Materials and Structures, 1995, 28, 357–365

RILEM DRAFT RECOMMENDATION
PROJETS DE RECOMMANDATION DE LA RILEM

107-GCS GUIDELINES FOR THE FORMULATION OF CREEP AND SHRINKAGE PREDICTION MODELS

Creep and shrinkage prediction model for analysis and design of concrete structures – model B3*

The text presented hereunder is a draft for general consideration. Comments should be sent to the TC Chairman, Professor Z. P. Bažant, W. P. Murphy Professor of Civil Engineering, Northwestern University, Evanston, IL 60208, USA, before 1 January 1996

1. SUMMARY

A model for the characterization of concrete creep and shrinkage in the design of concrete structures is recommended. It is simplier, agrees better with the experimental data and is justified better theoretically than the previous models. The model complies with the general guidelines recently formulated by RILEM TC 107. Justification of the model and various refinements are to be published shortly in two parts.

2. INTRODUCTION

During the last two decades, significant advances have been achieved in our knowledge about creep and shrinkage of concrete. They include: (i) vast expansion of the experimental data base on concrete creep and shrinkage; (ii) compilation of a computerized data bank; (ii) development of computerized statistical procedures for data fitting and optimization; and (iv) improved understanding of physical processes involved in creep and shrinkage such as ageing, diffusion processes, thermally activated processes and microcracking, and their mathematical modelling. These advances have made possible the formulation of the present model labelled model B3 (since it represents the third major update [1] of the models [2, 3] previously developed at Northwestern University). The model is formulated succinctly without any explanations, justifications, extensions and refinements. These are relegated to the two-part continuation paper [4], which does not have to be read by those who want merely to apply the model.

3. MAIN NOTATIONS AND APPLICABILITY RANGE

The following symbols are introduced.

t	time, representing the age of concrete in days
t'	age at loading, in days
t_0	age when drying begins, in days (only $t_0 \le t'$ is considered)
$J(t, t')$	compliance function = strain (creep plus elastic) at time t caused by a unit uniaxial constant stress applied at age t' (always given in 10^{-6} psi^{-1} here psi lb in^{-2} = 6895 Pa)
$C_0(t, t')$	compliance function for basic creep only
$C_d(t, t', t_0)$	compliance function for additional creep due to drying
ε_{sh}, $\varepsilon_{sh\infty}$	shrinkage strain and ultimate (final) shrinkage strain, always given in 10^{-6}; ε_{sh} considered negative (except for swelling, for which the sign is positive)
h	relative humidity of the environment (expressed as a decimal number, not as a percentage, $0 \le h \le 1$)
H	spatial average of pore relative humidity within the cross-section
$S(t)$	time function for shrinkage
τ_{sh}	shrinkage half-time (in days)
D	$2v/s$ = effective cross-section thickness in inches (inch = 25.4 mm)
v/s	volume-to-surface ratio in inches
c	cement content of concrete in lb ft^{-3} (lb ft^{-3} = 16.03 kg m^{-3})

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w/c ratio (by weight) of water to cementitious material
w $(w/c)c$ = water content of concrete in lb ft^{-3}
a/c ratio (by weight) of aggregate to cement
f'_c 28-day standard cylinder compression strength in psi (if only design strength f_{ck} is known, then $f'_c = f_{ck} + 1200$ psi)

The prediction of the material parameters of the present model is restricted to Portland cement concretes with the following parameter ranges:

$$2500 \le f'_c \le 10000, \quad f'_c \text{ in psi}, \quad 0.3 \le w/c \le 0.85 \quad (1)$$

$$10 \le c \le 45 \quad c \text{ in lb ft}^{-3}, \quad 2.5 \le a/c \le 13.5 \quad (2)$$

Formulae predicting model parameters from the composition of concrete have not been developed for special concretes containing various admixtures, pozzolans, microsilica, and fibres. However, if the model parameters are not predicted from concrete composition and strength but are calibrated by experimental data, the model can be applied even outside this range, for example, to high strength concretes, fibre-reinforced concretes, and mortars.

4. LINEARITY OF CREEP AND TIME DEPENDENT STRAIN COMPONENTS

The present prediction model is restricted to the service stress range (or up to about $0.4f'_c$) for which creep is assumed to be dependent linearly on stress. This means that for constant stress σ applied at age t'

$$\varepsilon(t) = J(t, t')\sigma + \varepsilon_{sh}(t) + \alpha \Delta T(t) \quad (3)$$

in which σ is the uniaxial stress, ε is the strain, $\Delta T(t)$ is the temperature change from reference temperature at time t, and α is the thermal-expansion coefficient. When stresses vary in time, the corresponding strain can be obtained from Equation 3 according to the principle of superposition [5]. Simplified design calculations can be done according to the age adjusted effective modulus method, which allows quasi-elastic analysis [6, 7] of the structure.

