

## Creep Deflections in Slab Buildings and Forces in Shores during Construction



by Manuel Aguinaga-Zapata and Zdeněk P. Bažant

*A method for analyzing forces in shores and reshores during construction of slab buildings and of long-time deflections is developed that accounts for creep of concrete, differences in its age, and the precise construction sequence. Numerical comparisons of computer solutions with the existing elastic solutions indicate that the effects of creep on the forces in the shores and the loads carried by the slab are small when no reshoring is done. In the case of reshoring, the effect of creep is more significant, but still not very large. The practical usefulness of the solution consists mainly in that long-time deflections as influenced by the early-age load history can be calculated.*

**Keywords:** age; concrete construction; concrete slabs; creep properties; deflection; loads (forces); shoring; structural analysis.

It is a common practice to support freshly cast concrete floors in buildings by a system of shores resting on the underlying floor. In the interest of economy and speed of construction, the contractor wants to remove the shores as soon as possible; however, a premature removal may endanger construction safety as well as cause excessive long-time deflections. In view of the many disasters experienced in the past and the intolerably large deformations sometimes observed, it is important for the designer to calculate accurately the internal forces during construction. Since concrete on which permanent load is applied at early age shows much higher long-time deformations than the same concrete loaded at a later age, predicting long-time deflections is also important.

Load distribution in a system of freshly cast floors mutually coupled by shores was analyzed by Nielsen in 1952.<sup>1</sup> He took into account the axial deformations of timber shores and their swelling, but his method was too complicated for practical purposes. The most important advance was made in 1963 by Grundy and Kabaila,<sup>2</sup> who presented an experimentally verified, fairly simple, and lucid analysis procedure in which the deformations of concrete slabs were considered as elastic, the axial deformations of the shores as well as building columns were assumed negligible, and the reactions of the shores were assumed as uniformly (continuously)

distributed. Grundy and Kabaila showed that the forces in the shores calculated in this manner agreed well with measurements.

Subsequent works<sup>3-17</sup> provided some further measurement results which supported Grundy and Kabaila's method of analysis as far as the forces in the shores are concerned. These works also refined the analysis in various directions. One refinement was the consideration of various construction sequences, including the reshoring technique, which achieves a reduction in the overloads of lower floors. Further refinements were the analysis of deflections and of structural reliability.<sup>15</sup>

Despite this significant progress, the present level of knowledge must be considered incomplete. We have a calculation method which is attractively simple and agrees with the measured forces in the shores, but is not entirely based on physically justified assumptions. Thus, the present method of analysis must be regarded as essentially empirical, yet unproven for applications outside the range of the existing measurements. The main physically unjustified assumption of the existing method of analysis is the neglect of creep.

The long-time creep deformations of concrete loaded at an early age are at least several times higher than the elastic deformations and are obviously important for serviceability. Except for a suggestion of a certain simplified method of analysis by Beresford,<sup>5</sup> creep has been neglected in previous works. Although some scant long-time deflection measurements have been made,<sup>13</sup> the reported data are not complete enough to carry out a realistic creep analysis (e.g., Reference 13 does not even mention the construction procedure.) Neglecting creep

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for the purpose of stress calculations might be theoretically justified if all the interacting parts of the structural system were of the same age, since internal stress redistributions due to creep do not occur when the creep properties of the structure are homogeneous (and the creep law is assumed to be linear). However, this is not the case for the present problem.

Therefore, the objective of the present study is to develop a method of analysis that takes creep into account in a realistic manner, and to check whether such analysis also agrees with the existing measurements. If so, the proposed method would probably have a broader applicability than the existing elastic analysis, and would allow predictions of long-time deformations caused by the early application of permanent loads on the floors.

Because the histories of individual floor loads are complicated, the creep law based on a history integral<sup>18,19</sup> will be used and the method of calculation will be similar to that recently developed for span-by-span construction of continuous bridge beams.<sup>20</sup> In solving the present problem we will attempt to calculate stresses and deformations of a concrete structure loaded from the very start of its hardening. Such a problem seems not to have been yet tackled in the literature. The aspect of aging obviously is paramount in this problem.

## MATHEMATICAL FORMULATION AND ASSUMPTIONS OF ANALYSIS

The present standard recommendations all introduce the assumption of linearity of the creep law, which means that the principle of superposition is assumed to be applicable. This assumption is reasonable when the stresses are within the service stress range, as is certainly the case in the present problem, and when there is no significant drying and cracking simultaneous with creep and no sudden unloading. Although the last two restrictions are not really applicable in practice, a realistic method to cope with them is not presently available and might be too complicated for our purposes since nonlinear structural analysis would be required.

