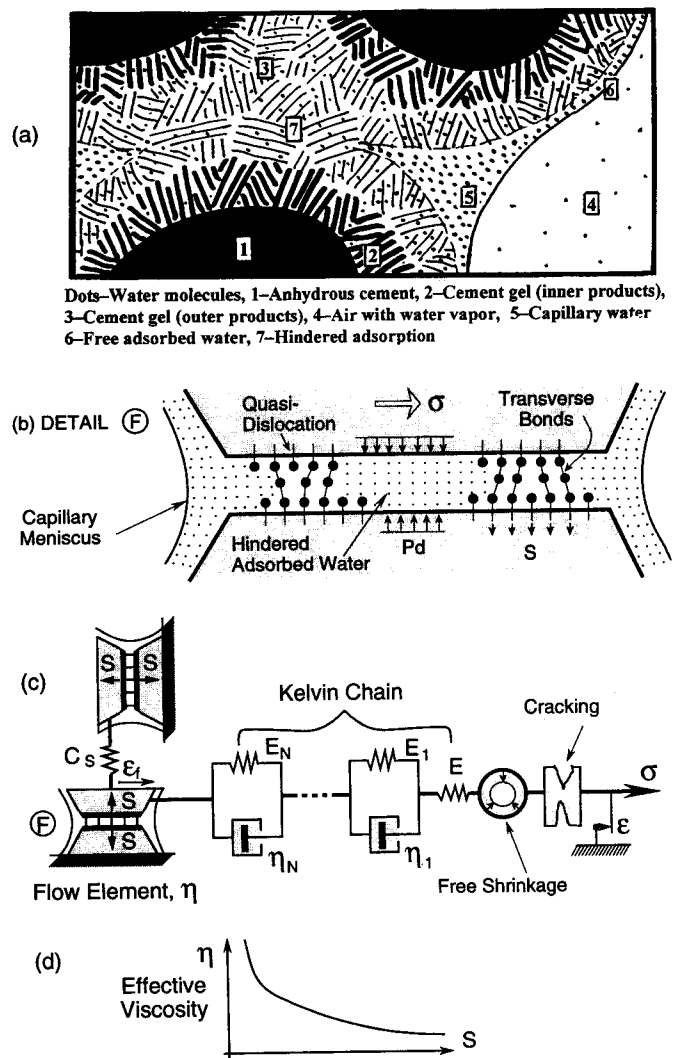


FIG. 1. (a) Idealized Microstructure of Hardened Cement Paste; (b) Micropore in Cement Gel, Disjoining Pressure p_d and Microstress S ; (c) Kelvin Chain Model with Flow Element and Microstress Relaxation Mechanism; (d) Dependence of Flow Viscosity on Microstress

humidity of water vapor in the capillary pore is $h = 0.4$ at 25°C , thermodynamic calculations (Bažant 1970) show that $p_c = 138 \text{ MPa}$ (1,360 atm), which is roughly the tensile strength of liquid water. The layers of adsorbed water on the pore walls exposed to water vapor are subjected to spreading pressure [e.g., Bažant (1972)], which reduces the surface tension of the solid γ_a [γ_a is a function of the mass of water molecules adsorbed per unit area of the surface; Fig. 1(a)]. The effective thickness δ_a of the adsorption layers (as well as the radius of the capillary menisci) is also a function of h . A monomolecular adsorption layer ($\delta_a = 0.263 \times 10^{-9} \text{ m}$) becomes full at $h = 0.12$ (Powers 1965). At saturation $h = 1$, δ_a reaches the maximum thickness of five molecules ($1.32 \times 10^{-9} \text{ m}$).