The compliance function may be decomposed further as

$$J(t, t') = q_1 + C_0(t, t') + C_d(t, t', t_0) \quad (4)$$

in which q_1 is the instantaneous strain due to unit stress, $C_0(t, t')$ is the compliance function for basic creep (creep at constant moisture content), and $C_d(t, t', t_0)$ is the additional compliance function due to simultaneous drying (all given in 10^{-6} psi^{-1}). Generalization to multiaxial stress may be based also on the principle of superposition. The creep Poisson ratio may be assumed to be constant and equal to the instantaneous Poisson ratio $\nu = 0.18$. (Tensile microcracking can cause the apparent Poisson ratio to be much smaller, but this is taken into account properly by a model for cracking.)

The instantaneous strain, as in previous models [2, 3], may be written as $q_1 = 1/E_0$ where E_0 is the asymptotic modulus. The use of E_0 instead of the conventional static modulus E is convenient because concrete exhibits pronounced creep, even for very short loads durations. E_0 should not be regarded as a real elastic modulus but merely an empirical parameter that can be considered age independent. The age independance of E_0 is demonstrated by the experimental fact that the short-time creep curves for various t' plotted as $J(t, t')$ versus $(t - t')^n$ (with $n \approx 0.1$) appear approximately as straight lines that all meet at $t - t' = 0$ approximately at the same point, regardless of t' (see Fig. 6(b) in Part 1 of [4]). As a rough estimate, $E_0 \approx 1.5E$. The value of the usual static elastic modulus E normally obtained in tests and used in structural analysis corresponds approximately to

$$E(t') = 1/J(t' + \Delta, t') \quad (5)$$

in which the stress duration $\Delta = 0.01$ day gives values approximately agreeing with ACI formula ($E = 57000\sqrt{f'_c}$). Equation 5 also gives the correct age dependence of the elastic modulus $E(t')$. The value $\Delta = 10^{-8}$ day gives approximately correct values of the dynamic modulus of concrete and its age dependence. The meaning of the value of $q_1 = 1/E_0$ is explained in Fig. 1, which also shows the typical curves of basic creep, shrinkage and drying creep according to the present model.

The creep coefficient, which represents the most convenient way to introduce creep into structural analysis, should be calculated from the compliance function, i.e.,

$$\phi(t, t') = E(t')J(t, t') - 1 \quad (6)$$

Note that for structural analysis it is not important which value of Δ corresponds to $E(t')$ in Equation 5, and not even whether some other definition of E is used in Equation 6. One can use the ACI formula, $E = 57000\sqrt{f'_c}$, or Equation 5 for any value of $\Delta \le 0.1$ day. For the results of structural analysis of creep and shrinkage (for $t - t' \ge 1$ day), the only important aspect is that E and ϕ together must give the correct total compliance $J(t, t') = [1 + \phi(t, t')]/E(t')$, as defined by model B3.

Note that if a prediction model would specify ϕ instead of $J(t, t')$, there would be a danger of combining ϕ with some incompatible value of $E(t)$ which would be giving wrong $J(t, t')$ values. There are many combinations of ϕ and E that give the same $J(t, t')$ and what matters for structural calculations is only the values of J, and not the values of ϕ and E that yield $J(t, t')$. Care in this regard must also be taken when updating the model parameters from some test data for which only the values of ϕ were reported. $J(t, t')$ cannot be calculated from such data using a definition of E, e.g., $E = 57000\sqrt{f'_c}$, which does not give values compatible with these ϕ values and gives $J(t, t')$ disagreeing with Equation 6. Conversions of such data from ϕ to J values must be based on short-time strains measured on the creep specimens themselves, or else such data cannot be used.

The relative humidity in the pores of concrete is initially 100%. In the absence of moisture exchange (as in sealed