When the principle of superposition (linearity) is assumed, the creep properties are fully characterized by the compliance function  $J(t, t')$ , also called the creep function, which is defined as the strain at age  $t$  caused by a unit constant uniaxial stress acting since age  $t'$ . Although the effect of reinforcement in the floors of building should properly be introduced by considering these floors as composite beams or slabs, we will assume for the sake of simplicity that the reduction of the overall creep due to reinforcement is approximately described by adjusting the compliance function  $J(t, t')$ , which may be based, for example, on the age-adjusted effective modulus and a transformed cross section. Possible warping of the floors due to asymmetric shrinkage or asymmetric creep properties will be also neglected, although these might not actually be very small in certain situations.

Following Grundy and Kabaila and other authors, we will consider the shores, reshores, and building columns as inextensible, and the reactions of the shores and reshores as uniformly distributed (Fig. 1).

The floors of the structure are numbered as  $i = 1, 2, \dots, n$  where  $n$  represents the floor cast most recently. Let  $t$  represent time, measured, e.g., from the moment when the construction of the building started.  $\tau_i$  is the time  $t$  when the  $i$ -th floor is cast;  $\tau_i + \delta_0$  is the time  $t$  when the  $i$ -th floor starts to resist deformations (typically  $\delta_0 = 0.1$  day); and  $\Delta = \tau_{i-1} - \tau_i =$  construction cycle, assumed to be constant. Although variable construction cycle  $\Delta$  could be accommodated within the

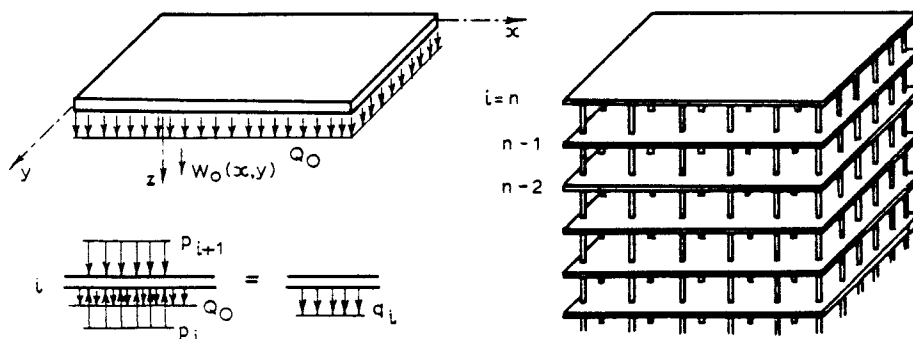


Fig. 1—Floor loads, floor numbering and shores in a slab building

present method of analysis, it will not be considered because it would introduce too many variables.

Recently the method of reshoring is frequently used in construction to reduce the overloads on the lower floors. If this procedure is used, the shores are removed (or slackened) after a certain number of cycles and are then reinstalled as reshores, without slack, while initially carrying no load. Alternatively, in this operation the shores may be moved to upper floors, and telescoping props may replace them as reshores. The time at slackening of the shores will be denoted as  $\tau_i + \delta_1$ , and the time at installation of reshores as  $\tau_i + \delta_2$ .

Fig. 2 illustrates a typical construction sequence in which the vertical shores are marked by crossed diagonals and the reshores by vertical lines.  $\delta_1$  represents the time into the cycle when the formwork is removed from the lowest shored floor to be reused for casting a new one.  $\delta_2$  represents the time into the cycle when the reshores are reinstalled (typically  $\delta_2 - \delta_1 = 0.2$  day). A special case of this construction sequence occurs when the reshores are installed almost immediately, in which case  $\delta_2 = \delta_1$  (and Stage III in Fig. 2 is deleted). Another special case occurs when the shores are moved to the top floor almost instantaneously, in which case  $\delta_2 = \Delta$  (and Stage IV in Fig. 2 is deleted). Finally, the case where no reshores are used is also a special case of the construction sequence in Fig. 2.

The lowest floor under a continuous train of props (shores and reshores) will be denoted as  $i = m$ ; note that floor  $m$  is changing during the construction sequence in Fig. 2. When the sequence of propped floors is interrupted by an unpropped floor, as in Stage III of Fig. 2, then  $m - 1$  is the top propped floor of the second train (reshored), and  $k$  is the lowest floor on which the props of the second train are resting (Stage III in Fig. 2).