Many pores are less than ten water molecules in width. In such small pores, called micropores (or gel pores), the full thickness of the adsorption layers cannot develop, i.e., the adsorption is hindered. Therefore, a transverse compressive stress, called the disjoining pressure (p_d), must be exerted by the hindered adsorbed layers on the micropore walls [(Deryagin 1933, 1940, 1963; Powers 1968; Bažant 1970, 1972; Bažant and Moschovidis 1973; Wittmann 1974, 1980, 1982); Fig. 1(b)]. Calculations of adsorption thermodynamics (Bažant 1972) show the disjoining pressures to be enormous; in a pore two water molecules wide, $p_d = 174 \text{ MPa}$ (1,720 atm) at saturation ($h = 1$) and temperature of 25°C . There are further

phenomena, such as the crystal growth pressure, that act similarly.

The solid microstructure of calcium silicate hydrates also contains water, called the interlayer hydrate water, which is to some extent mobile and can evaporate in a zero humidity environment (Feldman and Sereda 1968). This water no doubt acts similarly to the hindered adsorbed water and is mathematically indistinguishable from it at the present level of knowledge. Therefore, when speaking of the hindered adsorbed water and its disjoining pressure, we also have in mind the interlayer hydrate water and its analogous pressure.

The disjoining pressure (as well as the crystal growth pressure) must be balanced by tensile forces. These forces are carried partly by the solid framework around the micropore and partly by bridges or bonds between the opposite walls of the same micropore [see Fig. 1(b) but note that the walls are very rough, unlike schematic picture]. As a reaction to the disjoining pressure (or crystal growth pressure), these bridges (bonds) must transmit very large tensions. In other words, the solid part of microstructure of the hardened cement gel is permanently in a pretensioned state (which explains in part why the tensile strength is low but improves if concrete is dried without any cracking). The buildup of tensile microprestress in the bridges or bonds is also caused by high local volume changes near the micropore, induced by hydration or by drying.

Because the distances of water diffusion between the micropores and the adjacent capillary pore are very short, the hindered adsorbed water establishes thermodynamic equilibrium (i.e., the equality of chemical potentials) with the capillary water very fast (probably within seconds, as estimated by Bažant and Chern 1995). Thus all phases of pore water may be assumed locally to be almost in thermodynamic equilibrium. Therefore, the disjoining pressure responds to changes of capillary tension and surface tension almost instantly. This means that they are all given by an equation of the form [Bažant (1972), Eq. (44)]

$$\gamma = -C_1 R M^{-1} T \ln h + \gamma_1 \quad (1)$$

where R = universal gas constant; M = molecular weight of water; T = absolute temperature; and C_1 , γ_1 = constants. Thus the rate

$$\dot{\gamma} = -c_1 \dot{h}/h \quad (2)$$

where c_1 = a constant.

NEW CONCEPT: RELAXATION OF MICROPRESTRESS

A direct dependence of the viscosity on time could be explained by a gradual deposition of newly solidified layers on the pore walls. However, such a process would also have to increase the stiffness of the material. This process would lead directly to the solidification concept that, as already pointed out, cannot explain the long-term aging after the hydration has ceased.

The challenge is to find a mechanism of aging that does not require volume growth of the solidified constituent that has fixed properties in time. Obviously, such a mechanism cannot be associated with stiffness. However, it can be associated with a variation of viscosity of slip caused, in particular, by relaxation of a transverse normal stress (called microprestress). This is the key idea advanced here [recently suggested by Bažant, personal communication (1993)]. This idea will at the same time be also helpful for the modeling of the Pickett effect.

The idea cannot be described by the classical rheological models such as the Kelvin chain because they are uniaxial. The context of multiaxial stress is required. We consider that the creep of hardened cement gel originates in the gel mi-

crostructure from shear slips, which are driven by the shear stress and are influenced by normal stresses across the slip planes. So the rheological model [Fig. 1(c)] must reflect interactions between forces and deformations of two directions: axial and transverse.