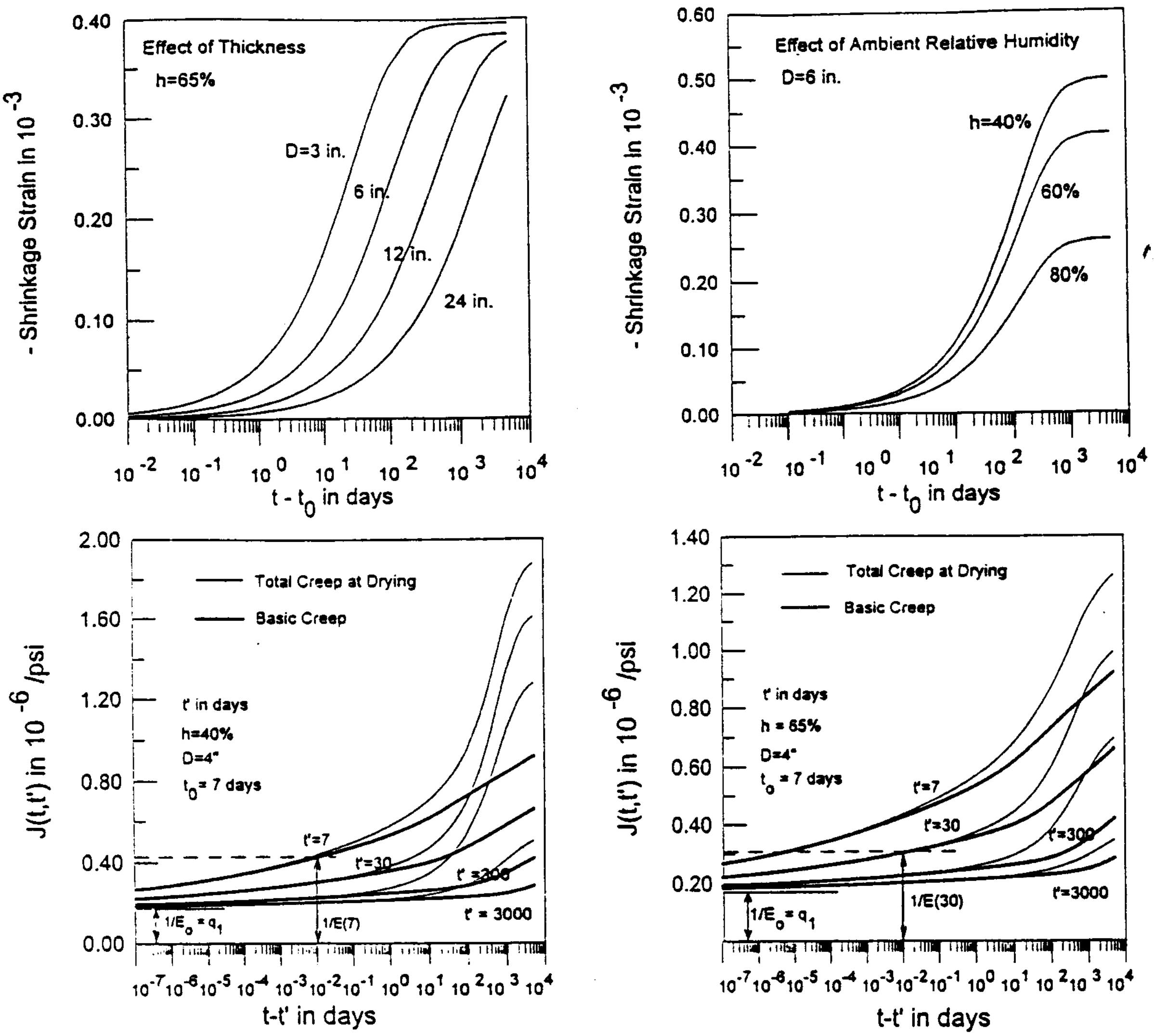

Fig. 1 Typical creep and shrinkage curves given by model B3.

concretes), a subsequent decrease of pore humidity (called self-desiccation) is caused by hydration, but this decrease in normal concretes is small (to about 96–98%). Exposure to the environment causes a long-term drying process (described by solutions of the diffusion equation), which causes shrinkage and additional creep. This means that the normal strain $J(t, t')\sigma$, representing the sum of the elastic and creep strains, is measured by subtracting the deformations of a loaded specimen from a load-free companion. For shear creep this is not necessary because shrinkage is strictly a volume change. In the absence of drying, there is another kind of shrinkage, called autogenous shrinkage, which is caused by the chemical reactions of hydration. This shrinkage usually is small compared with drying shrinkage for normal concretes. It does not occur if the relative humidity in the pores drops significantly below 100%. Further shrinkage (or expansion) may be caused by various chemical reactions, for example, carbonation. But in good concretes carbonation occurs only in a surface layer a few millimetres thick and can be neglected for normal structures. For concrete submerged in water ($h = 100\%$) there is positive ε_{sh}, that is swelling, which is predicted approximately by the present model upon substituting $h = 1$.

5. BASIC CREEP (MATERIAL CONSTITUTIVE (PROPERTY)

The basic creep compliance is more conveniently defined by its time rate than by its value:

$$\dot{C}_0(t, t') = \frac{n(q_2 t^{-m} + q_3)}{(t - t') + (t - t')^{1-n}} + \frac{q_4}{t}, \quad m = 0.5, n = 0.1 \tag{7}$$

in which $\dot{C}_0(t, t') = \partial C_0(t, t')/\partial t$, and q_2, q_3 and q_4 are empirical constitutive parameters which will be defined later. Note that the computer solutions of structural creep problems in small time steps require only the rate of compliance $\dot{J}(t, t')$, not the total value $J(t, t')$. By integration of Equation 7, the total basic creep