The distribution of deflections of the  $i$ -th floor may be described as  $w_i(x, y, t) = w_0(x, y) u_i(t)$ , where  $x$  and  $y$  are the horizontal coordinates of each floor,  $w_0(x, y)$  is the elastic deflection distribution of the floor based on elastic modulus  $E_0$ , and  $u_i(t)$  is the deflection parameter of the  $i$ -th floor. Deflections  $w_i$  may be assumed proportional to  $w_0$  in view of the assumptions of linearity of creep and of uniform distribution of the load and the reactions of the shores. The gravity load applied on floor  $i$  may be expressed as  $Q_i(t) = Q_0 q_i(t)$  where  $Q_0$  is the floor's own weight, assumed to be the same for all floors, and  $q_i$  is the overload ratio, or load parameter.

Because the creep properties of each floor are homogeneous, the contribution of load increment  $dq(t')$  to the deflection of floor  $i$  at time  $t$  is  $E_0 J(t - \tau_i, t' - \tau_i) dq(t')$ , in which  $t - \tau_i$  is the age of floor  $i$  at time  $t$  and  $t' - \tau_i$  was the age of this floor when load increment  $dq(t')$  was applied. Therefore, the total deflection of floor  $i$  at the time  $t$  is given by

$$u_i(t) = E_0 \int_{\tau_i - \delta_0}^t J(t - \tau_i, t' - \tau_i) dq_i(t') \quad (1)$$

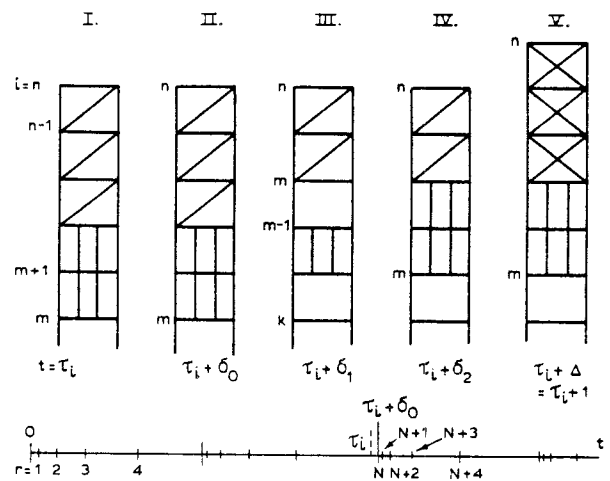


Fig. 2—Typical construction sequence with shores and reshores, and discrete subdivision of time

A more rigorous derivation of this equation is possible based on McHenry's analogy.<sup>18</sup> Due to the infinite stiffness of shores and reshores, the deflection increments of all interconnected floors must be the same, and so

$$du_i(t) = du_j(t) \text{ for } t \geq \tau_i \text{ (} i = m, m + 1, \dots, n \text{)} \quad (2)$$

The conditions of equilibrium of vertical forces applied on each of the floors  $i = m, m + 1, \dots, n$  are

$$\begin{aligned} q_n &= Q_0 - p_n \\ q_{n-1} &= Q_0 + p_n - p_{n-1} = 2Q_0 - q_n - p_{n-1} \\ q_{n-2} &= Q_0 + p_{n-1} - p_{n-2} = 3Q_0 - q_n - q_{n-1} - p_{n-2} \\ &\dots \\ q_m &= Q_0 + p_{m-1} - p_m = (n - m + 1) Q_0 - q_n - q_{n-1} \\ &\dots - q_{m+1} - P_m \end{aligned} \quad (3)$$

in which  $q_i$  represents the load resultant on floor number  $i$ ,  $p_i$  the upward reaction from the shores or reshores below,  $p_{i+1}$  the downward reaction from the shores or reshores above, and  $Q_0$  the weight of each floor. Summing Eq. (3), we obtain the total equilibrium condition

$$\sum_{i=m}^n q_i(t) = (n - m + 1) Q_0 \quad (4)$$

which means that the total weight of all interconnected floors equals the sum of the load resultants applied on all these floors. Furthermore, from Eq. (3), the forces in the shores or reshores under the floor  $i$  are

$$\begin{aligned} P_i(t) &= (n - i + 1) Q_0 - \sum_{j=i}^n q_j(t) \\ (i &= m + 1, m + 2, \dots, n) \end{aligned} \quad (5)$$