The microstructure of hardened cement gel contains dispersed, highly localized sites at which the atomic bonds are under a very high tensile stress. It is proposed that many of such highly stressed bonds are the bridges across the micropores containing the hindered adsorbed water and that the layers of hindered adsorbed water are the slip planes giving rise to the creep (or at least to the long-term flow part of creep). The creep is the macroscopic result of numerous interatomic bond breaks happening at different times at different overstressed sites (creep sites) in the hindered adsorbed layers. The bond breaks happen with a certain frequency determined by the kinetic energy of random thermal vibrations of atoms and by the magnitude of the activation energy barrier. Because the tensile stress reduces the activation energy barrier for bond rupture, this frequency increases as the tensile stress across the slip plane increases, as one could easily show according to the rate process theory. (A temperature rise, which increases the kinetic energy of the atomic vibrations, must obviously also increase the frequency of bond breakage, but the thermal effects are beyond the scope of this study.)

So we postulate that the microstructural source of the creep in the linear range is the shear slip at localized overstressed creep sites, represented by bridges across the micropores. The shear slip is the result of breakage of atomic bonds and restoration of bonds with adjacent atoms [Fig. 1(b)], which may be called quasidislocations because of similarity of the process to the movement of dislocations in crystals. Each bond breakage and restoration at one creep site relaxes the shear stress at that site. As the stress gets redistributed, another creep site may become overstressed, causing again a bond to break, etc. A progressive relaxation of the shear stress at the creep sites and exhaustion of the available overstressed creep sites causes the creep rate under a constant applied macrostress to decline.

Bond breakage without restoration is, of course, also possible. It leads to the nucleation and growth of tensile microcracks. Because the crack growth is time dependent and because the time dependence of crack growth has recently been shown to be significant even for rather slow rates of growth (Bažant and Gettu 1992; Bažant and Jirásek 1993), the result is also creep. However, because microcracks cause a decrease of stiffness on the macroscale, the result can only be a nonlinear creep, which occurs beyond the service stress range of concrete. So the source of the nonlinear part of creep of concrete is the time dependence of the growth of microcracks (Bažant 1993). This phenomenon is beyond the scope of the present study.

It would be wrong to infer that the shear slips could cause only the shear creep on the macroscale. Because of the high porosity and heterogeneity of the microstructure of the hardened cement gel, a hydrostatic macroscopic stress in the microstructure causes not only normal stresses but also shear stresses. Likewise (in view of the usual deformation calculation by the principle of virtual work), the slips in the microstructure must in general cause a change of volume (volumetric creep).

In view of the roughness of the pore surfaces in hydrated cement, the notion that new bonds can form in a hardened cement gel may at first seem questionable. However, the hardened cement gel with pore water is a metastable material whose surfaces are strongly hydrophilic and readily adapt their shape. That new bonds form quite easily is revealed by the classical experiment of Powers, in which cement and water were placed in a rotating vessel with steel balls. This mixture

produced a completely hydrated cement that was a powder. The powder was then stored in a container under a pressure of only a few psi. In a few days the powder solidified into a body whose strength was about the same as that of a normally cured paste. So the hydrated cement particles, when brought to contact, stick after a while. Another experiment revealing the capability to restore bonds was recently conducted at Northwestern University. Concrete specimens confined under enormous pressure (exceeding many times the uniaxial compression strength) were subjected to very large shear deformation such that the principal strain (precisely, the principal value of the Biot finite strain tensor) reached 35%. The compressive strength of the cores drilled from such deformed concrete was not zero but 25–30% of the strength of the cores drilled from virgin identically cured specimens.

The high tensile stress acting at the creep sites (i.e., across the bridges between the opposite micropore walls) is a locally self-equilibrated stress that is essentially independent of the applied macroscopic stress. It will therefore be called the microprestress [Figs. 1(b and c)]. The microprestress is generated as a reaction to the disjoining pressure in the micropore or to the high local shrinkage of small volumes in the material, which has recently been observed in an environmental electron microscope (Jennings, personal communication, 1996). An increase of microprestress may also be caused by the crystal growth pressures in the micropore [e.g., in the bridges in Fig. 1(b)], which are induced by the growth of crystalline hydration products precipitated from aqueous solution [large crystal growth pressures are documented by Schmidt-Döhl and Rosátsy 1995)].