Table 1 Values of function $Q(t, t')$ for $m = 0.5$ and $n = 0.1$

$\log(t - t')$	$\log t'$								
	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
−2.0	0.4890	0.2750	0.1547	0.08677	0.04892	0.02751	0.01547	0.008699	0.004892
−1.5	0.5347	0.3009	0.1693	0.09519	0.05353	0.03010	0.01693	0.009519	0.005353
−1.0	0.5586	0.3284	0.1848	0.1040	0.05846	0.03288	0.01849	0.01040	0.005846
−0.5	0.6309	0.3571	0.2013	0.1133	0.06372	0.03583	0.02015	0.01133	0.006372
0.0	0.6754	0.3860	0.2185	0.1231	0.06929	0.03897	0.02192	0.01233	0.006931
0.5	0.7108	0.4125	0.2357	0.1334	0.07516	0.04229	0.02379	0.01338	0.007524
1.0	0.7352	0.4335	0.2514	0.1436	0.08123	0.04578	0.02576	0.01449	0.008149
1.5	0.7505	0.4480	0.2638	0.1529	0.08727	0.04397	0.02782	0.01566	0.008806
2.0	0.7597	0.4570	0.2724	0.1602	0.09276	0.05239	0.02994	0.01687	0.009494
2.4	0.7652	0.4624	0.2777	0.1652	0.09708	0.05616	0.03284	0.01812	0.01021
0.3	0.7684	0.4656	0.2808	0.1683	0.1000	0.05869	0.03393	0.01935	0.01094
3.5	0.7703	0.4675	0.2827	0.1702	0.1018	0.06041	0.03541	0.02045	0.01166
4.0	0.7714	0.4686	0.2838	0.1713	0.1029	0.06147	0.03641	0.02131	0.01230
4.5	0.7720	0.4692	0.2844	0.1719	0.1036	0.06210	0.03702	0.02190	0.01280
5.0	0.7724	0.4696	0.2848	0.1723	0.1038	0.06247	0.03739	0.02225	0.01314

compliance is obtained in the form:

$$C_0(t, t') = q_2 Q(t, t') + q_3 \ln[1 + (t - t')^n] + q_4 \ln\left(\frac{t}{t'}\right) \qquad (8)$$

in which $Q(t, t')$ is a binomial integral which cannot be expressed analytically. It is given in Table 1 and it can also be calculated from an approximate explicit formula given by Equation A1 in Appendix A. Function $Q(t, t')$, of course, also can be obtained easily by numerical integration.

The terms in Equation 8 containing q_2, q_3 and q_4 represent the ageing viscoelastic compliance, non-ageing viscoelastic compliance, and flow compliance, respectively, as deduced from the solidification theory [8].

6. AVERAGE SHRINKAGE AND CREEP OF CROSS-SECTION AT DRYING

6.1 Shrinkage

Mean shrinkage strain in the cross-section:

$$\varepsilon_{sh}(t, t_0) = -\varepsilon_{sh\infty} k_h S(t) \qquad (9)$$

Time curve:

$$S(t) = \tanh\left(\frac{t - t_0}{\tau_{sh}}\right)^{1/2} \qquad (10)$$

Humidity dependence:

$$k_h = \begin{cases} 1 - h^3 & \text{for } h \le 0.98 \\ -0.2 & \text{for } h = 1 \text{ (swelling in water)} \\ \text{linear interpolation} & \text{for } 0.98 \le h \le 1 \end{cases} \qquad (11)$$

Size dependence:

$$\tau_{sh} = k_t (k_s D)^2 \qquad (12)$$

where $D = 2v/s$ = effective cross-section thickness and k_s is the cross-section shape factor:

$$k_s = \begin{cases} 1.00 & \text{for an infinite slab} \\ 1.15 & \text{for an infinite cylinder} \\ 1.25 & \text{for an infinite square prism} \\ 1.30 & \text{for a sphere} \\ 1.55 & \text{for a cube} \end{cases} \qquad (13)$$

The analyst needs to estimate which shape approximates his conditions best (but high accuracy in this respect is not needed and $k_s \approx 1$ can be assumed for simplified analysis).

Time dependence of ultimate shrinkage:

$$\varepsilon_{sh\infty} = \varepsilon_{s\infty} \frac{E(7 + 600)}{E(t_0 + \tau_{sh})} \qquad (14)$$

This empirical expression approximately describes the effect of ageing (hydration). Also (because τ_{sh} depends on D), it describes the fact that in thicker specimens drying leads to more microcracking and proceeds slower (thus allowing more hardening due to hydration), both os which reduce the final shrinkage. The value of elastic modulus $E(t)$ can be expressed either from Equation 5 of the present model or approximately from ACI equation $E(t) = E(28)[t/(4 + 0.85t)]^{1/2}$, which has been adopted for the present fitting of data (ε_{sh} is relatively insensitive to the precise definition of E and in either case the result is about the same). For simplified analysis one can assume $\varepsilon_{sh\infty} \approx \varepsilon_{s\infty}$. The typical values of $\varepsilon_{sh\infty}$ according to Equation 14 range from 300×10^{-6} to 1100×10^{-6}.

The present model does not describe well the autogenous shrinkage which occurs in sealed specimens (or mass concrete). Such shrinkage is caused by volume changes during the chemical reactions of hydration and is independent of the size of the specimen. This shrinkage is usually much smaller than the drying shrinkage. In exposed specimens there is some autogenous shrinkage, too, but still smaller because (i) most of it occurs before stripping of the mould and (ii) after stripping the mould it occurs only in the core and only until the drying front reaches the core. This part of autogenous shrinkage is included in the present model because the model was fitted to the total shrinkage data of drying specimens.