Since these props may slacken, we must impose the condition that the forces in the shores or reshores cannot become negative (tension), i.e.,  $p_i(t) \geq 0$ , and so

$$\sum_{j=i}^n q_j(t) \leq (n - i + 1)Q_0 \quad (6)$$

This condition should be checked for each time and all interconnected floors ( $m < i \leq n$ ). If, at any instant  $T$ , Eq. (6) is violated, the props will slacken and floors  $i$  and  $i-1$  will not be interconnected either until the next reshoring (assuming all props are tightened during reshoring), or as long as (for  $t > T$ )

$$u_i(t) - u_{i-1}(T) \geq u_i(t) - u_{i-1}(T) \quad (i = m+1, \dots, n) \quad (7)$$

If Eq. (7) is violated, floors  $i$  and  $i-1$  will become interconnected again, have equal deflection increments, and the train of interconnected floors will become longer. (In the present calculations, however, a naturally slackened shore did not again become tight before reshoring.)

When the shores under floor  $m$  are removed, a load increment  $\Delta q_m = -p_m$  is applied on floor  $m$ . This is then reflected in Eq. (4) by a reduction in the number  $m$ ; thus, the total load on the train comprising a smaller number of interconnected floors becomes smaller.

The foregoing equations represent a complete mathematical formulation of the problem. We will now show a step-by-step numerical solution.

### NUMERICAL STEP-BY-STEP SOLUTION

The method of numerical solution of creep problems with integral-type creep law [Eq. (1)] is well known<sup>18,19</sup> and is based on approximating the integral in Eq. (1) by a finite sum, which may be written as

$$u_{i,r} = E_0 \sum_{s=p(i)}^r J(t_r - \tau_s, \tau_{s-1/2} - \tau_s) \Delta_s q_i \quad (8)$$

Here  $t_r$  ( $r = 1, 2, \dots, N$ ) are chosen discrete times; subscript  $r$  refers to a value at time  $t_r$ ;  $\Delta_s q_i = q_i(t_s) - q_i(t_{s-1})$ ;  $t_{s-1/2}$  represents the middle of time interval ( $t_{s-1}, t_s$ ), which may be best taken as the geometric mean,<sup>18,19</sup> such that  $t_{s-1/2} - t_p = [(t_s - t_p)(t_{s-1} - t_p)]^{1/2}$ ; and  $p(i)$  represents the number of the time step ( $t_{p-1}, t_p$ ) during which floor  $i$  receives its first load. To do the calculations efficiently, the time steps  $\Delta_s t = t_r - t_{r-1}$  should start with a very small value after each load change and should be gradually increased until the next load change occurs (see Fig. 2). It is usually best to increase the time steps so that  $t_r - t_0$  represent a geometric progression.

Rewriting Eq. (8) for time  $t = t_{r-1}$

$$u_{i,r-1} = E_0 \sum_{s=p(i)}^{r-1} J(t_{r-1} - \tau_s, \tau_{s-1/2} - \tau_s) \Delta_s q_i \quad (9)$$

and subtracting this equation from Eq. (8), we obtain

$$\Delta_s u_i = \Delta_s u = E_0 J(t_r - \tau_s, t_{r-1/2} - \tau_s) \Delta_s q_i + \Delta u_i'' \quad (10)$$

in which

$$\Delta_s u_i'' = E_0 \sum_{s=p(i)}^{r-1} [J(t_r - \tau_s, t_{s-1/2} - \tau_s) - J(t_{r-1} - \tau_s, \tau_{s-1/2} - \tau_s)] \Delta_s q_i \quad (11)$$

Note that this last expression may be evaluated when all  $q_i$  have been solved up to time  $t_{r-1}$ . Therefore,  $\Delta_s u_i''$  may be considered as a fixed inelastic deflection increment (due to creep) for calculating time interval ( $t_{r-1}, t_r$ ). From Eq. (10) we may then solve

$$\Delta_s q_i = \frac{\Delta_s u_i - \Delta_s u_i''}{J(t_r - \tau_s, t_{r-1/2} - \tau_s) E_0} \quad (12)$$

Combining this with the equilibrium relation

$$\Delta_s Q = \sum_i \Delta_s q_i \quad (13)$$

we obtain

$$\Delta_s Q = \Delta_s u \sum_i \frac{1}{J(t_r - \tau_i, t_{r-1/2} - \tau_i)} - \sum_i \frac{\Delta_s u_i''}{J(t_r - \tau_i, t_{r-1/2} - \tau_i)} \quad (14)$$