On the other hand, the applied macroscopic stress or strain has only a negligible effect on the microprestress and the disjoining pressure. This result is corroborated by the conclusion that the disjoining pressure must respond to the changes in the capillary tension and surface tension almost instantaneously (Bažant and Chern 1985) and proportionally, and these in turn are functions of the relative vapor pressure h in the capillary pores, which is independent of the applied macroscopic stress (the other possible contributions to the microprestress, such as the high local volume changes and crystal growth pressure during hydration, are independent, too). This fact is confirmed by the famous experiments of Hansen (1960a,b), which defeated the consolidation theory of concrete creep. They show that identical loaded and load-free specimens dry equally fast. The solid framework of concrete and the hardened cement paste is so stiff that the applied load cannot change the pore space, and thus the relative humidity in the pore, by more than a fraction of one percent.

In some creep studies [e.g., Feldman and Sereda (1968)] it was surmised that the applied compressive macrostress causes the walls of the micropores to come into contact and stick by establishing new bonds. However, a quantitative analysis shows this to be impossible for the following two reasons: (1) Because the micropores with hindered adsorbed water and the interlayer spaces are not sparsely distributed but occupy a large fraction of any cross section through the cement gel, the relative change of thickness of the micropores and interlayer spaces that can be produced by the applied macrostress is of the same order of magnitude as the macroscopic strains, i.e., of the order of 0.1%, which is too small for establishing new bonds. (2) Squeezing the opposite walls of micropores together would cause much of the interlayer and hindered adsorbed water to be expelled, but this is contradicted by the experiments of Hansen (1960a,b).

Because the concrete creeps, it is natural to expect the tensile microprestress S at the creep sites (the bridges across the micropores) to relax with time. The relaxation reduces the rate of slip at the creep sites and also makes some of the creep

sites inactive. This inactivity causes a reduction of the macroscopic creep rate and is manifested as aging, which is the main idea advanced here (Bažant 1995). Obviously, the aging due to microprestress relaxation is not associated with the volume growth of the hydration products.

The macroscopic creep due to shear slip at the creep sites may be modeled by a viscous flow element coupled to the Kelvin chain, as shown in Fig. 1(c). The bonds across the slip plane in the flow element, representing a micropore filled by hindered adsorbed water, are subjected to the following two stresses: (1) The macroscopic applied stress σ causing shear slip, which acts in the figure horizontally; and (2) the tensile microprestress S , which acts in the figure vertically. The rate of strain in the flow element is

$$\dot{\epsilon}_f = \frac{\sigma}{\eta(S)} \quad (3)$$

where the effective viscosity $\eta =$ a decreasing function of S [Fig. 1(d)].

The relaxation of the microprestress S , acting in Fig. 1(c) vertically, is imagined to be the result of another similar flow element and a spring of stiffness C_s coupled in series, as shown in Fig. 1(c). Because the material is isotropic, and because the phenomena that generate the microprestress have no preferred orientation, the value of S (which is actually the macroscopic characteristic of the average microprestress at all creep sites) must be the same for all directions (the only anisotropic phenomenon is the macroscopic stress and strain tensors, but they cannot have any significant effect on the disjoining pressure, as already pointed out). Therefore the slip plane of the flow element governing the relaxation must also be subjected to normal stress S [horizontal in Fig. 1(c)], and so the viscosity must also be $\eta(S)$. Hence, the equation governing the relaxation of the microprestress may be written as

$$\frac{\dot{S}}{C_s} + \frac{S}{\eta(S)} = \dot{\gamma} \quad (4)$$

where $\dot{\gamma}$ = rate of the instantaneous microprestress induced by the capillary tension, surface tension, and crystal growth pressure.

We need to set up a suitable expression for $\eta(S)$. Because we do not know any characteristic value of S separating ranges of qualitatively different behavior, the function $\eta(S)$ should be self-similar with respect to changes of the scale by which S is measured. The only function satisfying this property is a power function. So we assume

$$\frac{1}{\eta(S)} = c p S^{p-1} \quad (5)$$

where c and p = positive constants; and $p > 1$ because $\eta(S)$ is a decreasing function.