6.2 Additional creep due to drying (drying creep)

$$C_d(t, t', t_0) = q_5[\exp\{-8H(t)\} - \exp\{-8H(t')\}]^{1/2}, \quad \text{for } t' \geq t_0 \qquad (15)$$

in which

$$H(t) = 1 - (1 - h)S(t) \qquad (16)$$

7. PREDICTION OF MODEL PARAMETERS AND UNCERTAINTIES

7.1 Estimation from concrete composition and strength

7.1.1 *Basic creep*

Predicting the creep and shrinkage properties of concrete from the composition of the concrete mix and the strength of the concrete is an extremely difficult problem for which no good theory has yet been developed. The following formulae which are partly empirical and partly reflect trends deduced theoretically from an understanding of the physical mechanisms, were calibrated by statistical analysis of the data in a computerized data bank involving about 15,000 data points and about 100 test series;

$$q_1 = 0.6 \times 10^6/E_{28}, \quad E_{28} = 57000(f_c')^{1/2} \qquad (17)$$

$$\left.\begin{aligned} q_2 = 451.1c^{0.5}(f_c')^{-0.9}, \quad q_3 = 0.29(w/c)^4 q_2,\\ q_4 = 0.14(a/c)^{-0.7} \end{aligned}\right\} \qquad (18)$$

7.1.2 *Shrinkage*

$$\varepsilon_{s\infty} = \alpha_1\alpha_2[26w^{2.1}(f_c')^{-0.28} + 270] \quad (\text{in } 10^{-6}) \qquad (19)$$

and

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$$k_t = 190.8t_0^{-0.08}f_c' \text{ days in}^{-2} \qquad (20)$$

where

$$\alpha_1 = \begin{cases} 1.0 & \text{for type I cement} \\ 0.85 & \text{for type II cement} \\ 1.1 & \text{for type III cement} \end{cases} \qquad (21)$$

and

$$\alpha_2 = \begin{cases} 0.75 & \text{for steam cured specimens} \\ 1.0 & \text{for specimens cured in water or at 100\% relative humidity} \\ 1.2 & \text{for specimens sealed during curing} \end{cases} \qquad (22)$$

7.1.3 *Creep at drying*

$$q_5 = 7.57 \times 10^5 f_c'^{-1} \varepsilon_{sh\infty}^{-0.6} \qquad (23)$$

7.1.4 *Parameter uncertainties to be considered in design*

The parameters of any creep and shrinkage model must be considered as statistical variables. The preceding formulae predicting the creep and shrinkage parameters from concrete composition and strength give the mean value of $J(t, t')$ and ε_{sh}. To take into account statistical uncertainties, the parameters $q_1, q_2, q_3, q_4, q_5, \varepsilon_{sh\infty}$ ought to be replaced by the values

$$\psi_1 q_1, \psi_1 q_2, \psi_1 q_3, \psi_1 q_4, \psi_1 q_5, \psi_2 \varepsilon_{sh\infty} \qquad (24)$$

in which ψ_1 and ψ_2 are uncertainty factors for creep and shrinkage, which may be assumed to follow roughly a normal (Gaussian) distribution with mean value 1. According to the statistical analysis of the data in the data bank, the following coefficients of variation of these uncertainty factors should be considered in design:

$$\left.\begin{aligned} \omega(\psi_1) &= 23\% \text{ for creep, with or without drying}\\ \omega(\psi_2) &= 34\% \text{ for shrinkage} \end{aligned}\right\} \qquad (25)$$

This means that, if the statistical distribution is approximated as Gaussian (normal), the 95% confidence limits for ψ_1 are $1 \pm 1.96 \times 0.23 = 1 \pm 0.45$, and for ψ_2 are $1 \pm 1.96 \times 0.34 = 1 \pm 0.67$.

The other input parameters of the model are also statistical variables. At least, the designer should consider the statistical variations of environmental humidity h and of strength f_c'. This can be done by replacing them with $\psi_3 h$ and $\psi_4 f_c'$ ahere ψ_3 and ψ_4 are uncertainty factors having a normal distribution with mean 1. In the absence of other information, the following coefficients of variation may be considered for these uncertainty factors [9]:

$$\left.\begin{aligned} \omega(\psi_3) &\approx 20\% \text{ for } h \to \psi_3 h\\ \omega(\psi_4) &\approx 15\% \text{ for } f_c' \to \psi_4 f_c' \end{aligned}\right\}. \qquad (26)$$

Factor ψ_3 is statistically independent of ψ_1, ψ_2, and ψ_4. Approximately, all the factors may be assumed mutually independent.

If the structure is exposed to a climate that is permanently hot or has prolonged high temperature extremes, it is advisable to take into account the temperature effect according to Appendix B. If this is not done, it is recommended to increase the aforementioned coefficients of variation ω by 10%.