From here we may solve the deflection increments for all interconnected floors, which are all the same

$$\begin{aligned} \Delta_s u_j &= \Delta_s u \\ &= \frac{\Delta_s Q + \sum_i \frac{\Delta_s u_i''}{J(t_r - \tau_i, t_{r-1/2} - \tau_i) E_0}}{\sum_i \frac{1}{J(t_r - \tau_i, t_{r-1/2} - \tau_i) E_0}} \\ (j &= m, \dots, n) \end{aligned} \quad (15)$$

If for some time steps there exists a second train of interconnected floors  $i = k, k+1, \dots, m-1$ , similar equations are again written for this set.

The calculation algorithm in each time step ( $t_{r-1}, t_r$ ) is as follows. First, we set the numbers  $n$  and  $m$  for the top and bottom floors of the train of interconnected floors, and possibly also the numbers  $m-1$  and  $k$  for the top and bottom floors of a possible second train of interconnected floors. Then, for all floors  $i = 1, 2, 3, \dots, n$  we calculate  $\Delta_s u_i''$  from Eq. (11),  $\Delta_s u$  from Eq. (10), and then  $\Delta_s q_i$  from Eq. (12). Then we increment the total load for each floor  $q_{i,r} = q_{i,r-1} + \Delta_s q_i$ , and we also calculate the increments of the prop reactions as  $\Delta_s p_i = \Delta_s Q - \sum_j \Delta_s q_j$ , in which the summation over  $j$  includes all interconnected floors of each train and  $\Delta_s Q$  is the change of any applied load. Finally we check the inequalities in Eq. (6) and (7), and use these checks in setting numbers  $m, n$ , and  $k$  for the next time step.

Note that as an alternative to Eq. (1), one could use the resolvent of this integral equation

$$q_i(t) = E_0^{-1} t_i + \delta_0 R(t - \tau_i, t' - \tau_i) du_i(t') \quad (16)$$

in which  $R(t, t')$  denotes the relaxation function which can be determined from the compliance function  $J(t, t')$ . Although a good approximate formula exists for this purpose,<sup>19</sup> it is unsuitable here because it has a non-negligible error for times less than about 1 day. Thus,  $R(t, t')$  would have to be determined numerically; however, this method of solution then might not be as easy to program as the present one. The fact that the solution based on  $R(t, t')$  is numerically much more efficient would make little difference because computer solution of the present problem is inexpensive anyway. Of concern is the programmer's time, not the computer's time.

The foregoing algorithm has been programmed for a computer and various numerical studies have been made.

### NUMERICAL STUDIES AND COMPARISONS WITH MEASUREMENTS

The compliance function is assumed in the form of the double power law<sup>9</sup>;

$$J(t, t') = \frac{1}{E_0} [1 + \phi_1 (t'^{-m} + \alpha)(t - t')^n] \quad (17)$$

in which the typical values of material parameters are  $m = 1/3$ ,  $n = 1/3$ ,  $\alpha = 0.05$ , and  $\phi_1 = 4.5$ ;<sup>5</sup>  $E_0$  represents the asymptotic elastic modulus, which is about 1.5 times larger than the conventional elastic modulus and is not needed for the present analysis (we may set  $E_0 = 1$ ). Although in a rigorous formulation the double power law should be used only for basic creep, it may be applied to creep at drying as an approximation, as we do it here. Note that the double power law agrees particularly well with test data for short load durations and for young ages at loading. For long times, the triple power law<sup>20</sup> would be more realistic but not as simple.

The computer program developed assumes an infinitely rigid foundation as the initial condition to start the process of calculation. The input data on the numbers of shores and reshores are summarized in Table 1. It is assumed for all the cases that the reshoring operation takes place in the middle of the cycle and that a 0.2-day time interval is allowed for the slab after unshoring to reach its natural deflected shape.

As numerical results for the cases with reshoring show, the overall effect of creep is to transfer part of the load from the train of shored floors to the train of reshored floors.