The disjoining pressure and the microprestress S develop during the initial hardening, beginning with the set of the cement paste. During the initial rapid hydration, the buildup of microprestress is doubtless caused by the crystal growth pressure and very large localized shrinkage at locations close to the micropore. Later, after the volume changes due to hydration have almost ceased, the microprestress changes are caused mainly by the changes of the disjoining pressure, which respond almost instantly to the changes in the capillary tension and surface tension and are expressed by (1).

According to (2), (4), and (5), we obtain the following nonlinear differential equation governing the microprestress relaxation:

$$\dot{S} + c_0 S^p = -c_1 \frac{\dot{h}}{h} \quad (6)$$

where we denoted $c_0 = C_s cp$. Note that, according to this equation, S is independent of the applied macroscopic stress σ and strain ϵ .

In the limiting case of no drying or wetting ($\dot{h} = 0$, sealed specimens, basic creep) we have

$$\dot{S} = c_0 S^p \quad (7)$$

the integration of which gives $S = S_0(t_0/t)^{1/(p-1)}$ where $t =$ time; $t_0 =$ initial time; and $S_0 =$ initial value of S at time t_0 . For $p = 2$, this gives long-term relaxation that causes the strain in the flow element to grow exactly as a logarithmic function of time. In numerical simulations, the value $p = 2$ was found to work well.

In the limiting case of $\dot{h} \rightarrow \infty$, (6) reduces to

$$\dot{S} = -c_1 \frac{\dot{h}}{h} \quad (8)$$

which implies $S = -c_1 \ln h + \text{const}$.

Eqs. (7) and (9) can also be derived in another way. Extensive calibrations of the solidification theory by test data (Bažant and Prasannan 1989; Bažant and Baweja 1995) have shown that the viscous flow rate would vary as $\dot{\epsilon}_r = q_4 \sigma / t$ where $t =$ age of concrete and $q_4 =$ empirical constant (solidification theory constants q_1, \dots, q_3 will be used later). However, time t is not allowed to appear in a fundamental approach because there is no growth of hydration products at long times and the time variation is what we are trying to explain physically. To get rid of t we first note that, according to (5), $1/\eta = cpS^{p-1} = q_4/t$ or $S^{p-1} = q_4/(cpt)$ or $1/t = cpS^{p-1}/q_4$. Then, differentiating this relation, we have $(p-1)S^{p-2}\dot{S} = -(q_4/cp)(1/t)^2$. Thus substituting for $1/t$ we get $(p-1)S^{p-2}\dot{S} = -S^{2p-2}cp/q_4$. Finally, denoting $c_0 = cp/[q_4(p-1)] = \text{const}$, we acquire (7) by a different route. Vice versa, the solution of differential equation (7) is $1/\eta = cpS^{p-1}$ or $1/\eta = q_4/t$. As for (6), it represents the simplest logical way of combining (7) with (8) to preserve them as the asymptotic cases.

In (7) we gained a microprestress relaxation law that does not involve time. This law replaces the age-dependence of viscosity. The long-term aging of flow viscosity is simply a consequence of relaxation of the microprestress.

Changes of the disjoining pressure, capillary tension, and surface tension must also cause elastic deformation of the solid skeleton of cement gel, which represents the free shrinkage (Powers 1968), i.e., the shrinkage that occurs at no macroscopic stress. Because $\dot{\gamma}$ is proportional to \dot{h} , the free shrinkage strain, which is additive to the creep strain [Fig. 1(c)], is [e.g., Bažant (1988)]

$$\dot{\epsilon}_{sh} = k_{sh}\dot{h} \quad (9)$$

where k_{sh} is an empirical shrinkage coefficient that can be taken approximately as constant.