The coefficients of variation in Equations 25 and 26 can result in similar or very different (much smaller or much larger) coefficients of variation of structural response such as the predicted maximum deflection or the maximum stress in the structure. Even if the safety against collapse is not threatened (as in long-time creep buckling of shell roofs), structures should not be designed for the mean effects of creep and shrinkage. Rather they should be designed for the response (deflection, stress, strain) values representing 95% confidence limits (this means that, if 20 identical structures were built and subjected to the same loading and environment, only one of them would be likely to suffer intolerable deflections or cracking damage, whereas the design for mean response means that 10 of them would be likely to suffer such a fate). One may assume the response values to have a normal (Gaussian) distribution, and then the 95% confidence limits may be estimated as (mean of X) $\times$ $(1 \pm 1.96\omega_X)$; ω_X can be calculated by generating in a proper way about 10 random samples of material parameters and then running for each sample a deterministic structural analysis [9].

7.2 Improved estimation: updating based on short-time tests

The large uncertainty in the prediction of creep and shrinkage of concrete, reflected in the values of the coefficients of variation in Equation 25, is caused mainly by the effect of the composition and strength of the concrete. This effect is very complicated and not sufficiently understood in quantitative terms. At present, the only way to reduce the uncertainty is to conduct short-time tests and use them to update the values of the material parameters in the model. This approach is particularly effective for creep but is more difficult for shrinkage [10]. A method to improve the prediction based on short-time tests is described in the continuation paper [4].

7.2.1 *Extension to special concretes*

Model B3 has been calibrated for normal concretes. This means that the formulae for the influence of concrete composition and strength are valid only for normal concretes. However, the model remains valid for special concretes if the model parameters are calibrated by tests. Short-time testing can be used for this purpose.

It must be pointed out in this context that for high strength concretes the autogenous shrinkage becomes relatively much more significant [14], because (due to very small water/cement ratios and larger cementitious material content) there is considerable self-desiccation in such concretes. This aspect of the shrinkage of high strength concretes is considered in the continuation paper [4], and a simple formula for autogenous shrinkage in high strength concretes is suggested.

7.2.2 *Levels of creep sensitivity of structures and type of analysis required*

Accurate and laborious analysis of creep and shrinkage is necessary for some types of structure but not for others. That depends on the sensitivity of the structure. Although more intensive studies are needed, the following approximate classification of sensitivity levels of structures can be made on the basis of general experience.

Level 1. Reinforced concrete beams, frames and slabs of spans under 20 m and heights up to 30 m, plain concrete footings, retaining walls.
Level 2. Prestressed beams or slabs of spans up to 20 m, high-rise building frames up to height 100 m.
Level 3. Medium-span box girder or arch bridges of spans up to 80 m, ordinary tanks, silos, pavements.
Level 4. Long-span prestressed box girder or arch bridges, large bridges built sequentially in stages by joining parts, large gravity dams, cooling towers, large roof shells, very tall buildings, large arch bridges.
Level 5. Record span bridges, nuclear containments and vessels, large offshore structures, large cooling towers, record span thin roof shells, record span slender arch bridges.

For the type of model and analysis the following recommendations can be made:

1. *The use of a model as realistic and sophisticated as B3:* recommended but not strictly required for level 3, mandatory for levels 4 and 5. For levels 1 and 2 simpler models are adequate.
2. *Method of structural creep analysis:* The age-adjusted effective modulus method is recommended for levels 3 and 4. The effective modulus method suffices for level 2. For level 1, creep and shrinkage analysis of the structure is not needed but a crude empirically based estimate is desirable. Level 5 requires the most realistic and accurate analysis possible, typically a step-by-step computer solution based on a constitutive law coupled with the solution of the differential equations for drying and heat conduction.
3. *Statistical analysis with estimation of 95% confidence limits:* (a) mandatory for level 5; (b) highly recommended for level 4; (c) for lower levels desirable but not necessary: however, the confidence limits for any response X (such as deflection, stress) should be considered, being estimated as $\bar{X} \times (1 \pm 1.96\omega)$ where $\bar{X}$ = mean estimate of X and ω is taken same as in Equation 25.
4. *Analysis of temperature effects and effects of cycling of loads and environment:* must be detailed for level 5 and approximate for level 4. It is not necessary though advisable for level 3 and can be ignored for levels 1 and 2 (except for heat of hydration effects).

Note: a more precise classification of the levels of sensitivity of a structure could be made on the basis of the coefficients of variation of responses such as deflection or stress, but such an approach might be cumbersome for practice.

APPENDIX A. APPROXIMATE FORMULA FOR FUNCTION $Q(t, t')$

Instead of Table 1, the values of this function can be obtained also from the following approximate formula (derived by Bažant and Prasannan, 1989 [8]) which is also valid for m and n values different from 0.5 and 0.1.

$$Q(t, t') = Q_f(t')\left[1 + \left(\frac{Q_f(t')}{Z(t, t')}\right)^{r(t')}\right]^{-1/r(t')} \tag{A 1}$$

with

$$r(t') = 1.7(t')^{0.12} + 8, \quad Z(t, t') = (t')^{-m} \ln[1 + (t - t')^n] \tag{A 2}$$

and

$$Q_f(t') = [0.086(t')^{2/9} + 1.21(t')^{4/9}]^{-1} \tag{A 3}$$

The error of this formula is under 1% for $n = 0.1$ and $m = 0.5$.