Table 1 shows the deflection ratios calculated with the consideration of creep. These results pertain to a converged repetitive solution, which applies to floors high above the foundation and is such that the time variations are periodically repeated from floor to floor, with delay  $\Delta$ . The results are given for the moment  $t_i$  just after the last props are retired from the floor and for the age of 1000 days after casting. The deflection

**Table 1 — Calculated Deflection Ratios**

Number of shores	Number of reshores	5-day cycle		10-day cycle		15-day cycle	
		$t_i$	1000 days	$t_i$	1000 days	$t_i$	1000 days
1	0	8.55	10.80	8.13	9.77	7.70	8.84
1	1	7.75	10.19	7.24	8.81	6.85	7.93
1	2	7.50	9.85	6.93	8.37	6.55	7.52
1	3	7.40	9.62	6.79	8.11	6.41	7.27
1	4	7.35	9.45	6.71	7.91	6.34	7.10
2	0	8.17	9.57	7.60	8.36	7.18	7.55
2	1	7.14	9.30	6.64	7.97	6.28	7.18
2	2	6.87	9.15	6.34	7.76	6.00	6.98
2	3	6.76	9.04	6.20	7.61	5.86	6.85
2	4	6.72	8.96	6.14	7.52	5.79	6.76
3	0	7.88	8.82	7.28	7.62	6.88	6.89
3	1	6.74	8.67	6.25	7.40	5.93	6.67
3	2	6.51	8.58	5.99	7.27	5.67	6.55
3	3	6.42	8.51	5.89	7.18	5.57	6.48
3	4	6.39	8.46	5.84	7.12	5.52	6.42

$t_i$  = time just after the last props are retired.

ratios in this table are given with regard to the asymptotic elastic deformation based on  $E_0$ ; the deflection ratios with regard to the conventional elastic deformation are about 1.5 times smaller. The constant unit load is assumed to act through the entire history; therefore, the effect of the later superimposed loads is not taken into account.

As expected, Table 1 indicates that the deflections increase with a decreasing number of shores, and also with a decreasing duration of the construction cycle.

The results of the numerical examples solved with the present program are shown in Fig. 3 through 7, which list given data for these examples. Fig. 3 shows the history of the load ratio of a floor cast after many construction cycles. The present creep solution does not differ significantly from the elastic solution according to Grundy and Kabaila's method. This seems typical of the construction sequences without reshoring. When reshoring is used, appreciable but not very large differences between the elastic and creep solutions are found; see Fig. 4. The influence of the duration of the construction cycle on the load ratio is shown in Fig. 5; within the range of typical cycle durations this influence is not very large.

A growing number of measurements on buildings can be found in the literature.<sup>1,3,5,7,10</sup> Comparison with Agarwal and Gardner<sup>3</sup> on Altavista Towers is shown in Fig. 6 and 7. The elastic solution does not deviate very significantly from the measured values, and for the present creep solution the comparison is neither better nor worse.

Thus it appears that the effect of creep on the forces in the shores, and to a somewhat lesser extent in the reshores, is essentially negligible. This confirms the validity of Grundy and Kabaila's method. From our limited number of examples it is not clear whether a situation exists where the difference would be significant. If so, the solution that takes creep into account would be the correct one.

In consequence of these observations, the practical value of the present solution with creep is that long-

time deflections of floors may be calculated taking into account the early age loading history, which is known to be capable of significantly influencing long-time deformations. Unfortunately, no measured data sufficient to verify our long-time predictions seem to exist at present.

The deviations from measurements seen in Fig. 6 and 7 may be due to the neglect of various further influencing factors in our assumptions. As indicated already by Nielsen,<sup>1</sup> shrinkage warping (due to asymmetric drying and cracking) may be an important factor which would tend to decrease the load on the younger floors and increase the loads on the older ones (shrinkage warping was even deemed responsible for observed vanishing of the forces in shores shortly after casting).

In interpreting the differences from measurements seen in Fig. 6 and 7, note that an overprediction of the

load of one floor is always connected with underprediction of the load in the shores of another floor. Thus, differences from measurements of either sign are cause for concern.

The calculations described above begin from the ground floor and give load histories in the shores which are different for each floor. After many floors, e.g., twenty or more, the numerical solution converges, yielding about the same load histories in the shores of every floor, periodically repeated with delay  $\Delta$ . The measurements at Altavista Towers were done, however, on the shores resting on the seventh and eighth floors, which appear to be far from the floors in which the solutions would converge to a periodic solution and would be independent of the foundation influence. Thus, as pointed out by Beresford,<sup>2</sup> the stiffness of the foundations and their possible settlements might have