The additional terms due to stress-induced shrinkage, which were introduced in Bažant and Chern (1985, 1987), do not belong in the present expression for the shrinkage. The concept of stress-induced shrinkage was introduced as a consequence of the assumption that η depends on \dot{h} , which is now replaced by a dependence of η on S and of \dot{S} on \dot{h} . So it transpires that the present formulation does not represent a negation but a refinement of the formulation of Bažant and Chern (1985, 1987) and Bažant and Xi (1994). Likewise, the present theory is an extension of the theory for the effect of hindered adsorption and disjoining pressure on drying creep presented in Bažant (1972, 1975) and Bažant and Moschovidis (1973). The model for aging due to volume growth (Bažant 1977) must also be included in the formulation, as we discuss in the next section.

AMALGAMATION OF MICROPRESTRESS AND SOLIDIFICATION MODELS

In addition to the aging resulting from relaxation of microprestress, there is also the aging resulting from volume growth of hydration products, which dominates at early ages. From extensive studies of test data (Bažant and Prasannan 1989b; Bažant and Baweja 1995) and theoretical analysis (Carol and Bažant 1992), it appears that this latter type of aging can be described by either a Kelvin chain [Fig. 1(c)] or a Maxwell chain spring-dashpot model in which the relaxation times τ_μ ($\mu = 1, 2, \dots$) are constant and all the spring moduli and dashpot viscosities vary in proportion. The simple proportional variation of all the spring moduli is a logical consequence of the fact that this type of aging is due to volume growth. The variation of the spring moduli, however, is not arbitrary. It is obtained as a result of the volume growth of cement gel, which itself is characterized by a Kelvin or Maxwell chain model of age-independent properties.

In general, the microprestress might affect not only the viscosity of the flow term of solidification theory but also the viscosities of the dashpots in the Kelvin units of the chain [Fig. 1(c)], which are coupled to parallel springs. Vice versa, the volume growth might affect the viscosity of the flow term and thus complicate the mathematical formulation. Fortunately, analysis of the available test data shows that such a possibility need not be implemented in the model. In other words, the effects of microprestress and of volume growth can be assumed to be separated, acting on different parts of the chain model.

Putting all the sources together, the compliance function of concrete can be expressed as

$$J(t, t') = q_1 + C_v(t, t') + q_4' S^{p-1} \quad (10)$$

where $J(t, t')$ = strain at time t caused by a unit uniaxial stress acting since time t' ; $C_v(t, t')$ = viscoelastic compliance affected by the volume growth of hydration products, characterized by function $\nu(t)$, $q_4' = cp = q_4/k_1 =$ new constant introduced for brevity of notation; and $q_1 =$ constant characterizing the asymptotic instantaneous strain [the age independence of q_1 is justified by Fig. 6(b) in Bažant and Baweja (1995)].

As shown in an earlier paper (Bažant and Prasannan 1989b; Bažant and Baweja 1995), it is more convenient to write the detailed expression in terms of the compliance rate $\dot{J}(t, t') = \partial J(t, t')/\partial t$. Co-opting for $C_v(t, t')$ the expressions previously formulated and justified for the solidification theory, the compliance rate may be written as (Bažant and Prasannan 1989b; Bažant and Baweja 1995)

$$\dot{J}(t, t') = \frac{n(q_2 \lambda_0^m t^{-m} + q_3)}{(t-t') + \lambda_0^n (t-t')^{1-n}} + q_4'(p-1)S^{p-2}\dot{S} \quad (11)$$

where q_2, q_3, m, n , and $\lambda_0 =$ empirical constants. According to extensive data fitting, one can always take $\lambda_0 q = 1$ day, $m = 0.5$ and $n = 0.1$. On the other hand, q_2 and q_3 depend on the type of concrete. They can be predicted on the basis of compression strength of concrete and the composition of concrete mix by using the formulas of the B3 model (Bažant and Baweja 1995), although it is preferable to determine at least q_2 on the basis of short-time creep tests of the given concrete by using the updating procedure specified also in the B3 model. The microprestress S must first be solved from the differential equation (6) before (11) can be evaluated.