APPENDIX B. EXTENSION TO BASIC CREEP AT CONSTANT ELEVATED TEMPERATURE

Equation 7 for the rate of basic creep compliance function is generalized as follows:

$$\dot{C}_0(t, t', T) = R_T\left[\left(q_2 \frac{\lambda_0}{t_T} + q_3\right) \frac{n\xi^{n-1}}{\lambda_0(1 + \xi^n)} + \frac{q_4}{t_T}\right] \tag{B 1}$$

where $\xi = t_T - t'_e$. In the foregoing equation, the age at loading and the stress duration are replaced by the following equivalent age and equivalent stress duration:

$$t'_e = \int_0^{t'} \beta_T(t'')\,dt'', \quad t_T - t'_e = \int_{t'}^{t} \beta'_T(t')\,dt' \tag{B 2}$$

The temperature dependent coefficients are defined by equations:

$$\left.\begin{aligned} \beta_T &= \exp\left[\frac{U_h}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right], \quad \beta'_T = \exp\left[\frac{U_c}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right], \\ R_T &= \exp\left[\frac{U'_c}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \end{aligned}\right\} \tag{B 3}$$

(which represent Arrhenius equations based on the activation energy theory); T is the absolute temperature, T_0 is the reference temperature (for all the data fits, $T_0 = 293\,K$); U_h is the activation energy of cement hydration, U_c is the activation energy of creep describing the acceleration of creep rate due to temperature increase, and R is the gas constant. Integration of Equation B1 yields the basic creep compliance function:

$$C_0(t, t', T) = R_T\left\{q_2 Q(t_T, t'_e) + q_3 \ln[1 + (t_T - t'_e)^n] + q_4 \ln\left(\frac{t_T}{t'_e}\right)\right\} \tag{B 4}$$

Fitting of the available test data indicates the following parameter values:

$$\left.\begin{aligned} \frac{U_h}{R} &= 5000\,K, \quad \frac{U_c}{R} = \overset{110}{\cancel{2.04}}[(w/c)(c)]^{-0.27}(f'_c)^{0.54}, \\ \frac{U'_c}{R} &= 0.18\,\frac{U_c}{R} \end{aligned}\right\} \tag{B 5}$$

APPENDIX C. FURTHER REFINEMENTS FOR HIGHLY SENSITIVE STRUCTURES

In the case of structures which are highly sensitive to creep, such as nuclear reactor structures or large spans slender bridges, several other influences on creep and shrinkage should be taken into account, as mentioned below.

C.1 Creep at elevated temperature and drying

The formulation given in [3] (1992, Part IV, Equations 9–17) consists of relatively simple explicit expressions which are compatible with the present model. However, it must be warned that explicit formulae can never be very accurate for drying concrete when the temperature varies. Only integration of the differential equations of the problem can fulfil such expectations.

C.2 Cyclic environment and cyclic loading

These influences can be described approximately by simple explicit expressions given in [3] (1992, Part V, Equations 1–10).

APPENDIX D. UNCERTAINTIES DUE TO TIME VARIATION OF STRESS AND METHOD OF STRUCTURAL ANALYSIS

The stresses in concrete structures often vary significantly with time. The cause can be changes in load, but even under constant applied loads significant changes of stress can be caused by imposed displacements (stress relaxation), by sequential construction with joining of previously disconnected load-carrying members, by changes of restraints of the structure, and by differences in shrinkage and creep of joined structural parts of different age, of parts made of different materials (different concretes, or concrete and steel), of parts of different thickness or hygrothermal conditions, etc. (see, e.g. [15–18]). In such cases the effect of stress variation is calculated according to the principle of superposition. This causes additional errors of two kinds: (i) error of the principle of superposition per se, and (ii) error in the approximate method of analysis compared with the exact solution, according to the principle of superposition [16]. Estimation of these errors is a complex question which needs further research, but the following simplified approach is better than ignoring these errors altogether.

Let ω_X be the coefficient of variation of some response such as deflection or maximum stress in the structure. Let $\lambda_L = \max(\Delta P / P_{max}, \Delta\sigma / \sigma_{max})$ where $\Delta P = P_{max} - P_{min}$ and $\Delta\sigma = \sigma_{max} - \sigma_{min}$ where P_{max}, P_{min}, σ_{max}, σ_{min} are rough estimates of the maximum and the minimum values of load P and stress σ during the lifetime of the structure. For a constant load, $\lambda_L = 0$. Then one should replace ω_X by $\omega_X + \Delta\omega_X$ where the increase $\Delta\omega_X$ of the coefficient of variation may be taken approximately as follows:

$$\left.\begin{aligned} \Delta\omega_X &= 0.05\lambda_L \text{ for class I methods} \\ &= 0.07\lambda_L \text{ for class II methods} \\ &= 0.25\lambda_L \text{ for class III methods} \end{aligned}\right\} \quad \text{(D 1)}$$

Coefficient λ_L takes into account errors of the first kind and the classes of method take into account the errors of the second kind. Class I methods are the computer methods of structural creep and shrinkage analysis which solve the problem accurately according to the principle of superposition (using either integral equations or the Maxwell or Kelvin chain approximations, in small time steps). Class II methods are the simplified methods of good accuracy such as the age-adjusted effective modulus method. Class III methods are crude simplified methods such as the effective modulus method, the rate-of-creep (Dischinger) method and the rate-of-flow method.