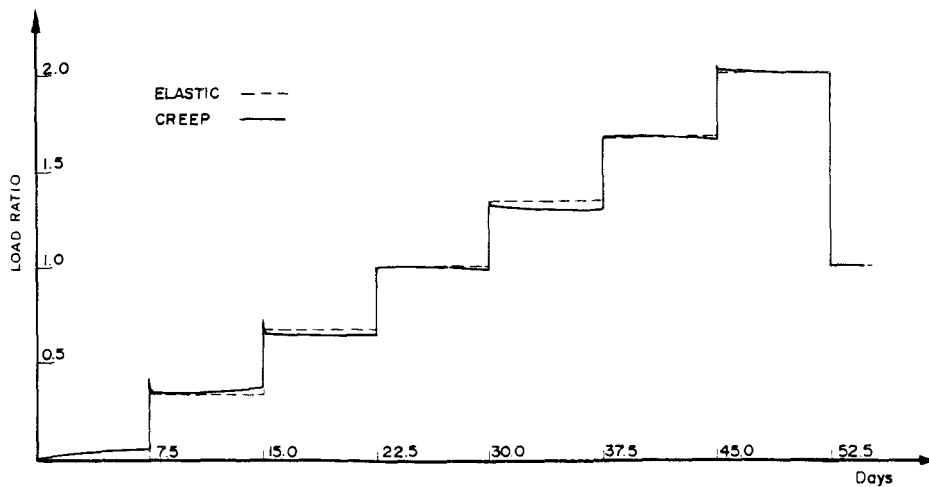


Fig. 3—Comparison of elastic and creep solutions of load ratio for three levels of shores

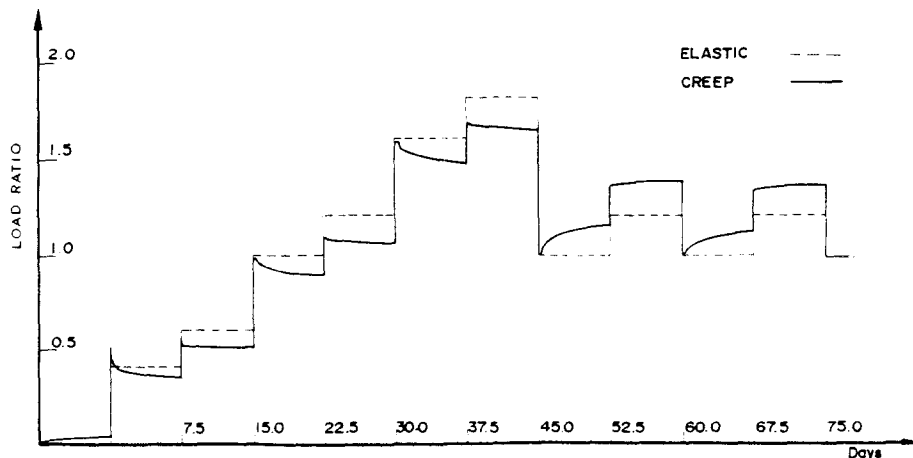


Fig. 4—Comparison of elastic and creep solutions for three levels of shores and two levels of reshores

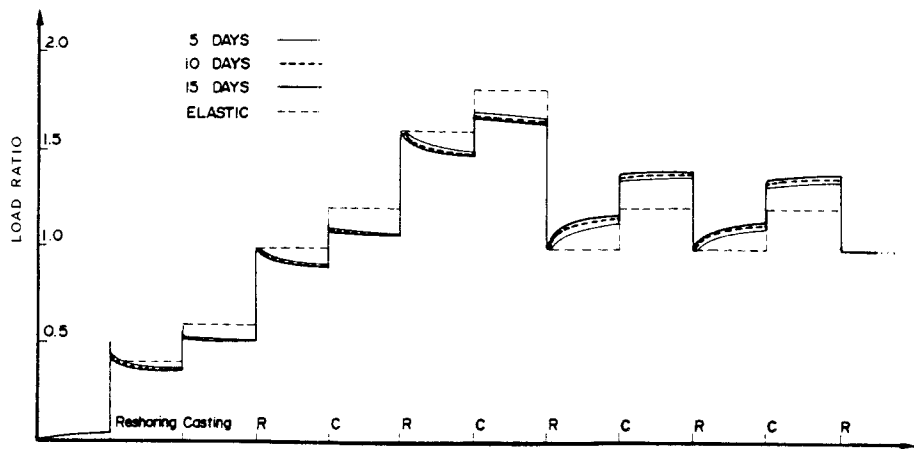


Fig. 5—Effect of the duration of construction cycle on the history of load ratio (Table 1 input data)