According to Bažant and Prasannan (1989a), the nonaging compliance function and the volume growth function of cement gel, from which the term with q_2 and q_3 in (11) was derived, are as follows:

$$J_g(t-t') = q_2 \ln \left[1 + \left(\frac{t-t'}{\lambda_0} \right)^n \right], \quad \frac{1}{v(t)} = \alpha + \sqrt{\frac{\lambda_0}{t}} \quad (12)$$

where α = material parameter.

RATE-TYPE CONSTITUTIVE RELATION BASED ON KELVIN CHAIN

Before writing the multiaxial creep law, note that the uniaxial Hooke's law $\epsilon = \sigma/E$ is generalized to multiaxial stress by the matrix relation $\epsilon = E^{-1}G\sigma$. For axisymmetric stress $\sigma = (\sigma_r, \sigma_\theta, \sigma_z, \sigma_{rz})^T$, $\epsilon = (\epsilon_r, \epsilon_\theta, \epsilon_z, \gamma_{rz})^T$ are column matrices of stress and strain components in cylindrical coordinates (r, θ, z), superscript T denotes matrix transpose and

$$G = \begin{bmatrix} 1 & -\nu & -\nu & 0 \\ -\nu & 1 & -\nu & 0 \\ -\nu & -\nu & 1 & 0 \\ 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \quad (13)$$

For the case of general stress $\sigma = (\sigma_x, \sigma_y, \sigma_z, \sigma_{xy}, \sigma_{yz}, \sigma_{zx})^T$, $\epsilon = (\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx})^T$, and

$$G = \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \quad (14)$$

Because the creep law is linear and the creep Poisson ratio ν can be assumed to be constant, the generalization to a multiaxial creep law may be accomplished by replacing in the uniaxial creep constitutive law σ with $G\sigma$ and ϵ with ϵ [i.e., the stresses and strains with the corresponding column matrices (Bažant 1988)]. Thus for a constant stress matrix σ the strain history is $\epsilon = GJ(t, t')\sigma$. Because the creep Poisson ratio can be assumed to be constant, the matrix generalization of the uniaxial constitutive law, $\epsilon(t) = \int_0^t J(t, t') d\sigma(t') + \epsilon_{sh}(t)$ may be written as

$$\epsilon(t) = \int_0^t J(t, t')G d\sigma(t') + \epsilon_{sh}(t) \quad (15)$$

where $\epsilon_{sh} = [\epsilon_{sh}, \epsilon_{sh}, \epsilon_{sh}, 0]^T$ for the axisymmetric case.

Like the physical explanation and derivation of the solidification theory, large-scale numerical computation is also best approached by approximating the compliance function with the viscoelastic Kelvin chain or the Maxwell chain rheological model. For uniaxial stress, the constitutive law based on Kelvin chain and the corresponding step-by-step solution algorithm were formulated in Bažant and Prasanna (1989b). We will now present their matrix generalization for multiaxial stress, either axisymmetric or general, using the same method of matrix generalization as for the age-dependent Kelvin or Maxwell chain (Bažant 1988).

In view of (15) and the analogy of $\epsilon = E^{-1}G\sigma$ with $\epsilon = \sigma/E$, the matrix generalization of the uniaxial formulation of the Kelvin chain model may be obtained by replacing ϵ with ϵ , σ with $G\sigma$, ϵ_{sh} with ϵ_{sh} , γ_μ with γ_μ , etc. According to this matrix analogy, the equations of the Kelvin chain model given by Bažant and Prasanna's (1989b) equation (1) are generalized by

$$E_\mu(\gamma_\mu + \tau_\mu \dot{\gamma}_\mu) = G\sigma; \quad \gamma = \sum_{\mu=1}^N \gamma_\mu \quad (16)$$

where γ = column matrix of strains of age-independent solid constituent = $[\gamma_r, \gamma_\theta, \gamma_z, \gamma_{rz}]^T$ for the axisymmetric case; $\gamma_\mu =$