Approximate knowledge of the coefficient of variation provides a rational basis to the designer for deciding how sophisticated a method should be used. Simple but crude methods for predicting creep and shrinkage of concrete can be used but the important point is that their coefficients of variation should be considered. If the designer regards the coefficient of variation (or the 95% confidence limits) of deflection or stress obtained for the effective modulus method as acceptable (not uneconomic), he can use that method and need not bother using a more complicated method of structural analysis. The coefficient of variation depends on the type of structure and type of response. For example, the deflection of a small-span non-prestressed reinforced concrete beam is a problem relatively insensitive to creep, and a very simple estimation based on the effective modulus method is adequate, as documented by a small coefficient of variation and small mean values of deflections. On the other hand, the deflection of a large span prestressed bridge is a creep-sensitive problem, which is manifested by a high coefficient of variation of deflection or maximum stress. For such problems it pays to use the most sophisticated method.

APPENDIX E. FREE SHRINKAGE AND THERMAL STRAIN AS A CONSTITUTIVE PROPERTY

Some sensitive structures are today being analysed by layered beam finite element programs or by two- and three-dimensional finite element programs. In such programs, the material properties used in each finite element must be constitutive properties, independent of cross-section dimensions and shape as well as environmental conditions (which represent the boundary conditions for the partial differential equation [19]). At drying, the constitutive properties cannot be measured directly, but they have been identified by fitting with a finite element program the overall deformation measurements on test specimens (as, e.g., in [20–23]). For this kind of analysis, only Equations 2, 3 and 7 are retained while 9–16 and B1–B5 must be deleted and replaced by the following constitutive relation [20] for the strain that must be added to the strain due to linear basic creep strain (with elastic deformation):

$$\dot{\varepsilon}_{s_{ij}} = \varepsilon_s^0 \frac{E(t_0)}{E(t)} [\delta_{ij} + s_h(r\sigma_{ij} + r'\sigma^v \delta_{ij})](\dot{h}_l + a_T \dot{T}_l) \quad \text{(E 1)}$$

Subscripts i, j refer to Cartesian coordinates $x_i (i, j = 1, 2, 3)$; superimposed dots denote time rates (i.e., $\partial/\partial t$); $\delta_{ij} = 1$ if $i = j$ and 0 if $i \neq j$ (Kronecker delta); σ_{ij} is the stress tensor; σ^v is the volumetric (or mean) stress, ε_s^0 is the final shrinkage (at material points) which has a similar but not exactly the same value as $\varepsilon_{sh\infty}$ for the cross-section average; $\dot{h}_l$ and $\dot{T}_l$ are the rates of local relative humidity and temperature in the pores of the concrete (which must be obtained by solving the diffusion equations); s_h is the sign of $(\dot{h}_l + a_T \dot{T}_l)$, which is 1 or -1; and a_T is the coefficient relating stress-induced thermal strain and shrinkage. When Equation E1 is used, the cracking or fracture must also be included in the analysis. The constitutive relation E1 is much simpler than Equations 9–16 and B1–B5 which it replaces. However, at present there are not enough data to predict the values of ε_s^0, r, r' and a_T from the composition and strength of concrete. They must be identified by fitting data for drying creep, shrinkage and thermal expansion.

APPENDIX F. SIMPLIFIED APPROXIMATE METHOD OF STRUCTURAL ANALYSIS FOR CREEP AND SHRINKAGE

Theoretically exact solutions according to the principle of superposition lead to integral or differential equations. Although computer methods for such equations are accurate and effective, for most practical problems it suffices to use a much simpler approximate method, namely the age-adjusted effective modulus method, which converts the problem to elastic structural analysis with modified values of elastic modulus and with inelastic strains. For more detailed information, see [6, 7].

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Errata

RILEM Draft Recommendation 'Creep and shrinkage prediction model for analysis and design of concrete structures-model B3' *Materials and Structures*, **28**, (1995), 357-365.

- Eq. (20) on page 361 should read:

$$k_t = 190 \cdot 8 t_0^{-0.08} f'^{\,-1/4}_c \quad \text{days in}^{-2}$$

(Exponent −1/4 correctly appeared in the proofs but was somehow lost afterwards)

- Eq. (A3) in appendix A on page 363 should read:

$$Q_f(t') = \left[0.086\,(t')^{2/9} + 1.21(t')^{4/9}\right]^{-1}$$

- The second equation in Eq. (B5) in appendix B on page 363 should read:

$$\frac{U_c}{R} = 110\left[(w/c)(c)\right]^{-0.27}(f'_c)^{0.54}$$