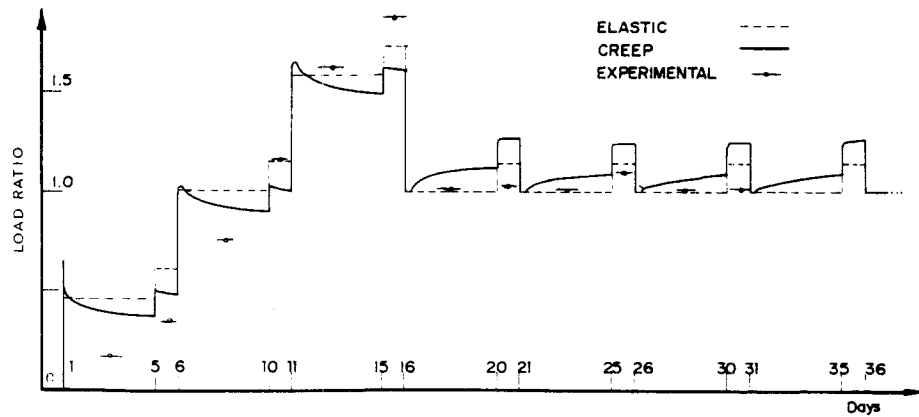


Fig. 6—Comparison of elastic and creep solutions of load ratio with Agarwal and Gardner's measurements on Altavista Towers, shores resting on Floor No. 7

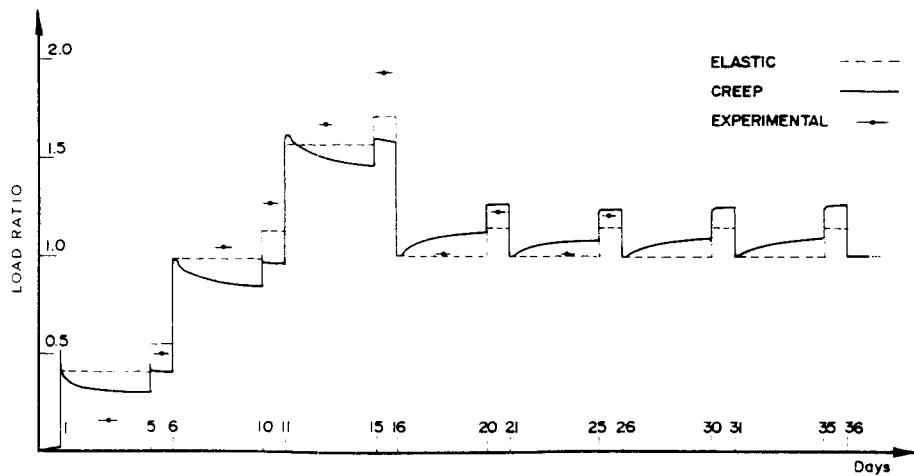


Fig. 7—Comparison of elastic and creep solutions with Agarwal and Gardner's measurements on Altavista Towers, shores resting on Floor No. 8

significantly influenced the load distributions for the measured floors, and the small deviations from calculations could in fact be also explained in this manner, although this cannot be said with certainty.

## CONCLUSIONS

1. The solution method that takes creep into account is theoretically better justified than the existing method which neglects creep.

2. Regarding the forces in the shores when no reshoring is done, the effect of creep appears to be small, and similar results are obtained as for the existing elastic solution. However, it is not clear whether this is true of all possible situations. As long as the elastic solution of floor loads is applicable, the elastically calculated history of the floor loads can, of course, be subsequently used to calculate approximate long-time creep deflections.

3. The effect of creep is more significant but still not very large when reshoring is done.

4. The practical usefulness of the present solution with creep consists in the possibility of calculating long-time deflections as affected by the early age load history. When programmed for a computer, the solution is quite easy and inexpensive.

Finally, it must be emphasized that the present solution should not be regarded as final and complete. To achieve a fully realistic model, it will eventually be necessary to take into account shrinkage warping, the effect of simultaneous drying on creep, microcracking, and nonlinearity of creep, and also to formulate the solution in probabilistic terms to obtain standard deviations of the internal forces and deflections. The statistical aspect is extremely important for creep and shrinkage, phenomena whose uncertainty is in fact much larger than that of strength, especially for loads applied at very young age.

On the other hand, it should be noted that the present state of affairs cannot be regarded as at all alarming. This is evidenced by the fact that the internal forces obtained here are not much different from the Grundy and Kabaila's elastic solution, as well as the available measured data.

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