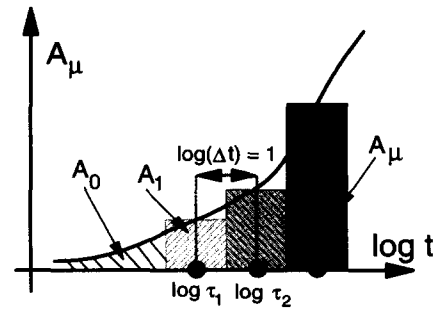


FIG. 2. Discretization of Continuous Retardation Spectrum of Solidifying Constituent

similar column matrices of partial strains corresponding to the individual Kelvin chain units ($\mu = 1, 2, \dots, N$) (the γ_μ s represent what are called internal variables in the thermodynamic theory of constitutive equations); and E_μ, τ_μ = spring modulus and retardation time of the μ th Kelvin unit in the chain [Fig. 1(c)], with $\tau_\mu = \eta_\mu/E_\mu$ where η_μ = viscosity of the dashpot of the μ th Kelvin unit ($\mu = 1, 2, \dots, N$). The value of τ_μ must be suitably chosen as described in Bažant (1988) and the corresponding values of E_μ are then determined by fitting the solution of (16) for unit uniaxial stress $\sigma = 1$ [i.e., $\epsilon(t) = \sum_{\mu} E_\mu^{-1}(1 - e^{-t/\tau_\mu}) + A_0$] to the compliance function $J_g(t-t')$ of the nonaging solidifying constituent. This determination is best done by discretizing a continuous retardation spectrum (Bažant and Xi 1995). Fig. 2 shows a graphical interpretation of this method. For each retardation time τ_μ the corresponding A_μ is given by the area of the rectangle centered at τ_μ , as shown in Fig. 2. To include the tail for $\log t \rightarrow -\infty$, an extra term A_0 is needed. Although only approximately 5 Kelvin units suffice for practical problems to avoid bumpy curves, the present calculations used 10 Kelvin units and resulted in $A_0 = 0.223$.

The viscoelastic strain ϵ^v is given for the uniaxial stress by in Bažant and Prasanna (1989a) equation (5), which for uniaxial stress and variable humidity in the pores reads $\dot{\epsilon}^v = \phi_h \dot{\gamma}/v(t)$ where ϕ_h is a function of h that decreases if h decreases because basic creep is smaller when the specimen has been brought to equilibrium at a lower environmental humidity. The previously verified form of function ϕ_h is

$$\phi_h(h) = \alpha_h + (1 - \alpha_h)h^2 \quad (17)$$

where α_h = material parameter; and $0 < \alpha_h < 1$.

According to the aforementioned matrix analogy, the multiaxial generalization of the equation for viscoelastic strains is

$$\dot{\epsilon}^v = \frac{\phi_h}{v(t)} \dot{\gamma} \quad (18)$$

The rate of the vector of total strains ϵ is

$$\dot{\epsilon} = \dot{\epsilon}^v + \dot{\epsilon}^f + \dot{\epsilon}_{sh} + \dot{\epsilon}_{cr} \quad (19)$$

where $\dot{\epsilon}^f$ = rate of the vector of flow strains; and $\dot{\epsilon}^v$ includes the instantaneous elastic strain rate. By generalizations of (3)

$$\dot{\epsilon}^f = \frac{1}{\eta(S)} G\sigma \quad (20)$$

where η is given by (5) and S is to be solved from (6). With the notation $\mathbf{I}_s = (1, 1, 1, 0)^T$, the rate of vector of shrinkage strains for the axisymmetric case is

$$\dot{\epsilon}_{sh} = k_{sh} h \dot{\mathbf{I}}_s \quad (21)$$

CONCLUSION

The formulation of the constitutive model of the microstress-solidification theory has been completed. What remains

is to incorporate cracking, formulate the numerical algorithm for finite-element applications, and verify the theory by comparisons with the main available test data. This process will be done in Part II of the companion paper (Bažant 1997). The conclusions (as well as acknowledgments) will be stated also in that paper.

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